replaced by the "lucky numbers." That is fitting, since this sequence was invented by the Los Alamos school of number theory. No good heuristic estimate was found for the number of "lucky" decompositions.

D. S.

1. A. SCHINZEL, "A remark on a paper of Bateman and Horn," *Math. Comp.*, v. 17, 1963, pp. 445–447, especially p. 446.

37[F].—THOMAS R. PARKIN & LEON J. LANDER, *Abundant Numbers*, Aerospace Corporation, Los Angeles, 1964, 119 unnumbered pages, 28 cm. Copy deposited in UMT File.

Leo Moser had shown [1] that every integer $> 83{,}160 = 88 \cdot 945$ can be expressed as the sum of two abundant numbers. This proof is first improved here to include all integers $> 28{,}121$. This is done by showing that every odd $N \geqq 28{,}123 = 89 \cdot 315 + 88$ can be written as $N = M \cdot 315 + B \cdot 88$ with $3 \leqq M \leqq 89$ and $B \geqq 1$. But $M \cdot 315$ and $B \cdot 88$ are both abundant. Further, it is easily shown that all even numbers > 46 can be written in the required manner [2].

The smallest odd N so representable is clearly 957, since 945 and 12 are the smallest odd and even abundant numbers, respectively. To examine the odd numbers between 957 and 28,123, the authors use two methods: (a) covering sets; and (b) trial and error based upon lists of abundant numbers. They thus find that 20,161 is, in fact, the largest integer not so decomposable. This had been previously found by John L. Selfridge.

The main table here (90 pages) gives a decomposition, if one exists, for every odd N satisfying $941 \leqq N \leqq 28{,}999$. There are, all in all, only 1455 integers not decomposable into a sum of two abundant numbers.

In their discussion of method (a) mentioned above, the authors erroneously state that a prime multiple of a perfect number is a *primitive abundant* number, where that is defined to be an abundant number that has no abundant proper divisor. A counterexample is $84 = 3 \cdot 28$, since this has the abundant number 12 as a divisor.

In connection with these computations (on a CDC 160A) a table of $\sigma(N)$ was computed up to $N = 29{,}000$ by the use of Euler's pentagonal number recurrence relationship. This table is reproduced up to $N = 1000$ in Appendix C. The authors planned to extend this table (on tape) up to 10^5 or 10^6, but believe that the use of the canonical factorization of the integers will be faster than Euler's method. Presumably that is because of the limited high-speed memory in the small computer which was being used.

D. S.

1. LEO MOSER, *Amer. Math. Monthly*, v. 56, 1949, p. 478, Problem E848.
2. F. A. E. PIRANI, *Amer. Math. Monthly*, v. 57, 1950, pp. 561–562, Problem E903.

38[F].—KARL K. NORTON, "Remarks on the number of factors of an odd perfect number," *Acta Arith.*, v. 6, 1961, pp. 372–373. Table in Section IV.

Let $\alpha(n)$ be defined by

$$\prod_{r=n}^{n+\alpha(n)-2} \frac{p_r}{p_r - 1} < 2 < \prod_{r=n}^{n+\alpha(n)-1} \frac{p_r}{p_r - 1},$$

where p_r is the rth prime. If an odd perfect number N has p_n as its smallest prime divisor, it follows that it has at least $\alpha(n)$ prime divisors. A table of $\alpha(n)$ for $n = 2(1)100$ was computed on the ILLIAC and is presented here; e.g., $\alpha(2) = 3$ and $\alpha(100) = 26308$. These may be compared with a theoretical formula:

$$\alpha(n) = \tfrac{1}{2}n^2 \log n + \tfrac{1}{2}n^2 \log \log n - \cdots .$$

Up to $n = 11$ and $n = 24$, the author could have used existing tables of $\prod_{p<x} (1 - 1/p)$ due to Legendre and Glaisher, respectively, instead of the ILLIAC, but he makes no mention of this. The later, and much more extensive table of Appel and Rosser [1] was not completely printed, and allows us only to determine such bounds as

$$5{,}730{,}105 < \alpha(1217) < 5{,}760{,}003.$$

D. S.

1. KENNETH I. APPEL & J. BARKLEY ROSSER, *Table for Estimating Functions of Primes*, IDA-CRD Technical Report Number 4, 1961; reviewed in *Math. Comp.*, v. 16, 1962, pp. 500–501, RMT 55.

39[G].—MARSHALL HALL, JR. & JAMES K. SENIOR, *The Groups of Order* 2^n ($n \leqq 6$), The Macmillan Company, New York, 1964, 225 pp., 36 cm. Price $15.00.

From the preface: "No single presentation of a group or list of groups can be expected to yield all the information which a reader might desire. Here, each group is presented in three different ways: (1) by generators and defining relations; (2) by generating permutations; and (3) by its lattice of normal subgroups, together with the identification of every such subgroup and its factor group. In this lattice the characteristic subgroups are distinguished.

"For each group, additional information is given. Here are included the order of the group of automorphisms and the number of elements of each possible order 2, 4, 8, 16, 32, and 64.....All the groups are divided into twenty-seven families, following Philip Hall's theory of isotopy.

"Chapters 3 and 4 give the theoretical background for the construction of the tables. But these chapters are not necessary for the use of those tables; for that purpose Chapter 2 is adequate. Chapter 5 draws attention to a number of the more interesting individual groups."

The preparation of these tables was begun by the "senior" author way back in 1935. For a while Philip Hall was directly involved, and though he later withdrew as a co-author, the classification used is still based largely upon his ideas.

The outsize pages (17″ × 14″) were necessary because of the lattice diagrams. Each of the 340 individual groups, for orders 2^n with $1 \leqq n \leqq 6$, is represented by such a lattice, and the more complicated diagrams require an entire page. The diagrams, and portions of the tables, will be understandable and of interest to a reader with even a causal knowledge of finite groups. Other portions of the tables and the theory underlying their construction require a much deeper understanding to appreciate. One value of the volume, indeed, is that it provides a vast amount of illustrative material that can be examined in the course of a study of these deeper aspects of the theory. It is probable that the tables will prove stimulating to many readers, and this may even lead to new developments.

As an example of such stimulation, consider the 14 groups of order 16 that are