The papers here are by Atle Selberg, R. R. Laxton and D. J. Lewis, W. J. LeVeque, Marshall Hall, Jr., Albert Leon Whiteman, Basil Gordon and E. G. Straus, Kenkichi Iwasawa, Morris Newman, E. C. Dade and O. Taussky, E. T. Parker, Gordon Pall, B. J. Birch, N. C. Ankeny, Paul T. Bateman and Roger A. Horn, Tom M. Apostol, S. Chowla and H. Walum, L. Carlitz, W. H. Mills, Leo Moser, P. Erdös, Harvey Cohn, and J. Lehner.

D. S.

112[F].—PAUL T. BATEMAN & ROGER A. HORN, "Primes represented by irreducible polynomials in one variable," *Theory of Numbers* (see previous review), pp. 119–132, in particular Tables II–V.

This paper is concerned with further development of a topic previously examined in this journal in references [1], [2]. The question is to estimate the number of integers n between 1 and N for which $f_i(n)$ $(i = 1, 2, \cdots, k)$ are simultaneously primes, where the f_i are distinct, irreducible polynomials. Under broad conditions, Bateman has conjectured that this number $P(N)$ satisfies

$$P(N) \sim c \frac{N}{(\log N)^k} \tag{1}$$

where the constant c is given by an explicit slowly convergent infinite product.

In a series of papers, [3]–[8], the reviewer had developed techniques of accurately computing these constants c for, say, $k = 1$ and $f_1 = n^4 + 1$ or $f_1 = n^2 + a$, and for $k = 2$ and $f_{1,2} = (n \pm 1)^2 + 1$. Bateman points out here that in all these cases the f_i are *abelian* polynomials, and he gives a general approach to the problem for any abelian polynomials. This general attack, like the specific ones mentioned, uses certain Dirichlet series, but it does not attain the degree of convergence which had been obtained in those special cases.

The authors also examine here (among others) eight *non-abelian* cases: $x^3 \mp 2$, $2x^3 \mp 1$, $x^3 \mp 3$, $3x^3 \mp 1$, and they give empirical counts of such primes for $x <$ 14000, 6000, 14000, and 8000, respectively. But for these non-abelian cases *no accurate way of computing the constants is known*. For example, the number of primes of either form $n^3 \mp 2$ is conjectured to satisfy

$$P(N) \sim \frac{1}{3} A \int_2^N \frac{dn}{\log n} \tag{2}$$

where

$$A = \prod_p \frac{p - \alpha(p)}{p - 1}, \tag{3}$$

the product being taken over all primes $p = 6m + 1$ with $\alpha(p) = 3$, or 0, according as p is, or is not, expressible as $a^2 + 27b^2$. The sequence of partial products here not only converges very slowly, but has an annoying, irregular "drifting" character that frustrates any standard acceleration technique. In the limit, there are twice as many primes p with $\alpha(p) = 0$ as with $\alpha(p) = 3$, (that is why the product converges), but the two types of p occur in a "random" manner, and this causes the sequence to drift up and down in a way that defies the instinct of any numerical analyst. Presumably, a Césaro sum would help some, but that is not very satisfactory. The authors tentatively suggest $A = 1.29$, which they obtain from (3) with $p < 1000$.

Since there are relatively few convergent sequences that arise naturally for which someone has not found an effective acceleration device, the evaluation of A must be considered a real and worthwhile challenge to analysts and number theorists. It is natural to hope that an analogue of the Gauss sums could be found for the appropriate algebraic integers and Dirichlet series present here, and with this one could proceed as before. Indeed, in the lecture by Birch mentioned in the previous review (on p. 107), there is a similar problem that has been solved by Birch and Davenport. But this is a difficult and incompletely presented paper, and the reviewer is unable to say whether similar techniques will work here. A *quite different* approach is to find a *theorem*, not a *conjecture*, in which the constant A enters. Then one could logically estimate A by empirical studies. The reviewer has, in fact, found [9] the following theorem. The number of positive odd numbers $<x$ of the form $4u^2 + 2uv + 7v^2$ is given by

$$\frac{\frac{7}{12}b_3\,x}{\sqrt{\log x}}\left[1 - \frac{d\sqrt[3]{\mathcal{L}(1)}}{\sqrt[3]{\log x}} + O\left(\frac{1}{\log x}\right)\right] \tag{4}$$

where b_3 is a known constant, [9] or [8, p. 136], where

$$d = \frac{18}{7}\frac{\sqrt[6]{\pi}}{\Gamma(\frac{1}{6})}\,(b_3)^{2/3}, \tag{5}$$

and where

$$\mathcal{L}(s) = \prod_p (1 - p^{-s})^{1-\alpha(p)}, \tag{6}$$

using the notation of (3). Thus $\mathcal{L}(1)$ could be estimated empirically from (4). Now

$$A = \frac{1}{\mathcal{L}(1)}\prod_p \left(1 - \frac{3}{p}\right)\left(1 - \frac{1}{p}\right)^{-3},$$

where this product is taken over all primes $p = a^2 + 27b^2$. The new product converges more rapidly, and monotonically. The error may therefore be bounded.

