35[A, C].—W. E. MANSELL, edited by A. J. THOMPSON, *Tables of Natural and Common Logarithms to* 110 *Decimals*, Royal Society Mathematical Tables, Volume 8, Cambridge University Press, New York, 1964, xviii + 95 pp., 28 cm. Price $7.50.

Two of the four main tables here give the natural and common logarithms of the integers 1 to 1000 to 110 decimals. The other two are the radix tables:

$$\log 1.0^k x \quad \text{for} \quad x = 1(1)9 \quad \text{and} \quad k = 4(1)11,$$

again to both bases, e and 10, and again to 110D. There is a fifth table of 25 con stants: e, π, γ, etc., also to 110D.

The four main tables were computed by hand by William Ernest Mansell, a retired accountant, sometime during the 1930's. He left a provision in his will to make thier publication possible. The extensive editing was done by A. J. Thompson, who checked the tables by comparing sums with $\log_e n!$ and $\log_{10} n!$ for $n = 100$, 500 and 1000. The latter were computed by Stirling's formula, and are included in the table of constants.

The introduction indicates several (desk-computer) techniques of computing the logarithms of larger integers or irrational numbers by the use of these tables together with factor tables and the Taylor series. For example,

$$3\,141593 = 13 \cdot 437 \cdot 553 \cdot 10^{-6}$$

and

$$\pi = (1 - x)\, 3.141593/1.0^6 1 \cdot 1.0^7 1 \cdot 1.0^9 2 \cdot 1.0^{10} 6 \cdot 1.0^{11} 5,$$

where $x = 0.0^{12}79 \cdots$. From a few terms of the series for $\log(1 - x)$ one therefore obtains $\log \pi$ to 40D.

No previously published table has the same combination of range and accuracy, although even more accurate tables exist for smaller ranges. See [1] for a complete listing of such tables. It is clear, of course, that modern machines can easily surpass these very extensive hand computations of Mansell.

The four main tables are reproduced photographically, and while the printing is much better than that of many tables so reproduced, it does not have the elegance usually found in the Royal Society Tables.

D. S.

1. A. FLETCHER, J. C. P. MILLER, L. ROSENHEAD & L. J. COMRIE, *An Index of Mathematical Tables*, Addison-Wesley, Reading, Mass., 1962.

36[F].—M. L. STEIN & P. R. STEIN, *Tables of the Number of Binary Decompositions of All Even Numbers* $0 < 2n < 200{,}000$ *into Prime Numbers and Lucky Numbers*, Volumes I & II, Los Alamos Scientific Laboratory Report LA-3106, 1964. Vol. I, 442 pp.; Vol. II, 426 pp., 28 cm. Price $5.00 each. Available from Office of Technical Services, U. S. Department of Commerce, Washington 25, D. C

The main table in Volume I (400 pages long) gives the number of solutions ν_{2n} of $2n = p_1 + p_2$, where p_1 and p_2 are primes, and $2n$ is an even number less than

200,000. Two conventions must be noted: the first (which is curiously old-fashioned) allows 1 to be considered as a prime, and the second (in distinction to some treatments) considers the decompositions $p_1 + p_2$ and $p_2 + p_1$ to be identical. Thus, $\nu_{14} = 3$, since

$$14 = 1 + 13 = 3 + 11 = 7 + 7.$$

In the introduction there is given a heuristic formula $E_{2n}(\nu)$ that is meant to estimate ν_{2n}. We will not reproduce its rather complicated definition here, but we do note that infinitely many approximations of $E_{2n}(\nu)$ are defined, depending upon an integral parameter k. The authors computed estimates based upon $k = 5$, for convenience, and compared these estimates with the actual counts ν_{2n}. They find that their estimates are usually a little too high. The average of the *absolute value* of the relative errors for the 85,000 cases: $30{,}000 \leqq 2n < 200{,}000$ is 2.60%. The worst single disagreement in this range is between $\nu_{33,038} = 224$ and $E_{33,038}(\nu) = 254.78$. But this error of 13.74% is rather exceptional, and in over 54,000 cases the error is less than 3%.

