REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

1[A-F, H-M, R, SX].—MILTON ABRAMOWITZ & IRENE A. STEGUN, Editors, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Applied Mathematics Series, No. 55, U. S. Government Printing Office, Washington, D. C., 1964, xiv + 1046 p., 27 cm. Price $6.50.

This elaborate, definitive handbook represents the impressive consummation of a decade of planning and preparation by many persons under the broad supervision of a committee originally elected by the participants in a Conference on Tables held at the Massachusetts Institute of Technology in September 1954.

The report of this Committee set forth the suggestion that, with the financial assistance of the National Science Foundation, the National Bureau of Standards undertake the production of "a Handbook of Tables for the Occasional Computer, with tables of usually encountered functions and a set of formulas and tables for interpolation and other techniques useful to the occasional computer." The existence of a real need was thus recognized for a modernized version of the classical tables of functions of Jahnke-Emde.

As early as May 1952 Dr. Abramowitz had mentioned preliminary plans for such an undertaking, and it was under his planning and supervision that active work on the project began at the Bureau of Standards in 1956. Following his untimely death in 1958, the project has been carried to its successful completion under the editorial direction of his former co-worker, Miss Stegun.

According to the Introduction, "this present Handbook has been designed to provide scientific investigators with a comprehensive and self-contained summary of the mathematical functions that arise in physical and engineering problems." To this end, this book extends the work of Jahnke-Emde by presenting more extensive and more accurate numerical tables and by giving more extensive compilations of mathematical properties of the tabulated functions. The number of functions considered has been increased by the inclusion of Coulomb wave functions, hypergeometric functions, parabolic cylinder functions, spheroidal wave functions, orthogonal polynomials, Bernoulli and Euler polynomials, arithmetic functions, Debye functions, Planck's radiation function, Einstein functions, Sievert's integral, the dilogarithm function, Clausen's integral, and vector-addition (Wigner or Clebsch-Gordan) coefficients.

The scope of this handbook may be inferred from the following enumeration of the titles and names of the contributors of the 29 chapters comprising the body of it.

1. Mathematical Constants—David S. Liepman,
2. Physical Constants and Conversion Factors—A. G. McNish,
3. Elementary Analytical Methods—Milton Abramowitz,
4. Elementary Transcendental Functions—Ruth Zucker,
5. Exponential Integral and Related Functions—Walter Gautschi & William F. Cahill,
6. Gamma Function and Related Functions—Philip J. Davis,
7. Error Function and Fresnel Integrals—Walter Gautschi,
8. Legendre Functions—Irene A. Stegun,

147

9. Bessel Functions of Integer Order—F. W. J. Olver,

10. Bessel Functions of Fractional Order—H. A. Antosiewicz,

11. Integrals of Bessel Functions—Yudell L. Luke,

12. Struve Functions and Related Functions—Milton Abramowitz,

13. Confluent Hypergeometric Functions—Lucy Joan Slater,

14. Coulomb Wave Functions—Milton Abramowitz,

15. Hypergeometric Functions—Fritz Oberhettinger,

16. Jacobian Elliptic Functions and Theta Functions—L. M. Milne-Thomson,

17. Elliptic Integrals—L. M. Milne-Thomson,

18. Weierstrasse Elliptic and Related Functions—Thomas H. Southard,

19. Parabolic Cylinder Functions—J. C. P. Miller,

20. Mathieu Functions—Gertrude Blanch,

21. Spheroidal Wave Functions—Arnold N. Lowan,

22. Orthogonal Polynomials—Urs W. Hochstrasser,

23. Bernoulli and Euler Polynomials, Riemann Zeta Function—Emilie V. Haynsworth & Karl Goldberg,

24. Combinatorial Analysis—K. Goldberg, M. Newman & E. Haynsworth,

25. Numerical Interpolation, Differentiation and Integration—Philip J. Davis & Ivan Polonsky,

26. Probability Functions—Marvin Zelen & Norman C. Severo,

27. Miscellaneous Functions—Irene A. Stegun,

28. Scales of Notation—S. Peavy & A. Schopf,

29. Laplace Transforms.

Within each chapter devoted to a function or a class of functions the material has been uniformly arranged to include mathematical properties, numerical methods, references, and tables, respectively.

