and

$$\Theta_n(w, z) = \sum_{m=0}^{\infty} (z/w)^{n+2m} I_{n+2m}(z),$$

where $I_n(z)$ is the modified Bessel function of the first kind of order n, for the following ranges: $w = 0.1(0.1)1$, $z = 0.1(0.1)1$ for $Y_1, Y_2, \Theta_0, \Theta_1$; $w = 1(1)z$, $z = 2(1)20$ for Y_1, Y_2; $w = 2(1)20$, $z = 1(1)w$ for Θ_0, Θ_1.

Lommel's functions of two variables are usually represented by the symbols $U_n(w, z)$ and $V_n(w, z)$; these are related to the above functions by the formulas $Y_n(w, z) = i^{-n} U_n(iw, iz)$ and $\Theta_n(w, z) = i^{-n} V_n(iw, iz)$.

Tables of U_n and V_n have been calculated by Dekanosidze [1] and Boersma [2].

Y. L. L.

1. E. N. Dekanosidze, *Tablit͡sy t͡silindricheskikh funkt͡siĭ ot dvukh peremennykh* (*Tables of cylinder functions*), Acad. Sci. USSR, Moscow, 1956. (See *MTAC*, v. 12, 1958, pp. 239–240, RMT **107**.) English translation published by Pergamon Press, New York, 1960. (See *Math. Comp.*, v. 16, 1962, p. 383, RMT **36**.)
2. J. Boersma, "On the computation of Lommel's functions of two variables," *Math. Comp.*, v. 16, 1962, pp. 232–238.

97[L, M].—Rory Thompson, *Table of* $I_n(b) = (2/\pi) \int_0^\infty ((\sin x)/x)^n \cos bx\, dx$, ms. of 26 computer sheets deposited in the UMT file.

The integral in the title is tabulated to 8D for $n = 3(1)100$, $b = 0(0.1)9$. Previous tables [1], [2] have been limited to the case $b = 0$. The method used in computing the present tables has been described by the author in [3].

In a marginal handwritten note the author notes 12 rounding errors detected by a comparison with the earlier tables, which extended to 10D. The presence of other rounding errors in this table is alluded to by the author; some of these are obvious among the early entries.

Apparently no attempt was made to edit the computer output constituting this table; for example, the fact that $I_n(b) = 0$ for $b \geqq n$ could have been used to reduce the number of entries shown for $n \leqq 8$. Furthermore, the obvious rounding errors referred to could have been removed in an improved copy.

Despite these flaws, this table is a valuable extension of the earlier, related tables.

A FORTRAN listing of the program used in the calculations is included.

J. W. W.

1. K. Harumi, S. Katsura & J. W. Wrench, Jr., "Values of $(2/\pi) \int_0^\infty ((\sin t)/t)^n dt$," *Math. Comp.*, v. 14, 1960, p. 379.
2. R. G. Medhurst & J. H. Roberts, "Evaluation of the integral $I_n(b) = (2/\pi) \int_0^\infty ((\sin x)/x)^n \cos (bx)\, dx$," *Math. Comp.*, v. 19, 1965, pp. 113–117.
3. Rory Thompson, "Evaluation of $I_n(b) = (2/\pi) \int_0^\infty ((\sin x)/x)^n \cos (bx)\, dx$ and of similar integrals", *Math. Comp.*, v. 20, 1966, pp. 330–332.

98[L, M].—Shigetoshi Katsura, Yuji Inoue, Seiji Hamashita & J. E. Kilpatrick, *Tables of Integrals of Threefold and Fourfold Products of Associated Legendre Functions*, The Technology Reports of the Tôhoku University, v. 30, 1965, pp. 93–164.

These extensive tables list the values, to accuracies varying from 11 to 15 significant figures, of the integrals of the product of three and four normalized associated Legendre functions. The integral of the product of three functions, $\Theta(l_1, m_1) \cdot \Theta(l_2, m_2)\Theta(l_3, m_3)$, is computed for all values of the l's from 0 to 8, and all permitted values of the m's satisfying the condition $m_1 + m_2 + m_3 = 0$, which is the condition for the nonvanishing of the integral of the product of three spherical harmonics. Similarly, the integrals of the product of four functions are tabulated for all values of the l's from 0 to 4 and for m's satisfying the condition $m_1 + m_2 + m_3 + m_4 = 0$.

The computations were carried out on a large-scale digital computer. It is stated that the Gaunt formula, which gives the integral of the threefold product in closed form, was not suitable for programming. Rather, the integrals were obtained in the following straightforward manner: The Legendre polynomials $P_l(x)$ are generated by means of the recursion formula. The associated polynomials $P_l^m(x)$ are obtained by m-fold differentiation. Then three or four of these polynomials, $(1 - x^2)$ to the appropriate power, and the normalizing factors are multiplied to obtain the integrand; the indefinite integral is taken and the limits $x = -1$ and $+1$ are substituted.

To the reviewer's knowledge, these are the first extensive tables of their kind. Previous computations involving the integral of three associated Legendre functions have tabulated either the Clebsch-Gordan coefficients or the $3j$-symbols, which are these integrals multiplied by certain other factors.

George Shortley

Booz, Allen Applied Research Inc.
Bethesda, Maryland

99[L, M].—Henry E. Fettis & James C. Caslin, *Elliptic Integral of the First Kind* and *Elliptic Integral of the Second Kind*, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, February 1966. Two mss., each of 36 computer sheets, deposited in the UMT File.

These companion tables consist of 10D values (without differences) of the elliptic integrals of the first and second kinds, respectively, in Legendre's form, namely, $F(\theta, k)$ and $E(\theta, k)$, for $\theta = 0°(1°)90°$ and $\arcsin k = 0°(1°)90°$.

The authors have informed this reviewer that essentially the same subroutine was used in computing these tables on an IBM 1620 with 16-digit arithmetic as was employed in the computation of their published tables [1] of elliptic integrals, wherein the modulus serves as one argument, rather than the modular angle as in the present tables.

It is interesting to note that these new tables in range and precision resemble rather closely the celebrated 9–10D tables of Legendre [2], [3], which may now be considered superseded after more than a century.

J. W. W.

1. Henry E. Fettis & James C. Caslin, *Tables of Elliptic Integrals of the First, Second and Third Kind*, Report ARL 64-232, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, December 1964. (See *Math. Comp.*, v. 19, 1965, p. 509, RMT **81**.)
2. A. M. Legendre, *Exercises de Calcul Intégral*, v. 3, Paris, 1816.
3. A. M. Legendre, *Traité des Fonctions Elliptiques et des Intégrales Eulériennes*, v. 2, Paris, 1826. A facsimile reproduction of Tables II and VIII therein appears in K. Pearson, *Tables of the Complete and Incomplete Elliptic Integrals*, reissued from Tome II of Legendre's *Traité des Fonctions Elliptiques*, London, 1934. (See also Alan Fletcher, "Guide to tables of elliptic functions," *MTAC*, v. 3, 1948, pp. 229–281.)