

The case $s = 1$ has been treated by H. Salzer. (See "Orthogonal polynomials arising in the numerical evaluation of Laplace transforms," *MTAC*, v. 9, 1955, p. 164-177, and "Additional formulas and tables for orthogonal polynomials originating from inversion integrals," *J. Math. Phys.*, v. 40, 1961, p. 72-86.) These latter sources give the zeros and weights to 15D for $n = 1(1)15$. Note that Salzer's quadrature formula is exact if $g(p)$ is a polynomial in $1/p$ of degree $2n$ such that $g(\infty) = 0$. In the booklet under review, the quadrature formula is exact if $g(p)$ is of degree $(2n - 1)$, but $g(\infty)$ need not vanish. Thus the Christoffel numbers in Salzer's work differ from those of the present author. However, the zeros are the same. Twice the negatives of the zeros of $P_n(x)$ have been tabulated mostly to 5 D by V. N. Kublanovskaia and T. N. Smirnova. (See "Zeros of Hankel functions and some related functions," *Trudy. Mat. Inst. AN, USSR* No. 53, 1959, p. 186-192. This is also available as Electronic Research Directorate, Air Force Cambridge Research Laboratories Report AFCRL-TN 60-1128, October 1960.)

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16[M, X].—E. L. ALBASINY, R. J. BELL & J. R. A. COOPER, *A Table for the Evaluation of Slater Coefficients and Integrals of Triple Products of Spherical Harmonics*, National Physical Laboratory Mathematics Division Report No. 49, 1963, xi + 163 pages.

This is a table of integrals

$$\int_{-1}^{+1} \Theta_{l_1}^{m_1}(x) \Theta_{l_2}^{m_2}(x) \Theta_{l_3}^{m_3}(x) dx$$

where

$$\Theta_l^m(x) = (-1)^m \left[\frac{(2l+1)(l-m)!}{2(l+m)!} \right]^{1/2} \frac{(1-x^2)^{m/2}}{l!2^l} \frac{d^{l+m}}{dx^{l+m}} (x^2-1)^l,$$

$$0 \leq m \leq l$$

is an associated Legendre function. These integrals are closely related to integrals which occur in molecular structure calculations (see, for example, [1]).

Values of the above integrals are tabulated to 12 decimal places for (m_i, l_i) integers)

$$m_1 \pm m_2 \pm m_3 = 0,$$

$$l_1 \leq l_2 \leq l_3,$$

$$l_1 + l_2 + l_3 \text{ even,}$$

$$|l_1 - l_2| \leq l_3 \leq l_1 + l_2,$$

$$l_1, l_2 \leq 12, l_3 \leq 24.$$

Under these conditions the integrand is a polynomial of degree ≤ 48 , and thus can be calculated exactly, using an n -point Gauss-Legendre quadrature formula [3, p. 107-111] for $n \geq 25$. The tables were computed using the 25-point formula tabulated by Gawlik [2] and recomputed as a check using the 26-point formula. The calculations were carried out on the ACE computer, which has a 46-bit floating-

point mantissa. The tabulated values are exact to within two units in the last place.

The above integrals are also known in closed form [4]. However, the expressions for them are not as convenient for computations as the quadrature formulas.

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1. A. R. EDMONDS, *Angular Momenta in Quantum Mechanics*, Princeton Univ. Press, Princeton, N. J., 1957, Chapters 3, 4.

2. H. J. GAWLIK, *Zeros of Legendre Polynomials of Orders 2-64 and Weight Coefficients of Gauss Quadrature Formulae*, Armament Research and Development Establishment Memorandum (B) 77/58, 1958. [See *Math. Comp.*, v. 14, 1960, p. 77, RMT 4.]

3. V. I. KRYLOV, *Approximate Calculation of Integrals*, Macmillan, New York, 1962.

4. J. MILLER, "Formulas for integrals of products of associated Legendre or Laguerre functions," *Math Comp.*, v. 17, 1963, p. 84-87.

17[M, X].—L. KRUGLIKOVA, *Tables for Numerical Fourier Transformations*, Academy of Sciences of USSR, Moscow, 1964, 30 p., 22 cm. Paperback. Price 13 kopecks.

This pamphlet contains Gaussian quadrature formulas of the form

$$\int_0^{\infty} (1 + \sin x)f(x) dx \cong \sum_{k=1}^n A_k f(x_k),$$

$$\int_0^{\infty} (1 + \cos x)f(x) dx \cong \sum_{k=1}^n A_k f(x_k),$$

which are exact whenever

$$f(x) = (1 + x)^{-s-i}, \quad i = 0, 1, \dots, 2n - 1.$$

Values of x_k and A_k are given for $n = 1, \dots, 8$ for the following values of the parameter s :

$$s = \frac{5}{4}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{7}{4}, 2, \frac{9}{4}, \frac{7}{3}, \frac{5}{2}, \frac{3}{3}, \frac{1}{4}, 3, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, 4.$$

The x_k are given to between 8 and 10 significant figures and the A_k to between 5 and 11.

These formulas can also be used to approximate integrals of the form

$$\int_0^{\infty} f(x) \sin \alpha x dx, \quad \int_0^{\infty} f(x) \cos \alpha x dx.$$

This is done by writing these as

$$\int_0^{\infty} \phi(y) \sin y dy, \quad \int_0^{\infty} \phi(y) \cos y dy,$$

$$\alpha x = y, \quad \phi(y) = \frac{1}{\alpha} f\left(\frac{y}{\alpha}\right),$$

and approximating