No reference is given to the classical Hardy-Littlewood conjecture for the number of such Goldbach decompositions, cf. Schinzel [1, conjecture C for $a = b = 1$], and possibly the authors did not realize that their approximations become asymptotic to the Hardy-Littlewood formula when their parameter k goes to infinity. Thus, Hardy and Littlewood give

$$(1) \qquad P(2n) \sim 2c_2 \prod_{p \mid 2n} \left(\frac{p-1}{p-2}\right) \int_2^{2n} \frac{dx}{\log^2 x},$$

where the product shown is taken over all odd primes that divide $2n$, and where

$$c_2 = \prod_{p=3}^{\infty} \left(1 - \frac{1}{(p-1)^2}\right) = 0.660162.$$

There is a factor of 2 in (1) because $p_1 + p_2$ and $p_2 + p_1$ are considered distinct there. Clearly, whether one allows 1 as a prime or not does not affect the asymptotic law. Aside from these two differences in the conventions adopted, the Stein-Stein estimate, for $k = 5$, is rather close to (1), except that the constant c_2 there is replaced (in effect) by

$$\prod_{p=3}^{p=11} \left(1 - \frac{1}{(p-1)^2}\right) = 0.676758.$$

Now, this latter constant is 2.51% larger than c_2, so if one allows for the fact that the 2.60% mentioned above is based upon the absolute value of the error, one concludes that the Hardy-Littlewood formula fits the empirical data very well.

The second table in Volume I (20 pages long) gives an inverse function: the number of values of $2n$ for which there are exactly k decompositions: $2n = p_1 + p_2$. For every $k \leqq 3931$ they give the smallest value of $2n$, the largest value of $2n$ (up to $2n = 200{,}000$), and the number of such values of $2n$. They discover that for all values of $k \leqq 2428$ there is at least one value of $2n$, and that for $k > 8$ the smallest value of $2n$ is always divisible by 6.

There are also several small statistical tables, which we will not describe.

In Volume II we have completely analogous tables where the primes have been replaced by the "lucky numbers." That is fitting, since this sequence was invented by the Los Alamos school of number theory. No good heuristic estimate was found for the number of "lucky" decompositions.

D. S.

1. A. SCHINZEL, "A remark on a paper of Bateman and Horn," *Math. Comp.*, v. 17, 1963, pp. 445–447, especially p. 446.

37[F].—THOMAS R. PARKIN & LEON J. LANDER, *Abundant Numbers*, Aerospace Corporation, Los Angeles, 1964, 119 unnumbered pages, 28 cm. Copy deposited in UMT File.

Leo Moser had shown [1] that every integer $> 83{,}160 = 88 \cdot 945$ can be expressed as the sum of two abundant numbers. This proof is first improved here to include all integers $> 28{,}121$. This is done by showing that every odd $N \geqq 28{,}123 = 89 \cdot 315 + 88$ can be written as $N = M \cdot 315 + B \cdot 88$ with $3 \leqq M \leqq 89$ and $B \geqq 1$. But $M \cdot 315$ and $B \cdot 88$ are both abundant. Further, it is easily shown that all even numbers > 46 can be written in the required manner [2].

The smallest odd N so representable is clearly 957, since 945 and 12 are the smallest odd and even abundant numbers, respectively. To examine the odd numbers between 957 and 28,123, the authors use two methods: (a) covering sets; and (b) trial and error based upon lists of abundant numbers. They thus find that 20,161 is, in fact, the largest integer not so decomposable. This had been previously found by John L. Selfridge.

The main table here (90 pages) gives a decomposition, if one exists, for every odd N satisfying $941 \leqq N \leqq 28{,}999$. There are, all in all, only 1455 integers not decomposable into a sum of two abundant numbers.

In their discussion of method (a) mentioned above, the authors erroneously state that a prime multiple of a perfect number is a *primitive abundant* number, where that is defined to be an abundant number that has no abundant proper divisor. A counterexample is $84 = 3 \cdot 28$, since this has the abundant number 12 as a divisor.

In connection with these computations (on a CDC 160A) a table of $\sigma(N)$ was computed up to $N = 29{,}000$ by the use of Euler's pentagonal number recurrence relationship. This table is reproduced up to $N = 1000$ in Appendix C. The authors planned to extend this table (on tape) up to 10^5 or 10^6, but believe that the use of the canonical factorization of the integers will be faster than Euler's method. Presumably that is because of the limited high-speed memory in the small computer which was being used.

D. S.

1. LEO MOSER, *Amer. Math. Monthly*, v. 56, 1949, p. 478, Problem E848.
2. F. A. E. PIRANI, *Amer. Math. Monthly*, v. 57, 1950, pp. 561–562, Problem E903.

38[F].—KARL K. NORTON, "Remarks on the number of factors of an odd perfect number," *Acta Arith.*, v. 6, 1961, pp. 372–373. Table in Section IV.

Let $\alpha(n)$ be defined by

$$\prod_{r=n}^{n+\alpha(n)-2} \frac{p_r}{p_r - 1} < 2 < \prod_{r=n}^{n+\alpha(n)-1} \frac{p_r}{p_r - 1},$$

where p_r is the rth prime. If an odd perfect number N has p_n as its smallest prime