

The three criteria for exclusion, (a), (b), and (c), are marked by placing the symbols $-/-$, $+/+$, and $/ /$, respectively, after the prime.

For primes $\equiv 1 \pmod{4}$, the only odd exponent which had to be considered was 1, as $\sigma(p)$ divides $\sigma(p^{2m+1})$. The prime with the odd exponent is preceded by the letter P .

With this result, Kanold's lower bound of 10^{20} for an odd perfect number can be raised. To produce a specific number as a bound, however, it is necessary to assemble various other restrictions upon odd perfect numbers. This is not being undertaken here, as M. Garcia has obtained (but not published) a yet higher bound.

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Solutions of the Diophantine Equations

$$x^2 + y^2 = l^2, y^2 + z^2 = m^2, z^2 + x^2 = n^2$$

By M. Lal and W. J. Blundon

Introduction. The solution of the system of three equations of the second degree in six unknowns i.e. $x^2 + y^2 = l^2$, $y^2 + z^2 = m^2$ and $z^2 + x^2 = n^2$ is a classical Diophantine problem [1, p. 112]. The geometrical significance of this problem is to find a rectangular parallelepiped whose edges and face diagonals are all rational integers. If x , y and z are relatively prime in pairs the above system has no solution; otherwise there are infinitely many solutions.

TABLE 1

X	Y	Z	X	Y	Z
44	117	240	2925	3536	11220
85	132	720	2964	9152	9405
140	480	693	2964	6160	38475
160	231	792	3696	9045	121940
187	1020	1584	4368	4901	13860
195	748	6336	4599	18368	23760
240	252	275	4900	17157	23760
429	880	2340	4928	10725	30780
495	4888	8160	5491	41580	46512
528	5796	6325	5643	43680	76076
780	2475	2992	5643	14160	21476
828	2035	3120	5720	8415	157248
832	855	2640	6072	16929	18560
935	17472	25704	7560	13728	35321
1008	1100	1155	7579	8820	17472
1008	1100	12075	7800	23751	29920
1080	1881	14560	7840	9828	10725
1105	9360	35904	7920	15232	26649
1155	6300	6688	8789	10560	17748
1188	16016	39195	9180	72611	206448
1560	2295	5984	9504	31372	61845
1575	1672	9120	10296	11753	16800
1755	4576	6732	10395	63364	327360
2079	44080	65472	10395	95004	220400
2163	15840	37100	12915	36720	290444
14112	15400	19305	49088	169575	360360
15939	18460	48720	61215	121264	141120
16016	100035	207900	61975	412920	425568
16380	62832	98549	62415	145464	362848
17325	100320	264404	65520	102765	394196
18525	71060	90576	66528	103095	446600
19175	112320	293832	67925	86580	332112
19604	55440	62205	68172	110979	141680
19635	21964	166320	72864	143640	474145
20163	33660	332384	80080	229824	500175
21328	25740	38571	81576	191065	1399200
23760	35075	35604	81840	122636	148005
23936	33120	67575	83804	108405	432960
24035	30636	70752	97152	198220	274275
27027	62700	573040	97812	188859	245440
27755	42372	62160	99603	295460	654720
28083	43056	105820	100776	166257	209440
28704	128205	247940	101952	272745	684400
30080	51129	85800	106227	154660	237120
30240	77805	141284	108031	212160	289800
34452	134064	406315	108108	250800	486875
34632	66976	299145	118755	149600	455532
35409	54288	79040	119680	209385	402696
38080	47736	177177	128520	459360	564311
41360	69513	83520	131157	167440	272580
168245	495264	533052	335825	426360	753984
180180	215760	495349	336300	758043	1902160
185925	191620	1276704	352275	485316	1608880
186615	321816	983680	373175	1055808	1932840

TABLE 1—Continued

<i>X</i>	<i>Y</i>	<i>Z</i>	<i>X</i>	<i>Y</i>	<i>Z</i>
200165	277200	970596	389367	426880	1190160
201300	204336	301645	392535	2069120	2816208
204160	576375	1175328	432432	839475	1947500
208692	869856	1073995	471276	613795	979440
215072	224025	434304	514080	631533	2257244
216720	272832	839575	526680	1133088	2342359
223440	1027675	1948716	706860	1997520	3461179
230112	256360	645975	933660	1407120	1645699
234780	240669	900592	985872	1654400	2812095
242535	560120	713952	1186328	1620465	3182400
264860	562848	611325	1354815	2574528	3626896

The smallest set of integral values of x, y, z which satisfies the above equations is $x = 44, y = 117$ and $z = 240$ [2, pp. 497–502]. The aim of the present investigation is primarily to find more such simple solutions. Incidentally, if we impose an additional restriction that $x^2 + y^2 + z^2 = k^2$, then such a system has no known solutions. In the present investigation we also attempt to find such a solution.

