10[F].—JOSEPH B. MUSKAT, *On Divisors of Odd Perfect Numbers*, proof of a pertinent theorem printed by a computer on 14 sheets and deposited in the UMT file.

This is the proof that includes and extends Muskat's Table 2 printed elsewhere in this issue of *Mathematics of Computation*. The entire proof, as given here, comprises approximately 650 lines, the first 44 of which are listed in Table 2.

Many questions concerning optimization would occur to interested parties. For example, in line 5 of Table 2 one finds the factors $5827 \cdot 6073$. The program selects 6073, and finds a contradiction in four lines. But suppose it chose 5827 instead? Presumably, a whole tree could be traced out, and the minimal branches then be selected. More subtle problems, involving the ordering of the primes, are also apparent. On the other hand, a quite simple redundancy is noted in the given proof. Thus, that proof examines the cases $\sigma(19^8)$, $\sigma(3^8)$, $\sigma(3^{14})$, $\sigma(3^{20})$, etc. These are not needed; the only non-redundant powers $\sigma(p^b)$ being those where $b + 1$ is a prime. The author does eliminate the cases $b = 2m + 1$, as he indicates in his text, and no doubt he knew of the existing redundancies also, but preferred not to complicate the program for the small savings possible.

D. S.

11[F, X].—A. YA. KHINTCHINE, *Continued Fractions*, Noordhoff, Groningen, 1963, Translated by P. WYNN, 101 pp., 23 cm. Price Dfl. 16.25.

12[F, X].—A. YA. KHINCHIN, *Continued Fractions*, University of Chicago, Chicago, Illinois, 1964. Translation edited by H. EAGLE, xii + 95 pp., 21 cm. Price $5.00.

Khintchine's classic is concerned with regular continued fractions, that is:

$$x = a_0 + \frac{1}{a_1 +}\,\frac{1}{a_2 +}\,\frac{1}{a_3 +} \cdots \tag{1}$$

with the a_i positive integers ($i \geqq 1$), with their use in representing real numbers, and with their "metric theory". These two translations into English appeared almost simultaneously. To compare the two, let us start with a paragraph from the first preface (1935).

Wynn writes:

"The main aim of this book is to close the gap in our literature described above; thus it is of necessity elementary and as compact as possible. By this reason its *style* is determined. Against this, its *content* slightly exceeds that which is absolutely necessary for the various applications. This is true above all with regard to the last chapter which is concerned with the fundamentals of the metric (or probability-theoretic) application of continued fractions. We are concerned here with an important new chapter which is virtually the exclusive creation of Soviet scientists. Many parts of Chapter II also go beyond the mentioned minimum, since I wished to discuss the fundamental rôle which continued fractions play in the investigation of irrational numbers, in as much detail as is possible within the framework of such an elementary introduction. If the theory of continued fractions is to be the subject of a special book, then it would seem to me futile to omit the very topics and interconnections of this theory which at the present time most occupy scientific thinkers."

Eagle writes:

"Since the basic purpose of this monograph is to fill the gap in our textbook literature, it necessarily had to be elementary and, to as great a degree as possible, accessible. Its *style* is in large measure determined by this fact. Its *content*, however, goes somewhat beyond the limits of that minimum absolutely necessary for any application. This remark applies chiefly to the entire last chapter, which contains the fundamentals of the measure (or probability) theory of continued fractions—an important new field developed almost entirely by Soviet mathematicians; it also applies to quite a number of items in the second chapter, where I attempted, to the extent possible in such an elementary framework, to emphasize the basic role of the apparatus of continued fractions in the study of the arithmetic nature of irrational numbers. I felt that if the fundamentals of the theory of continued fractions were going to be published in the form of a separate monograph, it would be a shame to leave unmentioned those highlights of the theory which are the subject of the greatest amount of contemporary study."

Aside from questions of style, and the reviewer is reluctant to indicate his preference here, one notes some differences in meaning. In all three cases: "compact" vs. "accessible", "slightly exceeds" vs. "goes somewhat beyond", "metric . . . application" vs. "measure . . . theory", the Eagle translation is surely more accurate. There are other, more serious translational errors in Wynn also, but the comparison is by no means one-sided.

