number of primes in the above expression for h_N. It should be borne in mind that the approximant vanishes at the endpoints of the interval $[0, \pi]$; consequently if the approximant does not have this property, we should modify it accordingly; this may involve subtracting a linear trend as suggested in similar circumstances by Lanczos [3, p. 236].

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1. P. J. Davis, *Interpolation and Approximation*, Blaisdell, New York, 1963. MR **28** #393.
2. I. P. Natanson, *Konstrucktive Funktionentheorie*, Akademie-Verlag, Berlin, 1955. MR **16**, 1100.
3. C. Lanczos, *Applied Analysis*, Prentice-Hall, Englewood Cliffs, N. J., 1956. MR **18**, 823.

A Note on Best Approximation in E^n†

By J. T. Day

Let D be a closed convex set with positive volume V in Euclidean n-dimensional space. Let f be a nonnegative function of class C^2 on D (see [2]), and Q be a linear polynomial on D, i.e.

$$Q(x) = a_0 + a_1x_1 + a_2x_2 + \cdots + a_nx_n, \qquad x \in D.$$

We consider the problem of "best" one sided approximation of f by Q in the sense that among all linear functions $Q(x)$ satisfying

$$(1) \qquad Q(x) \leqq f(x), \qquad x \in D,$$

we are looking for that one which maximizes $\int_D Q\,dx$.

Theorem 1. *The problem under consideration has a unique solution given by the tangent plane through the centroid p of D, provided that the eigenvalues of the Hessian matrix $(f_{ij}(x))$, $x \in D$, are nonnegative.*

The proof is by construction. Let the centroid p of D have cartesian coordinates $(p_1, p_2, \cdots, p_n)$. Then

$$(2) \qquad \int_D Q\,dx = V \cdot Q(p_1, p_2, \cdots, p_n)$$

for all linear polynomials Q (see [3]). Since $Q(p) \leqq f(p)$, we choose $Q^*(p) = f(p)$. Choose $Q_1^*(p) = f_1(p)$, $Q_2^*(p) = f_2(p), \cdots, Q_n^*(p) = f_n(p)$. Here $f_1(x) = (\partial f/\partial x_1)(x)$, etc. The above conditions determine $Q^*(x)$.

By Taylor's theorem we have $f(x) = Q^*(x) + R(x, p)$. The remainder $R(x, p)$ is nonnegative, since the eigenvalues of the Hessian matrix are nonnegative (see [2]). Thus $f(x) \geqq Q^*(x)$. We conclude that $Q^*(x)$ is a "best" approximate.

Suppose there were another "best" approximate $T(x)$. Then $T(p)$ must equal $f(p)$. Consider a point $x = (x_1, p_2, \cdots, p_n)$ where $x_1 > p_1$. By Taylor's theorem we have

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(3) $f(x) = f(p) + f_1(p)(x_1 - p_1) + f_{11}(p_1 + sh, p_2, \cdots, p_n)(x_1 - p_1)^2/2.$

Here $h = x_1 - p_1$, $0 < s < 1$.

$$T(x) = f(p) + T_1(p)(x_1 - p_1). \tag{4}$$

Since $f(x) \geqq T(x)$, we find that

$$f_1(p) - T_1(p) + f_{11}(p_1 + sh, p_2, \cdots, p_n)(x_1 - p_1)/2 \geqq 0. \tag{5}$$

The quantity $f_1(p) - T_1(p)$ must be nonnegative, for otherwise we could choose $(x_1 - p_1)$ so small that (5) could not hold. (We note here $f_{11}(x) \geqq 0$ for $x \in D$ by hypothesis.) A similar consideration in the case where $p_1 > x_1$ shows that $f_1(p) - T_1(p) \leqq 0$. Hence $f_1(p) = T_1(p)$. In the same manner one can show that $f_i(p) = T_i(p)$, $i = 2, \cdots, n$. Thus $Q^*(x)$ and $T(x)$ are identical.

The idea for this note occurred to the author after hearing a lecture by Prof. Ranko Bojanic [1] on "best" one sided approximation in the case of functions of one variable.

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1. R. Bojanic, "On polynomials of best one sided approximation." (To appear.)
2. W. H. Fleming, *Functions of Several Variables*, Addison-Wesley, Reading, Mass., 1965. MR **30** #4875.
3. A. H. Stroud, "Quadrature methods for functions of more than one variable," *Ann. New York Acad. Sci.*, v. 86, 1960, p. 784. MR **22** #10179.

A Close Approximation Related to the Error Function*

By Roger G. Hart

A function has been found that closely approximates the integral function

$$F(x) = \int_x^\infty \exp(-t^2/2)\, dt$$

for all real values of x.

Let

$$P(x) = \frac{\exp(-x^2/2)}{x}\left[1 - \frac{(1 + bx^2)^{1/2}/(1 + ax^2)}{P_0 x + [P_0^2 x^2 + \exp(-x^2/2)(1 + bx^2)^{1/2}/(1 + ax^2)]^{1/2}}\right]$$

$$\equiv P_0 + x^{-1}\{\exp(-x^2/2) - [P_0^2 x^2 + \exp(-x^2/2)(1 + bx^2)^{1/2}/(1 + ax^2)]^{1/2}\},$$

where $P_0 = (\pi/2)^{1/2} \cong 1.253314137$,

$$a = \frac{1 + (1 - 2\pi^2 + 6\pi)^{1/2}}{2\pi} \cong .212023887,$$

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