Method. We use the algorithm of Rignaux [2, p. 501],

$$x = 2mnpq; \quad y = mn(p^2 - q^2); \quad z = pq(m^2 - n^2)$$

then $x^2 + y^2 = \square$ and $x^2 + z^2 = \square$ and we restrain the parameters m, n, p, q to satisfy $y^2 + z^2 = \square$.

The above algorithm is highly symmetric. For example, m, n as well as p, q are symmetric. Furthermore, the pair (p, q) is symmetric to the pair (m, n) . Making use of the symmetry properties, we impose the following restrictions on the parameters.

Choose a large number N . Let $1 \leq m \leq N$ and $1 \leq n < m$. Take $p \leq m$: if $p = m$, let $1 \leq q \leq n$; if $p < m$, let $1 \leq q < p$.

The total number of calculations for a given N is

$$S_N = \frac{1}{8}(N^2 - N + 2)(N^2 - N).$$

For $N = 100$,

$$S_{100} = 12, 253, 725.$$

With our present speed of calculations of approximately 15000/hour and running the IBM 1620 computer 24 hours a day, it would take more than a month to do these calculations. Because of the excessive computing time required, we have performed calculations up to $N = 70$. The results of these calculations are summarized in Table 1. Only primitive solutions for which (x, y, z) have no common factors, are given. Values of x, y, z have been interchanged where necessary to put them in ascending order.

Results. (a) In the range of parameters which we have considered, no set for (x, y, z) has been found which satisfies $x^2 + y^2 = l^2, y^2 + z^2 = m^2, z^2 + x^2 = n^2$ and $x^2 + y^2 + z^2 = k^2$.

(b) With a slightly different algorithm i.e.

$$x = P^2 - Q^2 - R^2,$$

$$y = 2PQ,$$

$$z = 2PR.$$

We find for $x = 495, y = 840, z = 448,$

$$x^2 + y^2 + z^2 = 1073^2, \quad x^2 + y^2 = 975^2, \quad z^2 + y^2 = 952^2,$$

$x^2 + z^2$ not a square.

(c) The sets (1008, 1100, 1155) and (1008, 1100, 12075) have two numbers in common.

(d) There are several sets of (x, y, z) which have one value in common e.g. (2964, 9152, 9405), (2964, 6160, 38475) and (5643, 43680, 76076), (5643, 14160, 21476).

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Some Designs for Maximal (+1, -1)-Determinant of Order $n \equiv 2 \pmod{4}$

By C. H. Yang

When $n \equiv 2 \pmod{4}$, Ehlich [1] has shown that

(i) the maximal absolute value α_n of n th order determinant with entries ± 1 satisfies

$$\alpha_n^2 \leq 4(n - 2)^{n-2}(n - 1)^2 = \mu_n,$$

(ii) matrices M_n of the maximal n th order (+1, -1)-determinant whose absolute value equals $\mu_n^{1/2}$ exist for $n \leq 38$, provided that “ $(n - 1, -1)_p = 1$ (Hilbert's symbol) for any prime p ,” which is also equivalent to “any prime factor of squarefree part of $n - 1$ is not congruent to 3 (mod 4).”

It is found that M_{42}, M_{46} also exist by Ehlich's method and such maximal matrices M_n are likely to exist for all $n \equiv 2 \pmod{4}$ if $(n - 1, -1)_p = 1$ for any prime p . This means that for $n < 200$, all such matrices are likely to be found except for $n = 22, 34, 58, 70, 78, 94, 106, 130, 134, 142, 162, 166, 178,$ and 190.

The maximal matrix M_n such that

$$M_n M_n^T = \begin{pmatrix} P & 0 \\ 0 & P \end{pmatrix}, \quad \text{where } P = \begin{pmatrix} n & & & 2 \\ & \ddots & & \\ & & \ddots & \\ & & & \ddots \\ 2 & & & & n \end{pmatrix}$$