We are informed in the Introduction that the classification of mathematical functions and the organization of the chapters in the Handbook has been based on *An Index of Mathematical Tables*, by A. Fletcher, J. C. P. Miller, and L. Rosenhead, which has been published in a second, two-volume edition in 1962, with L. J. Comrie added as a co-author. The mathematical notations have followed those adopted in standard texts, in particular, *Higher Transcendental Functions*, Volumes 1–3, by A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi.

In the numerical tables no attempt has been made to fix the number of significant figures presented throughout the Handbook, because of the prohibitive labor and expense required to do so. However, the great majority of the tables provide at least five significant figures, at tabular intervals sufficiently small to permit linear interpolation accurate to four or five figures. Exceptions include certain tables designed to furnish key values, such as Table 9.4, entitled Bessel Functions—Various Orders, which gives $J_n(x)$ and $Y_n(x)$ to 10 significant figures for $n = 0(1)20(10)50, 100$ and $x = 1, 2, 5, 10, 50, 100$.

In those working tables of functions wherein linear interpolation is inadequate, Lagrange's formula or Aitken's method of iterative linear interpolation is recommended. These procedures, as well as others, are discussed in the Introduction and in Chapter 25. Tables are not provided with differences, so as to effect a saving of space that has been used for the tabulation of additional functions. However, at

the foot of most of the tables there appears a symbolic statement of the maximum error arising from linear interpolation and the number of function values required in Lagrange's formula or in Aitken's method to interpolate to full tabular accuracy, as illustrated on p. x in the Introduction.

The 184 numerical tables appearing in this volume are too numerous to describe individually in a review, although their extensive range may be inferred from the listing of chapters given above. Approximately one-third of these tables have been extracted or abridged, with appropriate acknowledgment, from the numerous well-known tabular publications of the National Bureau of Standards, another third were taken from tables of the British Association for the Advancement of Science, the Harvard Computation Laboratory, H. T. Davis, L. M. Milne-Thomson, A. J. Thompson, C. E. Van Orstrand, and many others, and the remainder are the results of new computations.

The claim is made on p. ix that the maximum end-figure error is 0.6 unit in all tables of elementary functions in the Handbook, and is 1 unit (or in rare cases, 2 units) in tables of higher functions. This reviewer has carefully examined Table 1.1 (Mathematical Constants) and discovered several errors exceeding this limit. These corrections and others submitted by other users are presented in the appropriate section of this issue.

Despite such minor flaws, which are almost unavoidable in a work of this magnitude, the Handbook is a truly monumental reference work, which should be in the possession of all researchers and practitioners in the fields of numerical analysis and applied mathematics.

J. W. W.

2[D].—Norton Goodwin, *Seven Place Cosines, Sines, and Tangents for Every Tenth Microturn*, Society of Photographic Scientists and Engineers, Washington, D. C., 1964, 79 p. (unnumbered), 26 cm. Price $2.00.

According to the author, these tables were designed primarily to facilitate desk-calculator transformations of the coordinates of artificial earth satellites. However, as he states, they should also prove useful in space navigation and in electrical engineering, where cyclical coordinate changes are encountered.

The tables consist of sine $2\pi x$ and cos $2\pi x$ for $x = 0(0.00001)0.25000$ and tan $2\pi x$ for $x = 0(0.00001)0.12500$, all to 7D. The (linearly interpolable) values of the sine and cosine are arranged semiquadrantally, without differences, on facing pages, each containing 500 distinct entries, arranged in the conventional ten columns, supplemented by an eleventh, which gives the same tabular value in any row as the first column in the succeeding row, thereby facilitating the use of the tables in obtaining functional values for complementary arguments. Economy of space is attained by separation of the first two decimal digits and listing only the last five decimal digits in all the columns after the first. Change in the second decimal place occurring within a line is signalled by boldface numerals.

The author has communicated to this reviewer the information that these tables were computed on an IBM 7090 system at The Rand Corporation, using double-precision arithmetic to evaluate the functions by Taylor series prior to final rounding.