48[L].—A. H. HEATLEY, *Tables of the Confluent Hypergeometric Function and the Toronto Function*, University of Waterloo, Waterloo, Ontario, Canada, October 1964, one typewritten sheet and four computer sheets deposited in UMT File.

In these tables the functions $e^{-x}M(\alpha, \gamma, x)$ and $T(m, n, r)$ are tabulated to 9S in floating-point form, based upon calculations performed to at least 12S on an IBM 1620 system [1].

For the confluent hypergeometric function, the ranges of parameters are:

$$\alpha = \tfrac{1}{4}(\tfrac{1}{4})1, \quad \gamma = -\tfrac{1}{2}, \tfrac{1}{2}(\tfrac{1}{2})3; \quad \alpha = \tfrac{9}{4}(\tfrac{1}{4})3, \quad \gamma = 3; \quad \alpha = \tfrac{5}{4}(\tfrac{1}{4})2, \quad \gamma = 2.$$

The values of x are such that $x^{1/2} = 0(0.2)4, 5$; except that when $\alpha = 1, \gamma = -\frac{1}{2}$, $\frac{1}{2}(\frac{1}{2})3$, we find $x^{1/2} = 0(0.1)3$.

For the Toronto function, the corresponding ranges are:

$$m = -\tfrac{1}{2}(\tfrac{1}{2})\tfrac{1}{2}, \quad n = -2(\tfrac{1}{2})2, \quad r = 0(0.2)4(1)6, 10, 25, 50; \text{ and}$$
$$m = 1, \quad n = -2(\tfrac{1}{2})2, \quad r = 0(0.1)3.$$

J. W. W.

1. *Math. Comp.*, v. 18, 1964, pp. 687–688, MTE **361**.

49[L].—J. R. JOHNSTON, *Tables of Values and Zeros of the Confluent Hypergeometric Function*, Report 31901, Aircraft Division, Douglas Aircraft Company, Inc., Long Beach, Calif., August 1964, 4 pp., 28 cm.

This report briefly describes the computational procedure followed in evaluating the confluent hypergeometric function ${}_1F_1(A, B, X)$ and its zeros by a FORTRAN IV program prepared for use on an IBM 7094 system.

Computation of the function and its zeros was carried to 7D precision for $A = -5(0.25) - 0.25$, $B = 0.25(0.25)4$, $X = 0.05, 0.1(0.1)1(0.2)10(0.5)20$, and for $A = -20(0.5)1$, $B = 2$, $X = 0.05, 0.1(0.1)1(0.2)20(0.5)50$. These internally computed values for each of these two ranges were rounded to 4S and written as separate files on a special output tape, of which a copy can be obtained upon request from the Technical Library, Aircraft Division, Dept. C-250, Douglas Aircraft Company, Inc.

No printed output is available except for an abbreviated table of zeros to 4S that appears on the last page of this report. The range represented therein is $A = -20(1) - 1$, $B = 2$, and, with a few exceptions corresponding to $A = -4(1) - 1$, the first five positive zeros are tabulated.

For a list of related tables the reader is referred to the publication of Slater [1].

J. W. W.

1. L. J. SLATER, *Confluent Hypergeometric Functions*, Cambridge Univ. Press, New York, 1960. [See *Math. Comp.*, v. 15, 1961, pp. 98–99, RMT **22**.]

50[L].—SIU-KAY LUKE & STANLEY WEISSMAN, *Bessel Functions of Imaginary Order and Imaginary Argument*, University of Maryland, Institute for Molecular Physics, Report DA-ARO(D)-31-124-G466 No. 1, 1964, College Park, Md.

This report gives a rather extensive tabulation of

$$G_q(v) = K_{iq}(v) = \frac{1}{2}\pi \frac{I_{iq}(v) - I_{-iq}(v)}{\sinh q\pi}, \qquad v = e^x,$$

where $I_\mu(z)$ and $K_\mu(z)$ are the modified Bessel functions of the first and second kinds, respectively. The range on q and x varies. For example, $q = 0.2(0.2)10$, $x = 1.0(0.01)2.22$; $q = 0.4(0.2)10$, $x = 2.23(0.01)2.29$; $q = 1.2(0.2)10$, $x = 2.30(0.01)2.39$; $q = 1.6(0.2)10$, $x = 2.40(0.01)2.49$. Roughly speaking, we have data for $q = 0.2(0.2)50$, where the tables were "cut at an x value for each set of q's where the oscillating amplitude appears to be a constant." When $x > \ln q$, the tables were "cut at its first zero after it passed the turning point." The entries were found by numerical integration of the differential equation. The authors expect the data to be good to at least 5S for $q < 40$ and to 4S for higher q. The only other tables of this kind known to us are by S. P. Morgan. [See *MTAC* v. 3, 1948–1949, pp. 105–107, RMT **504.**] There is some overlap.

Y. L. L.

51[L].—M. M. Stuckey & L. L. Layton, *Numerical Determination of Spheroidal Wave Function Eigenvalues and Expansion Coefficients*, AML Report 164, David Taylor Model Basin, Washington, D. C., 1964, 186 pp., 26 cm.

Spheroidal wave functions result when the scalar Helmholtz equation is separated in spheroidal coordinates, either prolate or oblate. The angular prolate spheroidal wave functions, for example, satisfy a differential equation of the form

$$\frac{d}{dz}\left[(1 - z^2)\frac{du}{dz}\right] + \left(\lambda_{mn} - c^2z^2 - \frac{m^2}{1 - z^2}\right)u = 0.$$

The solutions of this equation are much more complicated than either Bessel or Legendre functions, in which, in fact, series solutions of the spheroidal functions are most often expanded. The complexity arises from the fact that the spheroidal differential equation has an irregular singular point at ∞ and two regular ones at $z = \pm 1$, in contrast to the three regular ones of the Legendre equation and to the one regular and one irregular singularity of the Bessel equation.

The construction of tables of spheroidal wave functions involves the calculation of the eigenvalues λ_{mn} of the differential equation, that is, those values of λ for which there are solutions that are finite at $z = \pm 1$, and the calculation of the coefficients in expansions in terms of either Legendre or spherical Bessel functions. In the past, such calculations have been, for the most part, sporadic and in many cases not very accurate.

The tables of the spheroidal eigenvalues and expansion coefficients in this report from the David Taylor Model Basin are the most complete that have been made available so far. Values of λ_{mn} are given to 11S, in floating-point form, for $m = 0(1)9$, $n = m(1)m + 9$, for $c = 0.25(0.25)10(1)20$. Values of the expansion coefficients d_r^{mn} are given for $c = 0.25(0.25)10$, $m = 0$ and 1, $n = m(1)10$, $r = 1(2)29$ for $n - m$ odd, and $r = 0(2)28$ for $n - m$ even.

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52[L, M].—Andrew Young & Alan Kirk, *Bessel Functions. Part IV, Kelvin Functions*, Royal Society Mathematical Tables, Volume 10, Cambridge University Press, New York, 1964, xxiii + 97 pp., 27 cm. Price $11.50.