

B. van der Pol's "Forced Oscillations in a Circuit with Nonlinear Resistance (Reception with Reactive Triode)" discusses a highly interesting application by the author of the famous van der Pol equation.

The paper by N. Minorsky, entitled "Self-Excitation in Dynamical Systems Possessing Retarded Action", represents pioneer work on retarded action.

Here an appropriate main dish would have been the omitted paper by D. Bushaw entitled "Optimal Discontinuous Forcing Terms", which appeared in *Contributions to the Theory of Nonlinear Oscillations*, vol. IV, *Annals of Mathematics Studies*, no. 41, pp. 29-52. This is one of the earliest papers on optimization.

Remaining main dishes are: "An Extension of Wiener's Prediction Theory", by L. A. Zadeh and J. R. Ragazzini; and "On the Application of the Theory of Dynamical Programming to the Study of Control Processes", by Richard Bellman.

Attractive side dishes are: "Time Optimal Control Systems", by J. P. LaSalle, which is a high-grade contribution on optimization in the United States, originally published in 1959; and "On the Theory of Optimal Processes", by Boltyanskii, Gamkrelidze, and Pontryagin.

This reviewer hopes that his culinary description has not masked the fact that the editors have presented (à la earlier Bellman) quite a noteworthy collection of control papers—not an easy choice in such a popular field and from such varied directions. A suggestion to the editors for a future edition: Do not hesitate to present extracts from long, but classical, memoirs such as those of Poincaré and Liapunov.

Detailed references to many other articles are included in this collection.

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91[X].—HARRY M. COLBERT, *Asymptotic Expansion for the Characteristic Values of Mathieu's Equation*, ms. of 5 pp. + 1 table of 3 pp., deposited in UMT File.

The author repeats the well-known BWK procedure for obtaining the asymptotic expansion of

$$y'' + [A - k\phi^2(x)]y = 0.$$

He then specializes this equation to that of Mathieu, and uses the same procedure as Ince did to obtain the characteristic values. The equation is taken actually in the form

$$y'' + (a + 2q - 4q \sin^2 x)y = 0,$$

which becomes identical with the usual form of Mathieu's equation if q is replaced by $-q$.

For this specialized equation, the author tabulated G_n and c_m , occurring in the series

$$a_r \sim b_{r+1} = -q \sum_{n=1}^{\infty} \frac{\epsilon_n(\nu)}{2^{\sigma_n}} q^{-(n-1)/2},$$

$$\epsilon_n(\nu) = \sum_{m=1}^{(n+\sigma)/2} c_m \nu^{2m-1-\sigma},$$

where $\nu = 2r + 1$, $\sigma = 1$ if n is odd, and $\sigma = 0$ if n is even. The integer coefficients c_j are given to a maximum of 28 digits, corresponding to $n = 1(1)28$.

This reviewer doubts that the table will be very useful, since the asymptotic expansion gives little accuracy unless the order is very low or q is extremely large. It should be noted that for the first 15 orders, the characteristic values of Mathieu's equation have been tabulated from $q = 0$ to $q = \infty$. (See references 20.53 and 20.58 on p. 746 of the NBS *Handbook* [1].)

Nevertheless, no one can predict with certainty that a table will find no application.

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1. National Bureau of Standards, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Applied Mathematics Series, No. 55, U. S. Government Printing Office, Washington, D. C., June 1964.

92[X].—PETER HENRICI, *Elements of Numerical Analysis*, John Wiley & Sons, Inc., New York, 1964, xv + 328 pp., 23 cm. Price \$8.00.

The author's aim, as stated in the Preface, is to produce a textbook that will appeal to mathematicians. In order to do so, he has tried to suppress the *art* and emphasize the *mathematical discipline* by stressing unifying principles. To achieve a balance between the theoretical and practical, he has made a clear-cut distinction between algorithms and theorems.

The table of contents shows the range of material covered:

Introduction.

- Chapter 1. What is numerical analysis?
2. Complex numbers and polynomials.
3. Difference Equations.

Part One. Solution of Equations.

- Chapter 4. Iteration.
5. Iteration for Systems of Equations.
6. Linear Difference Equations.
7. Bernoulli's Method.
8. The Quotient-Difference Algorithm.

Part Two. Interpolation and Approximation.

- Chapter 9. The Interpolating Polynomial.
10. Construction of the Interpolating Polynomial: Methods Using Ordinates.
11. Construction of the Interpolating Polynomial: Methods Using Differences.
12. Numerical Differentiation.
13. Numerical Integration.
14. Numerical Solution of Differential Equations.

Part Three. Computation.

- Chapter 15. Number Systems.
16. Propagation of Round-off Error.

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