point mantissa. The tabulated values are exact to within two units in the last place.

The above integrals are also known in closed form [4]. However, the expressions for them are not as convenient for computations as the quadrature formulas.

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1. A. R. EDMONDS, *Angular Momenta in Quantum Mechanics*, Princeton Univ. Press, Princeton, N. J., 1957, Chapters 3, 4.

2. H. J. GAWLIK, *Zeros of Legendre Polynomials of Orders 2-64 and Weight Coefficients of Gauss Quadrature Formulae*, Armament Research and Development Establishment Memorandum (B) 77/58, 1958. [See *Math. Comp.*, v. 14, 1960, p. 77, RMT 4.]

3. V. I. KRYLOV, *Approximate Calculation of Integrals*, Macmillan, New York, 1962.

4. J. MILLER, "Formulas for integrals of products of associated Legendre or Laguerre functions," *Math Comp.*, v. 17, 1963, p. 84–87.

17[M, X].—L. KRUGLIKOVA, *Tables for Numerical Fourier Transformations*, Academy of Sciences of USSR, Moscow, 1964, 30 p., 22 cm. Paperback. Price 13 kopecks.

This pamphlet contains Gaussian quadrature formulas of the form

$$\int_0^\infty (1 + \sin x) f(x)\, dx \cong \sum_{k=1}^{n} A_k f(x_k),$$

$$\int_0^\infty (1 + \cos x) f(x)\, dx \cong \sum_{k=1}^{n} A_k f(x_k),$$

which are exact whenever

$$f(x) = (1 + x)^{-s-i}, \qquad i = 0, 1, \cdots, 2n - 1.$$

Values of x_k and A_k are given for $n = 1, \cdots, 8$ for the following values of the parameter s:

$$s = \tfrac{5}{4}, \tfrac{4}{3}, \tfrac{3}{2}, \tfrac{5}{3}, \tfrac{7}{4}, 2, \tfrac{9}{4}, \tfrac{7}{3}, \tfrac{5}{2}, \tfrac{8}{3}, \tfrac{11}{4}, 3, \tfrac{13}{4}, \tfrac{10}{3}, \tfrac{7}{2}, \tfrac{11}{3}, \tfrac{15}{4}, 4.$$

The x_k are given to between 8 and 10 significant figures and the A_k to between 5 and 11.

These formulas can also be used to approximate integrals of the form

$$\int_0^\infty f(x) \sin \alpha x\, dx, \qquad \int_0^\infty f(x) \cos \alpha x\, dx.$$

This is done by writing these as

$$\int_0^\infty \phi(y) \sin y\, dy, \qquad \int_0^\infty \phi(y) \cos y\, dy,$$

$$\alpha x = y, \qquad \phi(y) = \frac{1}{\alpha} f\left(\frac{y}{\alpha}\right),$$

and approximating

$$\int_0^\infty \phi(y)\, dy$$

by some other method. Then, for example,

$$\int_0^\infty \phi(y) \sin y\, dy = \int_0^\infty (1 + \sin y)\phi(y)\, dy - \int_0^\infty \phi(y)\, dy.$$

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18[P, X].—L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze & E. F. Mishchenko, *The Mathematical Theory of Optimal Processes*, John Wiley & Sons, Inc., New York, 1962, viii + 360 p., 23 cm. Price $11.95.

One of the major problems of the modern mathematical theory of control processes can be posed in the following terms: "Given a vector differential equation $dx/dt = g(x, y)$, $x(0) = c$, where x represents the state of a physical system at time t, the state vector, and $y(t)$ represents the control vector, determine $y(t)$ so as to minimize a given scalar functional $J = \int_0^T h(x, y, t)\, dt$, where x and y are subject to local constraints of the form $r_i(x, y) \leqq 0, i = 1, 2, \cdots, N$, global constraints of the form $\int_0^T h_i(x, y)\, dt \leqq k_i$, and terminal conditions of the form $f_i(x(T), y(T), T) \leqq 0$." In some cases of importance, T itself depends upon the history of the process, $T = T(x, y)$, and, indeed, may be the quantity we wish to minimize.

The book under review represents a fine and substantial contribution to a new mathematical domain. The major theme of the work is the "maximum principle," an analytic condition which provides important information concerning the structure of extremals, in the terminology of the calculus of variations, or of optimal policies, in the parlance of dynamic programming and control theory.

Since the book is an excellent one that will be widely read and used, it is worthwhile to analyze its objectives and results carefully within the framework of the classical theory of the calculus of variations, and with the desiderata of modern control theory in mind.

In the simplest version of classical variational theory, there are no local or global constraints. The first variation yields the Euler equation, generally a nonlinear differential equation, with two-point boundary conditions. For a variety of reasons, this direct approach is seldom effective computationally. If global constraints are present, Lagrange multipliers may be used to reduce the problem to one without constraints, at the expense of further computational difficulties.

If local constraints of the type indicated above are present, as they are in a large number of the most important classes of processes, the situation is even more complex. This is due to the fact that sometimes the Euler equation holds and sometimes the constraints determine the extremal, or policy. Hence, the analytic and computational difficulties that existed before, as far as effective algorithms for the solution are concerned, are now compounded.

Nevertheless, analogues and extensions of the classical results can be obtained. The pioneering work is that of Valentine [1]. Results of Valentine were used by Hestenes in some unpublished work on constrained trajectories in 1949. In 1961