

Haselgrove and Miller [2]. The integer coefficients are related to the Euler numbers and were calculated from recurrence relations.

## AUTHOR'S SUMMARY

1. D. H. LEHMER, "Extended computation of the Riemann zeta-function," *Mathematika*, v. 3, 1956, pp. 102-108.

2. C. B. HASEL GROVE & J. C. P. MILLER, *Tables of the Riemann Zeta Function*, Royal Society Mathematical Tables, v. 6, Cambridge Univ. Press, New York, 1960.

**81[L, M].**—HENRY E. FETTIS & JAMES C. CASLIN, *Tables of Elliptic Integrals of the First, Second and Third Kind*, Applied Mathematics Research Laboratory Report ARL 64-232, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, December 1964, iv + 93 pp.

This report contains 10D tables of both complete and incomplete elliptic integrals of all three kinds in Legendre's form. Table I consists of such decimal approximations to  $F(\phi, k)$  and  $E(\phi, k)$  for  $\phi = 5^\circ(5^\circ)90^\circ$  and  $k = 0(0.01)1$ , while Table II gives similar information for  $k^2 = 0(0.01)1$ . Table III gives 10D values of  $\Pi(\phi, \alpha^2, k) = \int_0^\phi (1 - \alpha^2 \sin^2 \theta)^{-1} (1 - k^2 \sin^2 \theta)^{-1/2} d\theta$  for  $\phi = 5^\circ(5^\circ)80^\circ(2.5^\circ)90^\circ$ ,  $k^2 = 0(0.05)0.9(0.02)1$ ,  $\alpha^2 = -1(0.1)-0.1, 0.1(0.1)1$  (except that when  $\alpha^2 = 1$ ,  $\phi$  extends only to  $87.5^\circ$ ). The authors' description of the tables contains some minor errors with reference to the ranges of the parameters.

To insure reliability in the final rounded values, the underlying calculations were performed to 16S on an IBM 1620, using a subroutine based on Gauss's transformation [1], which is given for all three integrals in the accompanying explanatory text. A discussion of the several checking procedures applied to the tabular entries is included; however, the problem of interpolating in the tables is not considered.

The authors refer to tables of elliptic integrals of the third kind by Selfridge & Maxfield [2] and by Paxton & Rollin [3], but appear to be unaware of the extensive 7S tables of Beliakov, Kravtsova & Rappaport [4].

The present tables contain the most accurate decimal approximations to the elliptic integral of the third kind that have thus far been published, and constitute a significant contribution to the tabular literature.

J. W. W.

1. HENRY E. FETTIS, "Calculation of elliptic integrals of the third kind by means of Gauss' transformation," *Math. Comp.*, v. 19, 1965, pp. 97-104.

2. R. E. SELFRIDGE & J. E. MAXFIELD, *A Table of the Incomplete Elliptic Integral of the Third Kind*, Dover, New York, 1959. (See *Math. Comp.*, v. 14, 1960, pp. 302-304, RMT 65.)

3. F. A. PAXTON & J. E. ROLLIN, *Tables of the Incomplete Elliptic Integrals of the First and Third Kind*, Curtiss Wright Corporation, Research Division, Quehanna, Pennsylvania, 1959. (See *Math. Comp.*, v. 14, 1960, pp. 209-210, RMT 33.)

4. V. M. BELIAKOV, R. I. KRAVTSOVA & M. G. RAPPAPORT, *Tablitsy ellipticheskikh integralov*, Tom I, Izdat. Akad. Nauk SSSR, Moscow, 1962. (See *Math. Comp.*, v. 18, 1964, pp. 676-677, RMT 93.)

**82[L, Z].**—JURGEN RICHARD MANKOPF, *Über die periodischen Lösungen der Van der Polschen Differentialgleichung  $\ddot{x} + \mu(x^2 - 1)\dot{x} + x = 0$* , Forschungsberichte des Landes Nordrhein-Westfalen, Nr. 1307, Westdeutscher Verlag, Opladen, 1964, 55 pp., 24 cm. Price DM 41.

This expository monograph analyzes the Van der Pol equation. The work is concerned primarily with the free-vibration equation. The case corresponding to large values of  $\mu$  is discussed in a very brief section.

One section of the investigation applies general results from nonlinear vibration theory to the Van der Pol equation. Another section develops a perturbational technique to obtain approximations to periodic solutions.

The final section discusses analogue computer techniques to generate solutions of the equation.

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**83[M, S].**—L. C. WALTERS & J. R. WAIT, *Computation of a Modified Fresnel Integral Arising in the Theory of Diffraction by a Variable Screen*, National Bureau of Standards Technical Note 224, U. S. Government Printing Office, Washington, D. C., October 1964, iii + 20 pp., 27 cm. Price \$0.20.

This report contains numerical tables of the real and imaginary parts of the diffraction integral

$$F = \int_{-\infty}^{\infty} f(z) \exp(-i\pi z^2/2) dz$$

in the following three cases:

I. Linear tapered aperture,

$$\begin{aligned} f(z) &= \frac{1}{2} + \frac{\beta}{2} (z - z_0), & z_0 - \beta^{-1} &\leq z \leq z_0 + \beta^{-1}, \\ &= 0, & -\infty &\leq z \leq z_0 - \beta^{-1}, \\ &= 1, & z_0 + \beta^{-1} &\leq z. \end{aligned}$$

II. Cubic tapered aperture,

$$\begin{aligned} f(z) &= \frac{1}{2} + \frac{\beta}{2} (z - z_0) - \frac{2\beta^3}{27} (z - z_0)^3, & z_0 - \frac{3}{2\beta} &\leq z \leq z_0 + \frac{3}{2\beta}, \\ &= 0, & -\infty &\leq z \leq z_0 - \frac{3}{2\beta}, \\ &= 1, & z_0 + \frac{3}{2\beta} &\leq z. \end{aligned}$$

III. Exponential tapered aperture,

$$f(z) = \{1 + \exp[-2\beta(z - z_0)]\}^{-1}, \quad -\infty \leq z \leq \infty.$$

In Tables 1 and 2, associated with Cases I and II, respectively, the integral  $F$  is tabulated to 5D for  $\beta = 0.2, 0.5, 1, 2, 5, 10$ , and  $z_0 = 0(0.1)5$ . Table 3, associated with Case III, consists of 5D approximations to  $F$  for  $\beta = 0.5, 1, 2(0.5)3(1)5, 10, \infty$ , and  $z_0 = 0(0.1)5$ .

The authors note that, in all three cases, as  $\beta$  tends to infinity,  $F$  approaches  $\frac{1}{2}(1 - i) - F_0(z_0)$ , where  $F_0(z) = \int_0^z \exp(-i\pi x^2/2) dx$  is the Fresnel integral. This fact permitted a check on the entries in Table 3 corresponding to  $\beta = \infty$ , through a comparison with the corresponding data in the tables of Wijngaarden and Scheen [1]. Complete agreement was found.