For instance, consider "systematic fractions"—a term not (generally) used in English. Wynn explains it as follows (p. 25):

. . . "For this reason the calculus of continued fractions may, at least in principle, make the same claims for consideration as those which may be made on behalf of decimal fractions, or quite generally for systematic fractions (i.e. those which use any fixed radix system).

"Now what are the essential advantages and disadvantages of continued fractions as a means of representing real numbers when compared with the far more widely established systematic fractions?"

But Eagle writes (p. 19):

. . . "Therefore, the apparatus of continued fractions can, at least in principle, claim a role in the representation of real numbers similar to that, for example, of decimal or of systematic fractions (that is, fractions constructed according to some system of calculation).

"What are the basic advantages and shortcomings of continued fractions as a means of representing the real numbers in comparison with the much more widely used systematic representation?"

Here Wynn's experience in computation reveals itself, while Eagle's "according to some system" is vague, even meaningless. Thus, neither translation is perfect.

The metric theory, which is the main point of the book, is largely Khintchine's invention. It culminates in his famous theorem that for almost all x in (1) with a_0 some integer, the geometric mean of the first n elements, that is, $(a_1 a_2 \cdots a_n)^{1/n}$, tends to an absolute constant $K = 2.6 \cdots$, as $n \to \infty$. Neither translator mentions that Khintchine's value of K as given here is incorrectly rounded. This was noted by D. H. Lehmer long ago [1]. Subsequently, more accurate values of K have been given in [2], [3], and [4].

"Almost all" means, of course, except for a set of measure zero. This latter set contains, however, *all* rational numbers, *all* quadratic surds, the number e, and presumably much more. An unsolved problem with a delightfully ironic flavor is whether K itself is in the "almost all".

D. S.

1. D. H. Lehmer, "Note on an absolute constant of Khintchine," *Amer. Math. Monthly*, v. 46, 1939, pp. 148–152.
2. D. Shanks, MTE **164**, *MTAC*, v. 4, 1950, p. 28.
3. D. Shanks & J. W. Wrench, Jr., "Khintchine's constant," *Amer. Math. Monthly*, v. 66, 1959, pp. 276–279.
4. J. W. Wrench, Jr., "Further evaluation of Khintchine's constant," *Math. Comp.*, v. 14, 1960, pp. 370–371.

13[G, X].—T. L. Saaty, Editor, *Lectures on Modern Mathematics*, Volume II, John Wiley & Sons, Inc., New York, 1964, ix + 183 pp., 22 cm. Price $5.75.

14[F, X].—T. L. Saaty, Editor, *Lectures on Modern Mathematics*, Volume III, John Wiley & Sons, Inc., New York, 1965, ix + 321 pp., 22 cm. Price $11.75.

These two volumes complete the series containing the 18 lectures given under O. N. R.—George Washington University sponsorship. For detailed, general remarks concerning this series see the review of Volume I [1]. Volume II contains the six chapters:

"Partial Differential Equations with Applications in Geometry," by L. Nirenberg,

"Generators and Relations in Groups—The Burnside Problem," by Marshall Hall, Jr.,

"Some Aspects of the Topology of 3-Manifolds Related to the Poincaré Conjecture," by R. H. Bing,

"Partial Differential Equations: Problems and Uniformization in Cauchy's Problem," by Lars Gårding,

"Quasiconformal Mappings and Their Applications," by L. Ahlfors,

"Differential Topology," by J. Milnor,

and the last volume has

"Topics in Classical Analysis," by Einar Hille,

"Geometry," by H. S. M. Coxeter,

"Mathematical Logic," by Georg Kreisel,

"Some Recent Advances and Current Problems in Number Theory," by Paul Erdös,

"On Stochastic Processes," by Michel Loève,

"Random Integrals of Differential Equations," by J. Kampé de Fériet.

Detailed individual reviews are not needed or appropriate here. The chapters vary in length from the extensive, 101-page contribution of Kreisel which attempts "to make out a case that real, if modest, progress has been made on foundational problems" to the brief, 14-page lecture by Ahlfors. That latter is the only one that does not include a list of references, even though it is particularly missed there since its subject began in "a small paper" of H. Grötzsch which "was buried in a small journal." The lectures are all difficult and highly-specialized. (If there exists even a single mathematician in the entire world who can read all 18 chapters with