3. Generating Functions, by John Riordan.
4. Lattice Statistics, by Elliott W. Montroll.
5. Pólya's Theory of Counting, by N. G. de Bruijn.
6. Combinatorial Problems in Graphical Enumeration, by Frank Harary.
7. Dynamic Programming and Markovian Decision Processes, with Particular Application to Baseball and Chess, by Richard Bellman.
8. Graph Theory and Automatic Control, by Robert Kalaba.
9. Optimum Multivariable Control, by Edwin L. Peterson.
10. Stopping-rule Problems, by Leo Breiman.
11. Combinatorial Algebra of Matrix Games and Linear Programs, by Albert W. Tucker.
12. Network Flow Problems, by Edwin F. Beckenbach.
13. Block Designs, by Marshall Hall, Jr.
14. Introduction to Information Theory, by Jacob Wolfowitz.
15. Sperner's Lemma and Some Extensions, by Charles B. Tompkins.
16. Crystallography, by Kenneth N. Trueblood.
17. Combinatorial Principles in Genetics, by George Gamow.
18. Appendices, by Hermann Weyl.

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90[X].—Richard Bellman & Robert Kalaba, Editors, *Selected Papers on Mathematical Trends in Control Theory*, Dover Publications, Inc., New York, 1964, vi + 200 pp., 24 cm. Price $2.00 (paperbound).

In this collection of papers we are offered a banquet whose menu includes a number of main dishes plus many side dishes and tidbits. That some of these offerings do, and some do not, appeal to the reviewer (the taster) is only to be expected.

In the opinion of the reviewer, the best major dish is Hurwitz' paper, "On the Conditions Under Which an Equation Has Only Roots with Negative Real Parts", which is a model of lucidity by a major mathematician and mathematical artist. The topic is also of practical importance for control technique. On the other hand, the worst tidbit seems to be the paper by the two banquet organizers on the work of Liapunov and Poincaré (four pages, one consisting of a list of references).

A few remarks will now be made concerning some of the main dishes.

With considerable propriety, the first paper in this "control" collection is Clerk Maxwell's "On Governors" (1868), the first in which control problems were dealt with scientifically and with appropriate mathematics. However, for lucidity we would have preferred the treatment by Pontryagin in his book on differential equations, or Vishnegradski's very early and independent treatment (1876) of Watt's steam-engine governor.

H. Bateman's "The Control of an Elastic Fluid" (1945) is the longest of the collection and is, in fact, too long for its content.

H. Nyquist's "Regeneration Theory" is not too lucid, but is interesting as the history of a noted application of mathematics to technology.

In H. W. Bode's paper "Feedback—The History of an Idea", the all important role of feedback in the development of long-distance telephony is interestingly described in full detail.

B. van der Pol's "Forced Oscillations in a Circuit with Nonlinear Resistance (Reception with Reactive Triode)" discusses a highly interesting application by the author of the famous van der Pol equation.

The paper by N. Minorsky, entitled "Self-Excitation in Dynamical Systems Possessing Retarded Action", represents pioneer work on retarded action.

Here an appropriate main dish would have been the omitted paper by D. Bushaw entitled "Optimal Discontinuous Forcing Terms", which appeared in *Contributions to the Theory of Nonlinear Oscillations*, vol. IV, *Annals of Mathematics Studies*, no. 41, pp. 29–52. This is one of the earliest papers on optimization.

Remaining main dishes are: "An Extension of Wiener's Prediction Theory", by L. A. Zadeh and J. R. Ragazzini; and "On the Application of the Theory of Dynamical Programming to the Study of Control Processes", by Richard Bellman.

Attractive side dishes are: "Time Optimal Control Systems", by J. P. LaSalle, which is a high-grade contribution on optimization in the United States, originally published in 1959; and "On the Theory of Optimal Processes", by Boltyanskii, Gamkrelidze, and Pontryagin.

This reviewer hopes that his culinary description has not masked the fact that the editors have presented (à la earlier Bellman) quite a noteworthy collection of control papers—not an easy choice in such a popular field and from such varied directions. A suggestion to the editors for a future edition: Do not hesitate to present extracts from long, but classical, memoirs such as those of Poincaré and Liapunov.

Detailed references to many other articles are included in this collection.

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91[X].—HARRY M. COLBERT, *Asymptotic Expansion for the Characteristic Values of Mathieu's Equation*, ms. of 5 pp. + 1 table of 3 pp., deposited in UMT File.

The author repeats the well-known BWK procedure for obtaining the asymptotic expansion of

$$y'' + [A - k\phi^2(x)]y = 0.$$

He then specializes this equation to that of Mathieu, and uses the same procedure as Ince did to obtain the characteristic values. The equation is taken actually in the form

$$y'' + (a + 2q - 4q\sin^2 x)y = 0,$$

which becomes identical with the usual form of Mathieu's equation if q is replaced by $-q$.

For this specialized equation, the author tabulated G_n and c_m, occurring in the series

$$a_r \sim b_{r+1} = -q \sum_{n=1}^{\infty} \frac{\epsilon_n(\nu)}{2^{G_n}} q^{-(n-1)/2},$$

$$\epsilon_n(\nu) = \sum_{m=1}^{(n+\sigma)/2} c_m \nu^{2m-1-\sigma},$$