REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

64[A–E, X, Z].—L. A. LYUSTERNIK, O. A. CHERVONENKIS & A. R. YANPOL′SKII, *Handbook for Computing Elementary Functions*, translated by G. J. Tee, Pergamon Press, New York, 1965, xiii + 251 pp., 23 cm. Price $10.00.

This handbook is a translation with corrections of *Matematicheskii analiz-Vychisleniye elementarnykh funktsii*, published in Moscow in 1963. It is Volume 76 in the International Series of Monographs in Pure and Applied Mathematics.

The authors point out in the preface the previous lack of any comprehensive collection of computational formulas for the elementary functions. As examples of earlier handbooks containing such formulas they cite specifically the collection of polynomial and rational approximations developed by Hastings and his coworkers [1] and the tables of Ryshik & Gradstein [2], which contain a large amount of material on the representation of functions by power series.

Included in the introduction to the present book is a concise discussion of the several ways of representing mathematical functions; namely, power series, series of orthogonal polynomials, continued fractions, interpolation formulas, best approximations (by polynomials and otherwise), iterative sequences, and differential equations. The great impetus given the search for numerical algorithms by the advent of electronic digital computers is noted by the authors.

The body of this book is divided into four chapters, followed by two appendices and a bibliography of 81 publications, together with a supplementary list of references for the appendices.

The first chapter deals with polynomials, rational functions, and power functions. The various methods of evaluating polynomials such as those of Horner, John Todd, Y. L. Ketkov, and V. Y. Pan are discussed. For the elementary rational functions there are presented power series expansions, expansions in Chebyshev polynomials, infinite products, and iterative formulas. Corresponding to power functions we find similar information, supplemented by continued fraction expansions, Padé approximations, and approximations by linear functions. Throughout the book virtually every formula is followed by a reference to the item in the bibliography that gives its source.

The second chapter is devoted to exponential and logarithmic functions. General information is supplied on each of these functions, followed in each case by the various types of expansions and approximations considered in the preceding chapter. A minor omission noted here is the source reference of iterative process 2.7, 2° on p. 75, which should be item 74 in the bibliography. An innovation is the inclusion of rational approximations to binary logarithms.

The trigonometric and hyperbolic functions and their inverses are treated in similar detail in the third chapter. Here we also find expansions of certain of the trigonometric functions in series of elementary rational functions. The numerical values of the coefficients in the expansion of the first four trigonometric functions (and the inverse of the first three) in series of Chebyshev and Legendre polynomials are tabulated to from 11 to 24 decimal places. Similar tables are presented for the coefficients of the best polynomial approximations (in the sense of Chebyshev) to certain trigonometric functions and their inverses. This information was extracted from the publications of Hastings [1] and of succeeding workers in the field of polynomial approximation, to which specific reference is made in the bibliography.

Chapter IV consists of brief descriptions of algorithms used for computing elementary functions on several Soviet computers, namely, Strela, BESM, M-2, M-3, and Ural.

The first appendix consists of an exposition of the definitions, mathematical properties, and various expansions of the gudermannian, harmonic polynomials, the hypergeometric function, and orthogonal polynomials (including those of Legendre, Chebyshev, Laguerre, and Hermite). This appendix is concluded with the tabulation to 6D of the zeros of the following polynomials: $P_n(x)$, $n = 1(1)40$; $L_n(x)$, $n = 1(1)15$; $H_n(x)$, $n = 1(1)20$; and $h_n(x)$, $n = 1(1)22$.

The second appendix consists exclusively of pertinent mathematical tables. Table 1, entitled Coefficients of Certain Series, gives for $n = 1(1)10$: n^{-1} and $\sum_{k=1}^{n} k^{-1}$ to 5D; exact values of $n!$, $(2n - 1)!!$, $(2n)!!$ and their reciprocals to from 5 to 11D; $n!/(2n - 1)!!$, $2^n n!/(2n + 1)!!$, $(2n - 1)!!/2^n n!$, all to 5D; $(2n - 1)!!/2^n n!(2n + 1)$, 6–7D; $(2n - 1)!!/2^{n+1}(n + 1)!$, 5–7D; and $(2n - 1)!!/2^{n+1}(n + 1)! \cdot (2n + 3)$, 6–8D. Table 2 gives to at most 8S the binomial coefficients $\binom{n}{m}$ for $n = 1(1)50$, while Table 3 gives the exact values of these coefficients $\binom{\nu}{m}$ for $m = 1(1)6$, $\pm\nu = \frac{1}{2}(1)\frac{7}{2}, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{1}{4}(\frac{1}{2})\frac{5}{4}$, and $-\nu = 1(1)5$. Table 4 gives sums $\sum_{k=1}^{n} k^r$ for $r = 1(1)5$, $n = 1(1)50$. In Table 5 the gudermannian, $\text{gd}(x)$, is given to 5D for $x = 0(0.01)5.99$ and to 6D for $x = 6(0.01)9$. The inverse gudermannian $\arg \text{gd}(x)$ is given in Table 6 to 5D for $x = 0(0.01)1.57$, and to 3D for $x = 1.47 \cdot (0.001)1.57$. Table 7 consists of 4D values of $P_n(x)$ for $n = 2(1)7$, $x = 0(0.01)1$. The normalized Laguerre polynomials $(1/n!)L_n(x)$ are tabulated to 4D in Table 8, corresponding to $n = 2(1)7$, $x = 0(0.1)10(0.2)20$. Finally, in Table 9 there are listed 4D values of the Hermite polynomials $(-1)^n h_n(x)$ for $n = 2(1)6$, $x = 0(0.01)4$.

In summary, this handbook constitutes the most complete compilation of formulas extant for the computation of the elementary mathematical functions, attractively arranged in a very convenient and accessible form. It can be recommended as a valuable accession to the libraries of all individuals and laboratories whose work involves numerical mathematics.

J. W. W.

1. C. Hastings, J. T. Hayward & J. P. Wong, *Approximations for Digital Computers*, Princeton Univ. Press, Princeton, N. J., 1955. See *MTAC*, v. 9, 1955, pp. 121–123, RMT **56**.

2. I. M. Ryshik & I. S. Gradstein, *Tables of Series, Products, and Integrals*, VEB Deutscher Verlag der Wissenschaften, Berlin, 1957. See *Math. Comp.*, v. 14, 1960, pp. 381–382, RMT **69**.

65[C, D].—L. A. Lyusternik, Editor, *Ten-Decimal Tables of the Logarithms of Complex Numbers and for the Transformation from Cartesian to Polar Coordinates*, Pergamon Press, New York, 1965, ix + 110 pp., 25 cm. Price $7.50.

This set of tables, constituting Volume 33 of the Mathematical Tables Series of Pergamon Press, is a reprint, with a translation by D. E. Brown, of *Desi͡atiznachnye tabli͡tsy logarifmov kompleksnykh chisel i perekhoda ot Dekartovykh koordinat k*