The counts of primes $x^3 \mp 2$, etc., were obtained here by a straight-forward trial-and-error method on a CDC 1604. No previously published tables of such counts are known to the reviewer, although in [10] W. A. Golubew indicated that he has made counts for $x^3 + 2$. It should be added that not only are the constants c much more difficult to compute in these cases, but that the counts of the primes themselves are also much more cumbersome. For primes of the form $x^2 + a$, for example, there is available the very efficient sieve method based upon the p-adic square roots of $-a$ wherein there is no trial-and-error whatsoever. Such an algorithm is not possible in these cubic cases, cf. [5].

D. S.

1. Paul T. Bateman & Roger A. Horn, "A heuristic asymptotic formula concerning the distribution of prime numbers," *Math. Comp.*, v. 16, 1962, pp. 363–367.

2. A. Schinzel, "A remark on a paper of Bateman and Horn," *Math. Comp.*, v. 17, 1963, pp. 445–447.

3. Daniel Shanks, "A note on Gaussian twin primes," *Math. Comp.*, v. 14, 1960, pp. 201–203.

4. DANIEL SHANKS, "On the conjecture of Hardy and Littlewood concerning the number of primes of the form $n^2 + a$," *Math. Comp.*, v. 14, 1960, pp. 321–332.
5. DANIEL SHANKS, "On numbers of the form $n^4 + 1$," *Math. Comp.*, v. 15, 1961, pp. 186–189; Corrigendum, *ibid.*, v. 16, 1962, p. 513.
6. DANIEL SHANKS, "Supplementary data and remarks concerning a Hardy-Littlewood conjecture," *Math. Comp.*, v. 17, 1963, pp. 188–193.
7. DANIEL SHANKS, "Polylogarithms, Dirichlet series, and certain constants," *Math. Comp.*, v. 18, 1964, pp. 322–324.
8. DANIEL SHANKS & JOHN W. WRENCH, JR., "The calculation of certain Dirichlet series," *Math. Comp.*, v. 17, 1963, pp. 136–154; Corrigenda, *ibid.*, p. 488.
9. DANIEL SHANKS & LARRY P. SCHMID, "Variations on a theorem of Landau," (to appear).
10. W. A. GOLUBEW, "Primzahlen der Form $x^2 + 3$," *Österreich. Akad. Wiss. Math.-Nat. Kl.*, 1958, Nr. 11, pp. 168–172.

113[F].—SIDNEY KRAVITZ & JOSEPH S. MADACHY, *Divisors of Mersenne Numbers*, $20{,}000 < p < 100{,}000$, ms. of 2 typewritten pages + 48 computer sheets, deposited in the UMT File.

The authors computed all prime factors $q < 2^{25}$ of all Mersenne numbers $M_p = 2^p - 1$ for all primes p such that $20{,}000 < p < 100{,}000$. The computation took about one-half an hour on an IBM 7090. There are 2864 such factors q. These are listed on 48 sheets of computer printout in the abbreviated form: k vs. p, where $q = 2pk + 1$. A reader interested in statistical theories of such factors may wish to examine the following summary that the reviewer has tallied from these lists. Out of the 7330 primes p in this range, M_p has 0, 1, 2, 3, or 4 prime divisors $q < 2^{25}$, according to the following table

0	1	2	3	4
4920	2006	356	46	2

The two values of p with four such factors are $p = 26{,}681$ and $68{,}279$.

The authors do not indicate whether or not any of these factors q is a multiple factor, that is, whether $q^2 \mid M_p$. Heuristically, the probability of a multiple factor here is quite low. Such a q has not been previously found [1], but, on the other hand, no convincing heuristic argument has ever been offered for the conjecture [1] that they do not exist. The alleged proof given in [2] is certainly fallacious, and for the quite closely analogous ternary numbers $\frac{1}{2}(3^p - 1)$ one finds a counterexample almost at once.

For earlier tables of factors of M_p see [1], [3] and the references cited there.

D. S.

1. JOHN BRILLHART, "On the factors of certain Mersenne numbers. II," *Math. Comp.*, v. 18, 1964, pp. 87–92.
2. E. KARST, "Faktorenzerlegung Mersennescher Zahlen mittels programmgesteuerter Rechengeräte," *Numer. Math.*, v. 3, 1961, pp. 79–86, esp. p. 80.
3. DONALD B. GILLIES, "Three new Mersenne primes and a statistical theory," *Math. Comp.*, v. 18, 1964, pp. 93–97.

114[F].—H. C. WILLIAMS, R. A. GERMAN & C. R. ZARNKE, *Solution of the Cattle Problem of Archimedes*, copy of the number T, 42 computer sheets, deposited in the UMT File.

There is deposited here the number T, the total number of cattle in Archimedes' problem, the computation of which is discussed elsewhere in this issue. This enor-