1. C. Heuman, "Tables of complete elliptic integrals," *J. Math. and Phys.*, v. 20, 1941, pp. 127–206, 336.
2. P. F. Byrd & M. D. Friedman, *Handbook of Elliptic Integrals for Engineers and Physicists*, Springer, Berlin, 1954, p. 344.

102[L, S].—Otto Emersleben, "Über lineare Beziehungen zwischen Madelung-konstanten", *Izvestiia na Matematicheskiia Institut B"lgarska Akademiia na Naukite*, Sofia, v. 2, 1957, pp. 87–120.

The electrostatic energy of a crystal structure, assumed to consist of positive and negative point charges $\pm e$, is proportional to e^2/a, where a is a characteristic length, e.g., for cubic crystals the edge of the smallest cube of periodicity (the "cell"), or the smallest distance between charges. In the expression of this energy, referred to the cell or to a neutral group of charges, the "molecule", the factor of e^2/a is called the Madelung Constant (here denoted by ψ). Its value is given by a lattice sum over the reciprocal distances of atoms in the structure; it depends on the structure, and, in a trivial way, on the normalizations mentioned above.

Beginning with his thesis with M. Born (1922) the author has used a specialized brand of Epstein's zeta functions for expressing the constant, namely

$$Z \begin{vmatrix} 000 \\ xyz \end{vmatrix} (1)_{\delta} = \Pi(x, y, z),$$

a function which Born interpreted as the "Grundpotential." Thus

$$\psi(\mathrm{NaCl}) = \frac{e^2}{a} \Pi(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2})$$

and

$$\psi(\mathrm{CsCl}) = \frac{e^2}{a} [\tfrac{3}{4}\Pi(0, 0, \tfrac{1}{2}) + \tfrac{1}{4}\Pi(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2})], \quad \mathrm{etc.}$$

Since, in a number of simple cubic structures the atomic positions are fixed by the intersections of symmetry elements, only multiples of $a/8$ occur as the atomic coordinates, and only a small number of arguments of Π occur in the expressions of ψ for these structures. This leads to linear relations between the ψ, as has been discussed in an exhaustive way by Pinhas Naor in *Zs. f. Krist.*, v. 110, 1958, pp. 112–126. In the present paper the author deduces the relations

$$2\psi(\mathrm{ZnS}) = 2\psi(\mathrm{CaF_2}) + \psi(\mathrm{NaCl}) = 2[2\psi(\mathrm{CsCl}) + \psi(\mathrm{NaCl})]$$

and

$$2\psi(\mathrm{CaF_2}) = 4\psi(\mathrm{CsCl}) + \psi(\mathrm{NaCl}) = \psi(\mathrm{ZnS}) + 2\psi(\mathrm{CsCl}).$$

Numerical calculations have previously been carried out by Emersleben for $\psi(\mathrm{NaCl})$ to 15 decimals, for a value of order 1; now $\psi(\mathrm{CsCl})$ is added to the same accuracy. The calculation is done in two parts in the way first used by this reviewer. The exponentials were first taken from the tables of Hayashi [1] and later from 17D tables of exp $(-n\pi/20)$, $0 \leq n \leq 200$, prepared in the Department of Applied Mathematics of the University of Greifswald. The error function was taken from the author's table in [2].

In Table 1 the Madelung constants for NaCl, CsCl, ZnS, CaF$_2$ are listed to better than 12 decimals with different normalizations.

P. P. EWALD

19 Fordyce Road
New Milford, Connecticut

1. K. HAYASHI, *Tafeln der Besselschen, Theta-, Kugel-, und anderer Funktionen*, Springer, Berlin, 1930.
2. O. EMERSLEBEN, "Numerische Werte des Fehlerintegrals für $\sqrt{n\pi}$," *Z. Angew. Math. Mech.*, v. 31, 1951, pp. 393–394.
EDITORIAL NOTE: For reviews of earlier, related papers by the same author, see *MTAC*, v. 5, 1951, pp. 77–78, RMT **871**; v. 11, 1957, pp. 109–110, RMT **56**; *ibid.*, p. 113–114, RMT **65**.

103[L, X].—LUCY JOAN SLATER, *Generalized Hypergeometric Functions*, Cambridge University Press, New York, 1966, xiii + 273 pp., 24 cm. Price $13.50.

The author of this valuable book remarks in the preface that the volume should really be attributed to both the late W. N. Bailey and Miss Slater. It was Professor Bailey's intention to write a comprehensive book on hypergeometric functions with the assistance of Miss Slater. The present work is dedicated to the memory of W. N. Bailey and is based in part on notes for a series of lectures which he gave during the years 1947–1950.

The ordinary hypergeometric or Gauss series is

$$\sum_{k=0}^{\infty} \frac{(a)_k (b)_k z^k}{(c)_k k!}, \qquad |z| < 1, \qquad (a)_k = \Gamma(a + k)/\Gamma(a),$$

and is usually represented by the symbol $_2F_1(a, b, c; z)$. A natural generalization is a series like the above but with an arbitrary number of numerator and denominator parameters. Thus (formally at least)

$$_pF_q\left(\begin{matrix} a_1, a_2, \cdots, a_p \\ b_1, b_2, \cdots, b_q \end{matrix} \middle| z\right) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_p)_k z^k}{(b_1)_k (b_2)_k \cdots (b_q)_k k!}.$$

The latter is called a generalized Gauss function or hypergeometric function, and where no confusion can result, is simply called a $_pF_q$. If in the above $_pF_q$, a numerator parameter is set to unity and the summation index sums from $-\infty$ to ∞, then such a series is called a bilateral series. It obviously can be expressed as a combination of two generalized hypergeometric functions.

The $_pF_q$ may be generalized by considering an obvious generalization of the double series

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(a)_{m+n} (b)_m (b')_m}{(c)_{m+n} m! n!} x^m y^m$$

to an arbitrary number of numerator and denominator parameters. These are known as Appell series or functions. We can also have triple, quadruple, or multiple sums which are known as Lauricella functions.

The theory of the $_pF_q$ is fundamental in the applications since many of the special functions (Bessel functions, Legendre functions, etc.) are special cases. Appell and Lauricella functions also appear in the applications.

A different view of the problem of generalizing the Gauss function was adopted