where $I_\mu(z)$ and $K_\mu(z)$ are the modified Bessel functions of the first and second kinds, respectively. The range on q and x varies. For example, $q = 0.2(0.2)10$, $x = 1.0(0.01)2.22$; $q = 0.4(0.2)10$, $x = 2.23(0.01)2.29$; $q = 1.2(0.2)10$, $x = 2.30(0.01)2.39$; $q = 1.6(0.2)10$, $x = 2.40(0.01)2.49$. Roughly speaking, we have data for $q = 0.2(0.2)50$, where the tables were "cut at an x value for each set of q's where the oscillating amplitude appears to be a constant." When $x > \ln q$, the tables were "cut at its first zero after it passed the turning point." The entries were found by numerical integration of the differential equation. The authors expect the data to be good to at least 5S for $q < 40$ and to 4S for higher q. The only other tables of this kind known to us are by S. P. Morgan. [See *MTAC* v. 3, 1948–1949, pp. 105–107, RMT **504.**] There is some overlap.

Y. L. L.

51[L].—M. M. STUCKEY & L. L. LAYTON, *Numerical Determination of Spheroidal Wave Function Eigenvalues and Expansion Coefficients*, AML Report 164, David Taylor Model Basin, Washington, D. C., 1964, 186 pp., 26 cm.

Spheroidal wave functions result when the scalar Helmholtz equation is separated in spheroidal coordinates, either prolate or oblate. The angular prolate spheroidal wave functions, for example, satisfy a differential equation of the form

$$\frac{d}{dz}\left[(1-z^2)\frac{du}{dz}\right] + \left(\lambda_{mn} - c^2z^2 - \frac{m^2}{1-z^2}\right)u = 0.$$

The solutions of this equation are much more complicated than either Bessel or Legendre functions, in which, in fact, series solutions of the spheroidal functions are most often expanded. The complexity arises from the fact that the spheroidal differential equation has an irregular singular point at ∞ and two regular ones at $z = \pm 1$, in contrast to the three regular ones of the Legendre equation and to the one regular and one irregular singularity of the Bessel equation.

The construction of tables of spheroidal wave functions involves the calculation of the eigenvalues λ_{mn} of the differential equation, that is, those values of λ for which there are solutions that are finite at $z = \pm 1$, and the calculation of the coefficients in expansions in terms of either Legendre or spherical Bessel functions. In the past, such calculations have been, for the most part, sporadic and in many cases not very accurate.

The tables of the spheroidal eigenvalues and expansion coefficients in this report from the David Taylor Model Basin are the most complete that have been made available so far. Values of λ_{mn} are given to 11S, in floating-point form, for $m = 0(1)9$, $n = m(1)m + 9$, for $c = 0.25(0.25)10(1)20$. Values of the expansion coefficients d_r^{mn} are given for $c = 0.25(0.25)10$, $m = 0$ and 1, $n = m(1)10$, $r = 1(2)29$ for $n - m$ odd, and $r = 0(2)28$ for $n - m$ even.

CARSON FLAMMER

Stanford Research Institute
Menlo Park, California

52[L, M].—ANDREW YOUNG & ALAN KIRK, *Bessel Functions. Part IV, Kelvin Functions*, Royal Society Mathematical Tables, Volume 10, Cambridge University Press, New York, 1964, xxiii + 97 pp., 27 cm. Price $11.50.

As noted in the title, this volume is the fourth in a series of tables devoted to Bessel functions that was initiated by the Mathematical Tables Committee of the British Association for the Advancement of Science and has been continued by its successor, the Mathematical Tables Committee of the Royal Society.

The first three parts of this tabulation of Bessel functions, which dealt with those functions of the first two kinds of real and of pure imaginary arguments, have been previously reviewed in this journal (*MTAC*, v. 1, 1943–1945, pp. 361–363; *ibid.*, v. 7, 1953, pp. 97–98; *Math. Comp.*, v. 15, 1961, pp. 214–215).

The present tables are concerned with Kelvin functions of integer orders and real arguments. These functions are defined in terms of Bessel functions of the first two kinds by the relations

$$\mathrm{ber}_n\, x + i\, \mathrm{bei}_n\, x = J_n(xe^{3i\pi/4})$$

and

$$\mathrm{ker}_n\, x + i\, \mathrm{kei}_n\, x = \frac{i\pi}{2}\{J_n(xe^{3i\pi/4}) + iY_n(xe^{3i\pi/4})\}.$$

Table I presents 15D values of the Kelvin functions of orders 0 and 1 for $x = 0(0.1)10$. Table II consists of 7S and 8S values of these functions and of their polar forms for $n = 0(1)2$ and $x = 0(0.01)2.5$. The main table is Table III, which generally gives 6S and 7S values of the functions and of their polar forms for $n = 0(1)10$ and $x = 0(0.1)10$, together with second and fourth central differences. The second differences are modified when appropriate and are so designated.

It should be remarked that Table III is especially useful and important because in all previously published tables of this kind, the orders of the functions have not exceeded 5.

Interpolation in certain parts of Tables II and III is not readily accomplished; there the user is referred to Table IV, where auxiliary functions are tabulated for $\mathrm{ker}_n\, x$ and $\mathrm{kei}_n\, x$ when $n = 0$ and 1, and where the modified functions $x^{-n}\, \mathrm{ber}_n\, x$, $x^{-n}\, \mathrm{bei}_n\, x$, $x^n\, \mathrm{ker}_n\, x$, $x^n\, \mathrm{kei}_n\, x$, and the similarly modified moduli, $x^{-n}M_n(x)$ and $x^nN_n(x)$, are uniquely tabulated, generally to 7S and 8S, for $n = 3(1)5$, $x = 0(0.1)2.5$; $n = 6(1)10$, $x = 0(0.1)5$. For $n = 2$, only $x^n\, \mathrm{ker}_n\, x$, $x^n\, \mathrm{kei}_n\, x$, and $x^nN_n(x)$ are given; these appear to 7D for $x = 0(0.01)0.2$. Throughout Table IV second central differences and fourth central differences, where required, are shown.

The numerical tables are preceded by a section of 12 pages entitled Functions and Formulae, wherein appear basic definitions and properties of the Kelvin functions, as well as their various expansions in series. Also listed therein are indefinite integrals involving these functions, after which there is a detailed description of the preparation of the tables.

The authors state that, prior to printing, all the tabular entries, originally computed on desk calculators, were recomputed, using double- and triple-precision arithmetical routines when necessary, on the DEUCE computer in Liverpool University. Because of these elaborate precautions to insure accuracy, it is claimed that each tabular value is correct to within a unit in the least significant recorded figure.

A bibliography, consisting of 34 titles, serves to round out this valuable and attractive addition to the tabular information on Bessel functions.

J. W. W.