divisor, it follows that it has at least $\alpha(n)$ prime divisors. A table of $\alpha(n)$ for $n = 2(1)100$ was computed on the ILLIAC and is presented here; e.g., $\alpha(2) = 3$ and $\alpha(100) = 26308$. These may be compared with a theoretical formula:

$$\alpha(n) = \tfrac{1}{2}n^2 \log n + \tfrac{1}{2}n^2 \log \log n - \cdots.$$

Up to $n = 11$ and $n = 24$, the author could have used existing tables of $\prod_{p<x} (1 - 1/p)$ due to Legendre and Glaisher, respectively, instead of the ILLIAC, but he makes no mention of this. The later, and much more extensive table of Appel and Rosser [1] was not completely printed, and allows us only to determine such bounds as

$$5{,}730{,}105 < \alpha(1217) < 5{,}760{,}003.$$

D. S.

1. KENNETH I. APPEL & J. BARKLEY ROSSER, *Table for Estimating Functions of Primes*, IDA-CRD Technical Report Number 4, 1961; reviewed in *Math. Comp.*, v. 16, 1962, pp. 500–501, RMT 55.

**39[G].**—MARSHALL HALL, JR. & JAMES K. SENIOR, *The Groups of Order* $2^n$ ($n \leqq 6$), The Macmillan Company, New York, 1964, 225 pp., 36 cm. Price $15.00.

From the preface: "No single presentation of a group or list of groups can be expected to yield all the information which a reader might desire. Here, each group is presented in three different ways: (1) by generators and defining relations; (2) by generating permutations; and (3) by its lattice of normal subgroups, together with the identification of every such subgroup and its factor group. In this lattice the characteristic subgroups are distinguished.

"For each group, additional information is given. Here are included the order of the group of automorphisms and the number of elements of each possible order 2, 4, 8, 16, 32, and 64.....All the groups are divided into twenty-seven families, following Philip Hall's theory of isotopy.

"Chapters 3 and 4 give the theoretical background for the construction of the tables. But these chapters are not necessary for the use of those tables; for that purpose Chapter 2 is adequate. Chapter 5 draws attention to a number of the more interesting individual groups."

The preparation of these tables was begun by the "senior" author way back in 1935. For a while Philip Hall was directly involved, and though he later withdrew as a co-author, the classification used is still based largely upon his ideas.

The outsize pages ($17'' \times 14''$) were necessary because of the lattice diagrams. Each of the 340 individual groups, for orders $2^n$ with $1 \leqq n \leqq 6$, is represented by such a lattice, and the more complicated diagrams require an entire page. The diagrams, and portions of the tables, will be understandable and of interest to a reader with even a causal knowledge of finite groups. Other portions of the tables and the theory underlying their construction require a much deeper understanding to appreciate. One value of the volume, indeed, is that it provides a vast amount of illustrative material that can be examined in the course of a study of these deeper aspects of the theory. It is probable that the tables will prove stimulating to many readers, and this may even lead to new developments.

