for the "easy" reading of the essentially self-contained treatise include a year of calculus, and, say, the first-half of either Feller's *Introduction to Probability Theory and its Applications* (Wiley, New York, 1957) or else Riordan's *Introduction to Combinatorial Analysis* (Wiley, New York, 1958).

The research monograph (it is *not* a textbook) commences with numerous preliminary examples; e.g., the well-known cases of static maximal flow, minimal cost flow, multiterminal network flow, and multicommodity network flow. That discussion leads the author into the principal concern of his investigation, namely, the behavior of connected networks subjected to stochastic flow capable of accommodating queues. His most crucial measure of performance for such nets is the average time for a message to arrive at its destination. The remainder of the text is primarily concerned with optimization of performance for various nets. For example, supposing a fixed-cost constraint, he optimizes network performance relative to a prescribed assignment of channel capacities to the branches.

The author's format is, for the most part, the concise theorem-proof style supplemented by occasional concrete examples. Further, the book is replete with figures and diagrams as heuristic aids for the reader. For those readers unfamiliar with elementary queueing theory there is an appendix which includes the results needed in the text-proper. The author even provides a discussion on the simulation of communication nets on Lincoln Laboratory's TX-2 digital computer as a means of experimentally checking his theoretical results. Finally, he includes a brief section on suggestions for further research.

Investigators and students interested in communication theory and operations research should find this well-indexed and well-documented tract of considerable use.

ALBERT A. MULLIN

Lawrence Radiation Laboratory
University of California
Livermore, California

23[L].—G. BLANCH & DONALD S. CLEMM, *Tables Relating to the Radial Mathieu Functions*, Vol. 2: *Functions of the First Kind*, U. S. Government Printing Office, Washington, D. C., 20402, 1965, xxiii + 481 pp., 27 cm. Price $4.00.

The volume under review represents the second of a two-volume set. Volume 1 was reviewed earlier in this journal (Volume 18, 1964, pp. 159–160). The functions tabulated are solutions of the equation

$$(1) \qquad \frac{d^2f}{dx^2} - (a(q) - 2q \cos 2x)f = 0, \quad \text{where } a(q) \text{ is an eigenvalue}$$

corresponding to which the related equation

$$(2) \qquad \frac{d^2g}{dx^2} + (a(q) + 2q \cos 2x)g = 0$$

has solutions of period π or 2π. The tabulated solutions depend on three parameters; namely, q, x, and the order of the eigenvalue r.

The solutions of (2) fall into four categories; namely, even or odd, and periodicity π or 2π. When the variable x is replaced by ix in these solutions they become solutions of (1). Then even solutions are denoted by $Mc_r^{(1)}(x, q)$, and the odd ones by $Ms_r^{(1)}(x, q)$. For $q = 0$ the eigenvalues of (2) are double eigenvalues. Then $Mc_r^{(1)}(x, o)$ and $Ms_r^{(1)}(x, o)$ become linearly independent solutions of the same differential equation. They are in this case hyperbolic cosines and sines respectively.

Volume 1 was devoted to the functions mentioned above and their derivatives. Volume 2 extends the range of values tabulated for these functions, and also tabulates a second linearly independent solution and its derivatives. As in Volume 1, the functions are not tabulated directly. Thus, we have, for example, $Mc_r^{(2)}(x, q) = e^{-rx}Tc_r(x, q)/rM_r(q)$.

The extraction of the factor e^{-rx} leads to a function $Tc_r(x, q)$. These functions are readily interpolable in both x and q. Necessarily, therefore, this table must be used in conjunction with a table of exponential functions. Tabulated data for these functions are provided for $x = 0(0.02)1$; $q = 0(0.05)1$; $r = 0(1)7$; 7D, and $x = 0(0.01)1$; $q = 0(0.05)1$; $r = 8(1)15$; 7D.

An auxiliary set of tables is provided for a second set of linearly independent solutions and their derivatives denoted by $Dc_r(x, q)$, $Ds_r(x, q)$, $Ec_r(x, q)$, $Es_r(x, q)$.

Tables V–XII provide data by means of which all solutions can be computed for $x = 1(0.02)2$, $\sqrt{q} = 0.5(0.02)1$, to 7D. These are tabulated in terms of a class of functions denoted by $Fc_r^{(j)}(x, q)$, $Fs_r^{(j)}(x, q)$, $Gc_r^{(j)}(x, q)$, and $Gs_r^{(j)}(x, q)$, where $j = 1, 2$. These again are so defined as to lead to smooth data but must be used in conjunction with a table of Bessel functions.

These tables in conjunction with the asymptotic formulas provided in the introduction to Volume 2, provide a thorough numerical knowledge of the solutions of equation (1).

HARRY HOCHSTADT

Polytechnic Institute of Brooklyn
Brooklyn, New York

24 [L].—ROBERT SPIRA, *Check Values, Zeros and FORTRAN Programs for the Riemann Zeta Function and its First Three Derivatives*, University Computing Center, Report No. 1, University of Tennessee, Knoxville, Tennessee. Copy deposited in the UMT file.

This report contains documented FORTRAN programs for calculating $\zeta^{(k)}(s)$, $k = 0(1)3$, and also the following tables:

(a) $\zeta(.5 + it)$, $|Z(t)|$: $t = 500, 1000, 2000$; 18D.
(b) $\zeta^{(k)}(\sigma + it)$: $k = 1(1)3$: $\sigma = -2(1)0(.5)1(1)3$,
$t = 0, 10, 50, 100, 200$ (except for pole);
also $\sigma = 0(.5)1$, $t = 500, 1000$;
Accuracy: Nearly all 10D or 10S.
(c) Zeros of $\zeta(s)$: First 30, 13D.
Zeros of $\zeta'(s)$, $\zeta''(s)$: $-1 \leqq \sigma, 0 < t \leqq 100$, 10D.
(d) $-b_\mu = B_{2\mu+2}/B_{2\mu}$: $\mu = 1(1)50$, 30D.

Cross checking was accomplished by also computing the derivatives by a differences program that is included. The zeros of $\zeta'(s)$ and $\zeta''(s)$ were obtained from previously known approximations by a Newton's method program (also included).

AUTHOR'S SUMMARY