

100[L, M].—HENRY E. FETTIS & JAMES C. CASLIN, *Table of the Jacobian Zeta Function*, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio. Ms. of 36 computer sheets deposited in the UMT File.

Using the notation of Byrd & Friedman [1], the authors tabulate 10D values of the product $K(k)Z(\beta, k)$ of the complete elliptic integral of the first kind and the Jacobian zeta function, for $\beta = 0^\circ(1^\circ)90^\circ$ and $\arcsin k = 1^\circ(1^\circ)89^\circ$. No provision is made for interpolation; indeed, interpolation to the full precision of the table is not generally feasible because of the large number of successive differences that would be required in both arguments.

By means of the well-known relation $K(k)Z(\beta, k) = K(k)E(\beta, k) - E(k)F(\beta, k)$, the tabular entries before rounding to 10D were derived from values of both complete and incomplete elliptic integrals of the first and second kinds that were initially calculated to about 16S by use of double-precision arithmetic. (See the preceding review.)

Errors detected in the corresponding 6D table in [1], as the result of the present authors' comparison thereof with their table, are listed separately in this issue.

J. W. W.

1. P. F. BYRD & M. D. FRIEDMAN, *Handbook of Elliptic Integrals for Engineers and Physicists*, Springer-Verlag, Berlin, 1954.

101[L, M].—HENRY E. FETTIS & JAMES C. CASLIN, *Heuman Lambda Function*, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio. Ms. of 36 computer sheets deposited in UMT File.

The Heuman lambda function, usually designated $\Lambda_0(\alpha, \beta)$, is the product of $2/\pi$ and the complete elliptic integral of the third kind, namely

$$\int_0^{\pi/2} \frac{1}{1 - p \sin^2 \phi} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}},$$

in the circular case, $k^2 < p < 1$. The variable α is the modular angle (so that $k = \sin \alpha$) and the variable β is defined implicitly by the relation

$$p = \sin^2 \alpha / (\sin^2 \alpha + \cos^2 \alpha \cos^2 \beta).$$

The authors adopt the notation $\Lambda_0(\theta, k)$ in the present 10D table; thus, the range of parameters can be expressed as $\theta = 0^\circ(1^\circ)90^\circ$ and $\arcsin k = 0^\circ(1^\circ)90^\circ$. No differences are provided.

These tabulated values of the lambda function were derived from the computer data underlying the authors' 10D manuscript tables of the elliptic integrals of the first and second kinds by means of the relation

$$\Lambda_0(\theta, k) = (2/\pi)\{E(k) \cdot F(\theta, k') + K(k) \cdot [E(\theta, k') - F(\theta, k')]\}$$

where k' represents the complementary modulus.

This table may be considered a valuable extension of the original 6D table of Heuman [1], which has been abridged in Byrd & Friedman [2].

J. W. W.

1. C. HEUMAN, "Tables of complete elliptic integrals," *J. Math. and Phys.*, v. 20, 1941, pp. 127-206, 336.

2. P. F. BYRD & M. D. FRIEDMAN, *Handbook of Elliptic Integrals for Engineers and Physicists*, Springer, Berlin, 1954, p. 344.

102[L, S].—OTTO EMERSLEBEN, "Über lineare Beziehungen zwischen Madelungkonstanten", *Izvestiia na Matematicheskiia Institut B'lgarska Akademiia na Naukite*, Sofia, v. 2, 1957, pp. 87-120.

The electrostatic energy of a crystal structure, assumed to consist of positive and negative point charges $\pm e$, is proportional to e^2/a , where a is a characteristic length, e.g., for cubic crystals the edge of the smallest cube of periodicity (the "cell"), or the smallest distance between charges. In the expression of this energy, referred to the cell or to a neutral group of charges, the "molecule", the factor of e^2/a is called the Madelung Constant (here denoted by ψ). Its value is given by a lattice sum over the reciprocal distances of atoms in the structure; it depends on the structure, and, in a trivial way, on the normalizations mentioned above.

Beginning with his thesis with M. Born (1922) the author has used a specialized brand of Epstein's zeta functions for expressing the constant, namely

$$Z \begin{vmatrix} 000 \\ xyz \end{vmatrix} (1)_\delta = \Pi(x, y, z),$$

a function which Born interpreted as the "Grundpotential." Thus

$$\psi(\text{NaCl}) = \frac{e^2}{a} \Pi\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$$

and

$$\psi(\text{CsCl}) = \frac{e^2}{a} \left[\frac{3}{4} \Pi\left(0, 0, \frac{1}{2}\right) + \frac{1}{4} \Pi\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \right], \text{ etc.}$$

Since, in a number of simple cubic structures the atomic positions are fixed by the intersections of symmetry elements, only multiples of $a/8$ occur as the atomic coordinates, and only a small number of arguments of Π occur in the expressions of ψ for these structures. This leads to linear relations between the ψ , as has been discussed in an exhaustive way by Pinhas Naor in *Zs. f. Krist.*, v. 110, 1958, pp. 112-126. In the present paper the author deduces the relations

$$2\psi(\text{ZnS}) = 2\psi(\text{CaF}_2) + \psi(\text{NaCl}) = 2[2\psi(\text{CsCl}) + \psi(\text{NaCl})]$$

and

$$2\psi(\text{CaF}_2) = 4\psi(\text{CsCl}) + \psi(\text{NaCl}) = \psi(\text{ZnS}) + 2\psi(\text{CsCl}).$$

Numerical calculations have previously been carried out by Emersleben for $\psi(\text{NaCl})$ to 15 decimals, for a value of order 1; now $\psi(\text{CsCl})$ is added to the same accuracy. The calculation is done in two parts in the way first used by this reviewer. The exponentials were first taken from the tables of Hayashi [1] and later from 17D tables of $\exp(-n\pi/20)$, $0 \leq n \leq 200$, prepared in the Department of Applied Mathematics of the University of Greifswald. The error function was taken from the author's table in [2].