105[P, X].—SOLOMON LEFSCHETZ, *Stability of Nonlinear Control Systems*, Academic Press, New York, 1965, xi + 150 pp. Price $7.50.

This monograph gives an up-to-date, essentially self-contained, treatment of the problem of absolute stability of closed-loop control systems. It discusses in detail the sufficient conditions based on the construction of suitable Lyapunov functions as well as Popov's sufficient conditions utilizing Fourier-transform techniques. It is the first such monograph written outside the Soviet Union, by an author who has been for more than two decades the main force in making Lyapunov's approach to stability theory widely known and used in this country.

The book consists of nine chapters and two appendices. The first chapter is introductory, the last contains supplements on Jordan canonical forms, Lyapunov's matrix equation and the basic stability theorems. The construction of a Lyapunov function is discussed in Chapters 2, 3 for the case of indirect controls, in Chapter 4 for direct controls and in Chapter 5 for systems represented by a single equation. Discontinuous characteristics are briefly taken up in Chapter 6. The theorems of Popov are stated and proved in Chapter 7, and compared with the preceding results. Chapter 8 concerns essentially a strengthened version of a lemma of Yacubovich from which a somewhat less general necessary and sufficient condition than Kalman's is deduced very simply. The appendices give an application of multiple feedback control and an example from the theory of nuclear power reactors. The book closes with a bibliography of the most important papers and texts on the subject.

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106[P, X].—C. T. LEONDES, Editor, *Advances in Control Systems*, Vol. 2, Academic Press, New York, 1965, x + 313 pp., 24 cm. Price $13.00.

The present volume consists of five contributions.

The first article, "The Generation of Lyapunov Functions" by D. G. Schultz summarizes the methods of constructing Lyapunov functions for autonomous systems as proposed by Aizerman, Szegö, the author and Gibson. The usefulness and relative merits of these methods are discussed in detail and illustrated by numerous examples. A brief discussion of generating Lyapunov functions for nonautonomous systems is also included.

The paper by F. T. Smith, entitled "The Application of Dynamic Programming to Satellite Intercept and Rendezvous Problems," discusses the use of dynamic programming in synthesizing the optimal control and optimal estimation problem. A wealth of numerical data is presented for comparison purposes.

The article "Synthesis of Adaptive Control Systems by Function Space Methods" by the late H. C. Hsieh describes various control problems such as final value and minimum effort problems in the setting of functional analysis. It includes a general discussion of the minimization problem in Hilbert space, the steepest-descent method and its variants, and the least square estimation problem.

The paper "Singular Solutions in Problems of Optimal Control" by C. D. Johnson is essentially concerned with the solution of two-point boundary value problems for systems of ordinary differential equations containing a discontinuity of relay type, as they arise in control theory. A good number of examples is discussed, illustrating the construction of singular trajectories.

The final article "Several Applications of the Direct Method of Liapunov" by R. A. Nesbit describes in terms of Lyapunov functions and linear bounds various sufficient conditions for the stability of an equilibrium point of a nonlinear system of ordinary differential equations. Numerical examples are presented to illustrate the usefulness of these bounds.

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107[S, V, X, Z].—LEWIS F. RICHARDSON, *Weather Prediction by Numerical Process*, Dover Publications, Inc., New York, 1965, xvi + 236 pp., 24 cm. Price $2.00.

The author is the English scientist (1881–1953) known most widely for his iterative finite-difference method of solving elliptic differential equations and for the criterion ("Richardson number") concerning the onset of turbulence in stratified shear flow. This remarkable book, published originally by Cambridge University Press in 1922, describes in detail a visionary plan for the numerical forecasting of weather. The numerical process is the solution of an initial-value problem by finite-differences, the initial data for which are to come from an international meteorological network. Richardson also works out a sample test case, calculating the value, at $t = 0$, of the time rates of change of wind, pressure, and temperature at a limited grid network in Europe. The results differ greatly from the observed values (especially that for surface pressure), and he concludes that the initial data then available are too inaccurate.

Although the book made considerable impression when first published, it then appears to have been almost completely ignored until the late 1940's, when J. Charney and J. von Neumann began the modern era of numerical weather prediction at Princeton. (None of the books on dynamic meteorology published before 1948 discuss Richardson's book, and only two even mention its existence.) *Weather Prediction by Numerical Process* was thus 25 years ahead of time in 1922. The technical developments of electronic computers and radiosondes, and the theoretical developments of atmospheric hydrodynamics and the concept of computational stability, were all necessary before Richardson's basic idea could be put into successful operation. Although the book's attraction today is primarily one of historical interest, it still makes stimulating reading for meteorologists and, I should think, applied mathematicians. Its reissue now in an inexpensive form is therefore very welcome. Sydney Chapman has written an *Introduction* to this edition which conveys much of the dedication and passion which seem to have characterized Richardson.

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108[X].—WIKTOR ECKHAUS, *Studies in Non-Linear Stability Theory*, Springer-Verlag, New York, 1965, viii + 117 pp., 24 cm. Price $5.50.

This monograph is concerned with the problem of stability of solutions of nonlinear partial differential equations of the form $u_t = L(u) + F(u)$, with boundary