Reference is also made to the treatise by Luke [2], which includes a discussion of this integral, and gives an equivalent expression in closed form when $b = 1$.

J. W. W.

1. NORIO KATO, "Integrated intensities of the diffracted and transmitted X-rays due to ideally perfect crystals (Laue case)," *J. Phys. Soc. Japan*, v. 10, 1955, p. 46–55.
2. YUDELL L. LUKE, *Integrals of Bessel Functions*, McGraw-Hill Book Co., 1962, p. 122 and Chapter X.

15[L, M].—N. SKOBLIÂ, *Tables for the Numerical Inversion of Laplace Transforms*, Academy of Sciences of USSR, Moscow, 1964, 44 p., 22 cm. Paperback. Price 13 kopecks.

Consider the Laplace transform pair (which we assume exists)

$$(1) \qquad p^{-s}g(p) = \int_0^\infty e^{-pt}f(t)\,dt, \qquad f(t) = (2\pi i)^{-1}\int_{c-i\infty}^{c+i\infty} p^{-s}e^{pt}g(p)\,dp$$

where $c > 0$ and c lies to the right of all singularities of $g(p)$. Suppose that $g(p)$ is known and can be represented by a polynomial in $1/p$. Then an approximation formula for $f(t)$ is readily constructed from the second formula in (1). Now it may be shown that

$$(2) \qquad \int_{c-i\infty}^{c+i\infty} e^p p^{-s} P_n(p^{-1}) P_m(p^{-1})\,dp = \delta_{mn} h_n, \qquad h_n = \frac{(-1)^n n!}{(2n-1+s)\Gamma(n-1+s)}$$

where δ_{mn} is the Kronecker delta, and in hypergeometric notation,

$$(3) \qquad \begin{aligned} P_n(x) &= {}_2F_0(-n, n+s-1; x) \\ &= (2-2n-s)_n\, x^n\, {}_1F\left(-n; 2-2n-s; \frac{1}{x}\right). \end{aligned}$$

This shows that numerous properties of $P_n(x)$ follow from known results on confluent hypergeometric functions. In view of (2), we have the approximation

$$(4) \qquad f(t) \sim (2\pi i)^{-1} \sum_{k=1}^{n} A_{k,n} g(p_k)$$

where

$$(5) \qquad P_n(p_k^{-1}) = 0, \qquad k = 1, 2, \cdots, n,$$

and the weights, $A_{k,n}$ are the Christoffel numbers. Thus, the approximation is exact if indeed $g(p)$ is a polynomial in $1/p$ of degree $(2n - 1)$. A convenient formula for the weights is

$$(6) \qquad A_{k,n} = \sum_{m=0}^{n-1} \{P_m(p_n^{-1})\}^2/h_m .$$

The pamphlet gives some properties of $P_n(x)$, though (3) and (6) are not among them. The following are tabulated to 7S: p_k, $A_{k,n}$ for $k = 1(1)n$, $n = 1(1)10$, and $s = 0.1(0.1)3.0$.

The case $s = 1$ has been treated by H. Salzer. (See "Orthogonal polynomials arising in the numerical evaluation of Laplace transforms," *MTAC*, v. 9, 1955, p. 164–177, and "Additional formulas and tables for orthogonal polynomials originating from inversion integrals," *J. Math. Phys.*, v. 40, 1961, p. 72–86.) These latter sources give the zeros and weights to 15D for $n = 1(1)15$. Note that Salzer's quadrature formula is exact if $g(p)$ is a polynomial in $1/p$ of degree $2n$ such that $g(\infty) = 0$. In the booklet under review, the quadrature formula is exact if $g(p)$ is of degree $(2n - 1)$, but $g(\infty)$ need not vanish. Thus the Christoffel numbers in Salzer's work differ from those of the present author. However, the zeros are the same. Twice the negatives of the zeros of $P_n(x)$ have been tabulated mostly to 5 D by V. N. Kublanovskaia and T. N. Smirnova. (See "Zeros of Hankel functions and some related functions," *Trudy. Mat. Inst. AN*, USSR No. 53, 1959, p. 186–192. This is also available as Electronic Research Directorate, Air Force Cambridge Research Laboratories Report AFCRL-TN 60-1128, October 1960.)

Y. L. L.

16[M, X].—E. L. Albasiny, R. J. Bell & J. R. A. Cooper, *A Table for the Evaluation of Slater Coefficients and Integrals of Triple Products of Spherical Harmonics*, National Physical Laboratory Mathematics Division Report No. 49, 1963, xi + 163 pages.

This is a table of integrals

$$\int_{-1}^{+1} \Theta_{l_1}^{m_1}(x)\Theta_{l_2}^{m_2}(x)\Theta_{l_3}^{m_3}(x)\, dx$$

where

$$\Theta_l^m(x) = (-1)^m \left[\frac{(2l+1)}{2}\frac{(l-m)!}{(l+m)!}\right]^{1/2} \frac{(1-x^2)^{m/2}}{l!2^l}\frac{d^{l+m}}{dx^{l+m}}(x^2-1)^l,$$

$$0 \leqq m \leqq l$$

is an associated Lengendre function. These integrals are closely related to integrals which occur in molecular structure calculations (see, for example, [1]).

Values of the above integrals are tabulated to 12 decimal places for (m_i, l_i integers)

$$m_1 \pm m_2 \pm m_3 = 0,$$

$$l_1 \leqq l_2 \leqq l_3,$$

$$l_1 + l_2 + l_3 \text{ even},$$

$$|l_1 - l_2| \leqq l_3 \leqq l_1 + l_2,$$

$$l_1, l_2 \leqq 12, l_3 \leqq 24.$$

Under these conditions the integrand is a polynomial of degree $\leqq 48$, and thus can be calculated exactly, using an n-point Gauss-Legendre quadrature formula [3, p. 107–111] for $n \geqq 25$. The tables were computed using the 25-point formula tabulated by Gawlik [2] and recomputed as a check using the 26-point formula. The calculations were carried out on the ACE computer, which has a 46-bit floating-