

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

85[A, J, L, M].—I. S. GRADSHTEYN & I. M. RYZHIK, *Table of Integrals, Series, and Products*, Academic Press, New York, 1965, xiv + 1086 pp., 23 cm. Price \$10.50. (Translated from the 4th Russian edition by Scripta Technica and Alan Jeffrey. The 4th Russian edition had actually been prepared by Yu. V. Geronimus and M. Yu. Tseytlin after the deaths of both original authors.)

This is the first American edition. It is “translated” from the much enlarged, fourth Russian edition—that is, the text is translated, while the thousands of mathematical formulas are reproduced photographically. For detailed reviews of earlier editions, and for table errata listed in this journal, see *MTAC*, v. 1, 1945, RMT **219**, pp. 442–443; *Math. Comp.*, v. 14, 1960, RMT **69**, pp. 381–382, and MTE **293**, pp. 401–403; *Math. Comp.*, v. 17, 1963, MTE **326**, p. 102; *Math. Comp.*, v. 20, 1966, MTE **392**, p. 468. See also, if you wish, the numerous notices in *Math. Reviews*: MR **14**, p. 643; MR **22**, #3120; MR **28**, #1326; MR **28**, #5198; and MR **30**, #5458.

This fourth edition has “more than twice as many formulas as any of the previous editions” and is advertised as “the most comprehensive table of integrals ever published.” The main increases over the third edition have been in the tables of definite integrals of elementary functions (four times as long), and of special functions (ten times as long). But the chapter entitled Indefinite Integrals of Elementary Functions has also been doubled, and new material on special functions, e.g., Mathieu, Struve, Lommel, etc. has been added. On the other hand, some numerical tables in the third edition have been dropped, namely Lobachevskiy’s function, values of $\zeta(n)$, and numerical coefficients involving factorials.

Because of its inclusiveness, one is tempted to refer to this volume as the definitive reference book of its type, but unfortunately it is flawed in several directions: mediocre printing, imperfect translation, and persistence in repeating errors that had been pointed out long ago.

Photographic reprinting is not bad if the proper care is taken, but some pages here, e.g., pp. 1065, 1073, are actually shoddy. (As an aside, when so much of the volume is photographic, and no authorization by the Russian authors is indicated, the reviewer is curious about the legal status of the copyright, and particularly the warning printed there: “No part of this book may be reproduced in any form, by photostat, . . .”).

Translation of a table of integrals might seem to make minimal demands upon a translator, but some crude errors are found here. On p. 909 elliptic functions are called “rational” instead of “meromorphic,” and are falsely stated to have “no more than two simple and one second-order pole in such a parallelogram.” Statement 6 there is not expressed clearly, and in Statement 8 the stipulation of non-constancy is omitted. On p. 933, $\Gamma(z)$ is a “fractional” analytic function, and on p. 1074 it is alleged that an “uncountable” set of zeros of $\zeta(z)$ have been proven to have real part $\frac{1}{2}$.

Of the errors indicated in the MTE **293** mentioned above, about one-half of them remain, although now on different pages. Specifically, still uncorrected are those

errors in MTE **293** previously on pp. 2, 24, 149, 186, 274, 301, 303 together with the erroneous values of Euler's constant and B_{34} .

Particularly charming, and in a way a lesson to us all, is the erroneous

$$\prod_{k=1}^{\infty} \Gamma\left(\frac{k}{3}\right) = \frac{640}{3^6} \left(\frac{\pi}{\sqrt{3}}\right)^3$$

on p. 938. The upper limit of the product should be 8, not ∞ . The persistence of this error should be an inspiration to everyone. For many years it continued as a misprint in Whittaker and Watson, and though it was finally corrected there, and referred to in MTE **293**, it has managed to elude the combined scrutiny of Ryzhik, Gradshteyn, Geronimus, Tseytlin, Lapko, Scripta Technica, and Jeffrey, and that, in spite of the fact that it is so blatantly false that no mathematician examining it with even casual attention should fail to note that an error is present.

In summary, then, we have a mass of useful information here, but the editing was not of that quality which it deserved.

D. S.

86[F].—CARL FRIEDRICH GAUSS, *Disquisitiones Arithmeticae*, Yale University, New Haven, Connecticut, 1966. Translated into English by Arthur A. Clarke, S.J., xx + 472 pp., 24 cm. Price \$12.50 (paperback \$2.95).

Several years ago [1], the reviewer had occasion to emphasize that Gauss's *Disquisitiones* was still not available in English. At the suggestion of Dr. Herman Goldstine, Professor Arthur A. Clarke, S.J., now offers us a translation, and thus somewhat rectifies this 165-year-old anomaly. For this, English-speaking mathematicians will be somewhat grateful. We say only "somewhat," however, since the translation has unfortunately many defects: peculiar and inaccurate terminology, awkward and undesirable notation, some serious typographical errors, and frequent confusing and inadequate translations. Of course, these are serious charges—which must therefore be documented. Here are some samples.

On p. 168, instead of *convergent fractions*, we find first *approaching fractions*, and two lines later, *approximating fractions*. On p. 342, *trigonal* numbers replace the usual *triangular* numbers, and on p. 360 we find *middle* determinants instead of *mean* determinants. Many similar peculiarities exist.

On p. 240, we find *equation* (I) referring to four equations; on p. 360 *only one class* means *only one genus*; and on pp. 373–374 one finds two examples with inexplicably contradictory terminology: the first, which (correctly) has *four positive genera*, is immediately followed by the second with *eight positive categories*.

For awkward symbolism see f'''''' on p. 162, the undisplayed (I): (1), (3), (5), $\dots L \dots$ on p. 170, etc. Unlike Gauss, and (all?) modern writers: Mathews, Dickson, Cohn, etc., Clarke (p. 265) uses + instead of \times to represent the operation on classes called *composition*, and thus, for example, he writes $2K$ instead of K^2 for the *duplication* of a class. This is not only historically wrong, and at variance with customary usage, and in contradiction to earlier symbolism on, say, p. 258, where F is transformable into ff' , but it is intrinsically wrong, since, again on p. 258, if a is represented by f and a' by f' then their *product* aa' is represented by the composition class F . Further, this unfortunate symbolism destroys the artistry of Gauss's