

25[L, M].—H. E. SYRET & M. W. WILSON, *Computation of Fresnel Integrals to 28 Figures: Approximations to 8 and 20 Figures*, Computer Science Department, University of Western Ontario, London, Ontario, ms of 65 pp. + 96 pp. (un-numbered) of tables, deposited in UMT file.

The main tables in this unpublished report are those of the Fresnel integrals $S(x)$ and $C(x)$ to 28 S in floating-point form for $x = 0(0.001)2(0.01)10$.

The prefatory textual material is presented in ten chapters devoted, respectively, to definitions of the Fresnel integrals, expansions in Taylor series and in asymptotic series (with 12S and subsequently 28S tables of the first 24 coefficients of the latter), generating functions as introduced by Boersma [1], approximation by finite series of Chebyshev polynomials (with tables of coefficients to 10S and 20S), description of the main tables (including a discussion of the use of Lagrange interpolation), and a list of six references.

On p. 28 we find a brief list of pertinent constants, mostly to 28S; included are π , $\pi/2$, $(2/\pi)^{1/2}$, and $2^{1/2}/\pi$. Rather surprisingly, terminal-digit errors of a unit occur in the first and third of these.

The present tables of Fresnel tables are by far the most precise of their kind, inasmuch as previously published tables such as those by Watson [2], Pearcey [3], and Corrington [4] extend to at most 8D.

The user will observe the occasional omission of leading figures in the third group of six digits in the tabular entries. This is due to the unfortunate suppression of zeros in these places in the course of the computer editing of these data.

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1. J. BOERSMA, "Computation of Fresnel integrals," *Math. Comp.*, v. 14, 1960, p. 380.
2. G. N. WATSON, *A Treatise on the Theory of Bessel Functions*, second edition, Cambridge University Press, Cambridge, 1944, pp. 744-745.
3. T. PEARCEY, *Table of the Fresnel Integral to Six Decimal Places*, Cambridge University Press, Cambridge, 1957. (See *MTAC*, v. 11, 1957, pp. 210-211, RMT 87.)
4. M. S. CORRINGTON, *Tables of Fresnel Integrals, Modified Fresnel Integrals, the Probability Integral, and Dawson's Integral*, Radio Corporation of America, R.C.A. Victor Division. (See *MTAC*, v. 7, 1953, p. 189, UMT 166.)

26[L, X].—N. N. LEBEDEV, *Special Functions and their Applications*, translated from the Russian by RICHARD A. SILVERMAN, Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1965, xii + 308 pp., 24 cm. Price \$12.00.

A subject basic to applied mathematics and the mathematics of computation is that of special functions. The subject, unfortunately, has little academic status in the mathematical curriculum, though some topics are often covered as a by-product of courses in theoretical and mathematical physics. Thus, by far and large, it is necessary to learn the subject by self-study. The present volume is ideal for this purpose, and indeed should prove suitable for an academic text.

The book presupposes that the reader is familiar with the elements of real and complex variable theory, although the author attempts to keep the required background material to a minimum. Of course, a better understanding of the subject is also facilitated by some knowledge of differential equations and asymptotics. This material is not introduced in any systematic fashion, but is presented to the reader as the need arises in connection with certain special functions. The arrangement of the material in the various chapters is dictated by the desire to make the chapters

independent of one another as much as possible, so that one can study the simpler functions without becoming involved in more general functions. In illustration, the subject of hypergeometric functions is deferred to Chapter 9, though the transcendents of Chapters 2–8 are for the most part special cases of the generalized hypergeometric function. To reinforce the theoretical development, considerable attention is devoted to applications.

The gamma function is treated in Chapter 1. Chapters 2 and 3 take up the important special cases of the incomplete gamma function. Orthogonal polynomials and expansions in series of these functions are studied in Chapter 4. Chapters 5 and 6 are devoted to Bessel functions and their applications, while Chapters 7 and 8 take up spherical harmonics and applications. An introduction to the Gaussian hypergeometric function is given at the start of Chapter 7. As already implied, a more systematic development is presented in Chapter 9, where the confluent hypergeometric function is also taken up in some detail. Here the connection between the functions of the previous chapters and hypergeometric functions is noted, and a short introduction to generalized hypergeometric functions is given.

The exposition is clear and rigorous, and careful attention is paid to conditions of validity. This is an excellent volume. Each chapter contains a list of problems, which should facilitate use of the book as a text or for self study,

Y. L. L.

27[M, X].—E. T. COPSON, *Asymptotic Expansions*, Cambridge University Press, New York, 1965, 120 pp., 23 cm. Price \$6.00.

Certain important functions may often be represented by asymptotic series which are usually divergent. Nevertheless, the functions may be calculated to some level of accuracy by taking the sum of a suitable number of terms. In some situations, the sequence obtained by a certain weighting of the sequence of partial sums of an asymptotic series converges. Solutions of ordinary differential equations can often be expressed in the form of a definite integral or a contour integral. Thus, the subject of asymptotics is very important to both pure and applied mathematicians.

This volume gives an excellent treatment of asymptotic expansions of transcendents defined by integrals. After an introductory account of the properties of asymptotic expansions (Chapters 1 and 2), the standard methods of deriving asymptotic expansions are explained in detail and illustrated with special functions. These techniques include integration by parts (Chapter 3), the method of stationary phase (Chapter 4), Laplace's approximation (Chapter 5), Laplace's integral and Watson's lemma (Chapter 6), the method of steepest descent (Chapter 7) and the saddle-point method (Chapter 8). Chapter 9 treats Airy's integral by various methods. For the most part, the expansions discussed are not uniform. Uniform asymptotic expansions is the subject of Chapter 10.

Professor Copson's volume presupposes only a knowledge of the more elementary notions of real and complex variable theory. The subject matter is within the capabilities of undergraduate students.

The volume is very readable and suitable for self-study or as an academic textbook. In this connection, the utility of the text would have been considerably enhanced by the inclusion of exercises.

Y. L. L.