As an example of such stimulation, consider the 14 groups of order 16 that are

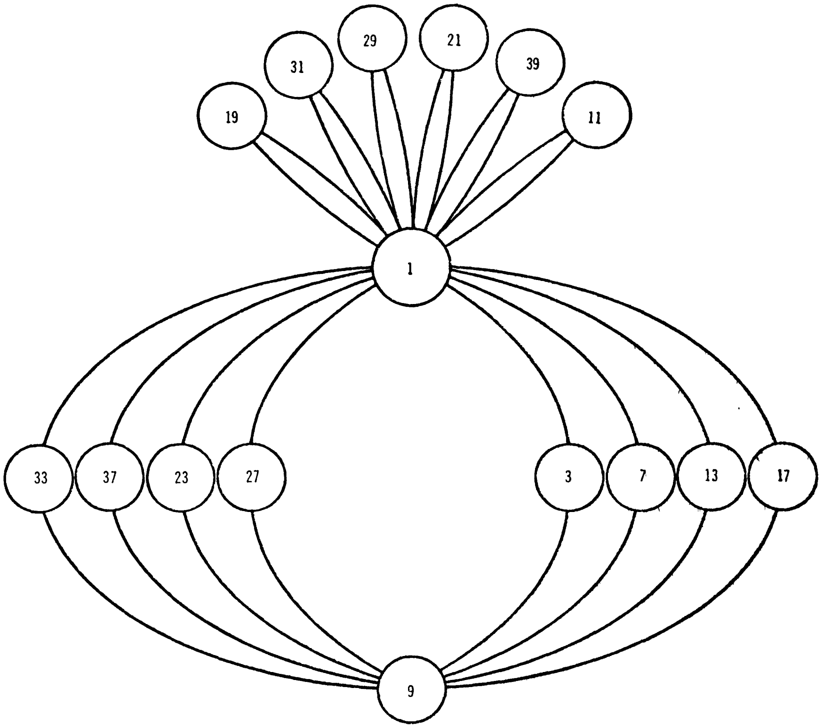


FIGURE 1. Cycle Graph of $\mathfrak{M}_{40}$

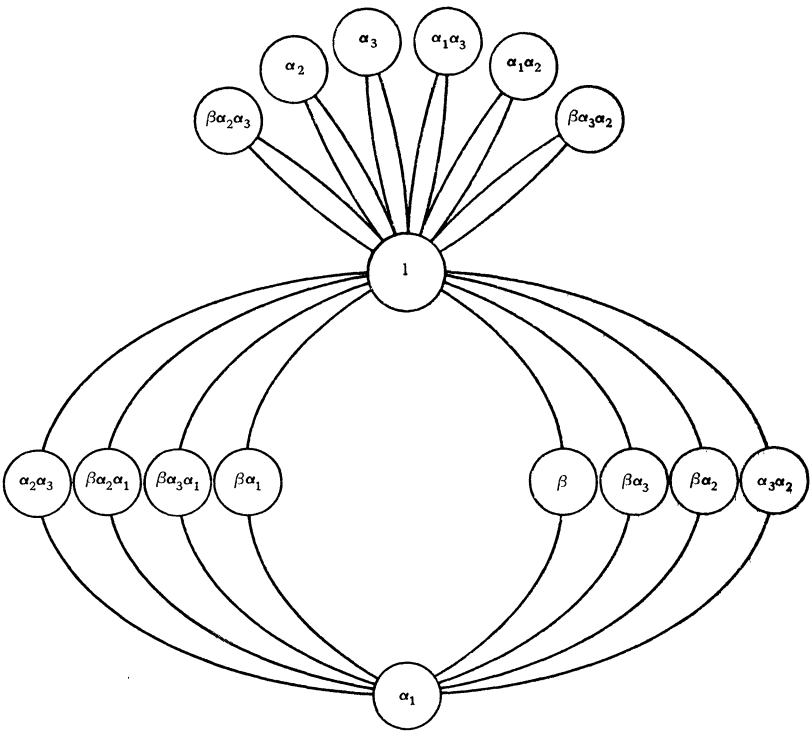


FIGURE 2. Cycle Graph of 16 $\Gamma_2 b$

tabulated here on pages 37, 39, and 45. Two of them, namely those designated as (31) and 16 $\Gamma_2 d$, are *conformal* to each other, that is, they contain an equal number of elements of each order. Likewise, the three groups $(2^2)$, 16 $\Gamma_2 a_2$, and 16 $\Gamma_2 c_2$ are conformal, and so are the three groups $(21^2)$, 16 $\Gamma_2 b$, and 16 $\Gamma_2 c_1$. The remaining 6 groups are conformal to no group.

By examination, we now note that although (31) and 16 $\Gamma_2 d$ are not isomorphic, they do have a lesser degree of similarity that we may call *isopotent.* We say that two groups are isopotent if their elements may be put into 1–1 correspondence: $a \leftrightarrow \alpha$, in such a way that all powers are also in correspondence: $a^n \leftrightarrow \alpha^n$. In distinction to this pair of isopotent groups, no two of the three conformal groups: $(2^2)$, 16 $\Gamma_2 a_2$, and 16 $\Gamma_2 c_2$ are isopotent. In the second set of three groups, $(21^2)$ is isopotent to 16 $\Gamma_2 b$, but they are not isopotent to 16 $\Gamma_2 c_1$.

It is clear that isopotence implies conformality, but not conversely. The exact relationship between the two concepts is not known to the reviewer at this time. If two groups are isopotent they have the same *cycle graph* [1]. We illustrate this in Figures 1 and 2. The group $\mathfrak{M}_{40}$ represents the 16 residue classes prime to 40 under multiplication modulo 40. It is isomorphic to $(21^2)$. The nonabelian group 16 $\Gamma_2 b$ is generated by the permutations:

$$\beta = (abcd)(efgh)$$

$$\alpha_2 = (eg)(fh)$$

$$\alpha_3 = (ae)(bf)(cg)(dh)$$

with $\alpha_1 = \beta^2$. An isopotent correspondence is that indicated diagrammatically: $3 \leftrightarrow \beta$, $31 \leftrightarrow \alpha_2$, etc.

The concept of isopotence may already be known, and it may, or may not, be of significance. The reviewer has not examined these questions, but they are not relevant here, since we merely wished to indicate that the tables can be stimulating.

The tables are nicely printed. The lattice diagrams, however, were not drawn by a professional draftsman, and exhibit much shaky lettering and uneven inking. This economy on the part of the publisher is somewhat regrettable, especially since the groups will be with us forever. Nonetheless, the diagrams are legible, and their interest and value are not negated by their lack of artistic perfection.

Apparently the tables were constructed entirely by hand. It would be an interesting challenge to an experienced programmer with the requisite algebraic knowledge and interest to attempt to reproduce and extend these tables with a computer.

Only one typographical error was noted. It is recorded on page 362 of this issue of *Mathematics of Computation.*

D. S.

1. Daniel Shanks, *Solved and Unsolved Problems in Number Theory*, Vol. 1, 1962, Spartan, Washington, pp. 83–103, 112–115, 206–208.

**40[G, X].**—L. Fox, *An Introduction to Numerical Linear Algebra*, Clarendon Press, Oxford, 1964, xi + 295 pp., 24 cm. Price $8.00.

This is a welcome addition to the growing number of textbooks on matrix computation. It should be quite accessible to students at the junior-senior level, although as a textbook it suffers from having no exercises. There are, however, numerous illustrative examples.