

of relay type, as they arise in control theory. A good number of examples is discussed, illustrating the construction of singular trajectories.

The final article "Several Applications of the Direct Method of Liapunov" by R. A. Nesbit describes in terms of Lyapunov functions and linear bounds various sufficient conditions for the stability of an equilibrium point of a nonlinear system of ordinary differential equations. Numerical examples are presented to illustrate the usefulness of these bounds.

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107[S, V, X, Z].—LEWIS F. RICHARDSON, *Weather Prediction by Numerical Process*, Dover Publications, Inc., New York, 1965, xvi + 236 pp., 24 cm. Price \$2.00.

The author is the English scientist (1881–1953) known most widely for his iterative finite-difference method of solving elliptic differential equations and for the criterion ("Richardson number") concerning the onset of turbulence in stratified shear flow. This remarkable book, published originally by Cambridge University Press in 1922, describes in detail a visionary plan for the numerical forecasting of weather. The numerical process is the solution of an initial-value problem by finite-differences, the initial data for which are to come from an international meteorological network. Richardson also works out a sample test case, calculating the value, at $t = 0$, of the time rates of change of wind, pressure, and temperature at a limited grid network in Europe. The results differ greatly from the observed values (especially that for surface pressure), and he concludes that the initial data then available are too inaccurate.

Although the book made considerable impression when first published, it then appears to have been almost completely ignored until the late 1940's, when J. Charney and J. von Neumann began the modern era of numerical weather prediction at Princeton. (None of the books on dynamic meteorology published before 1948 discuss Richardson's book, and only two even mention its existence.) *Weather Prediction by Numerical Process* was thus 25 years ahead of time in 1922. The technical developments of electronic computers and radiosondes, and the theoretical developments of atmospheric hydrodynamics and the concept of computational stability, were all necessary before Richardson's basic idea could be put into successful operation. Although the book's attraction today is primarily one of historical interest, it still makes stimulating reading for meteorologists and, I should think, applied mathematicians. Its reissue now in an inexpensive form is therefore very welcome. Sydney Chapman has written an *Introduction* to this edition which conveys much of the dedication and passion which seem to have characterized Richardson.

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108[X].—WIKTOR ECKHAUS, *Studies in Non-Linear Stability Theory*, Springer-Verlag, New York, 1965, viii + 117 pp., 24 cm. Price \$5.50.

This monograph is concerned with the problem of stability of solutions of non-linear partial differential equations of the form $u_t = L(u) + F(u)$, with boundary

conditions, where $L(u)$ and $F(u)$ are linear and nonlinear operators, respectively, involving u and the derivatives of u with respect to the space variables, and the order of the derivatives in F is lower than of those in L . Both cases, of one and two space variables, are discussed. The basic idea throughout is to illustrate how expansions in series of eigenfunctions (the eigenfunctions being obtained from the linear problem) with time-varying coefficients can be used to discuss stability (or instability) and oscillatory phenomena in the solution of such boundary-value problems, as well as to obtain asymptotic forms for the solutions. The discussion centers around the case where the instability in the linear equation is determined by only one eigenvalue. There are also applications of the results to some particular problems that arise in fluid dynamics and a comparison of his results with those of other contributors. The author has certainly systematized this method to a point where in principle it can be applied to many problems, especially the scaling techniques necessary to determine relative magnitudes of the Fourier coefficients.

The book is fairly easy to read and should serve as a guide to further research in this area.

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109[X].—RICHARD COURANT & FRITZ JOHN, *Introduction to Calculus and Analysis*, Vol. I, John Wiley & Sons, Inc., New York, 1965, xxiii + 661 pp., 24 cm. Price \$10.50.

We briefly report on this masterfully written volume on "calculus" for functions of a single variable, because of its treatment of selected topics in the field of numerical methods. The following subjects are presented maturely and in a clear, mathematically sound and practically useful manner:

- i. interpolation by polynomials (including the case of coincident points of interpolation),
- ii. approximation by algebraic and trigonometric polynomials,
- iii. computation of integrals (Simpson's rule),
- iv. calculus of errors,
- v. solution of simultaneous (nonlinear) equations (Newton's method, false position method, iteration method),
- vi. Bernoulli polynomials (numbers),
- vii. Euler-Maclaurin summation formula.

Only topics iii-v are treated in the short separate chapter on numerical methods. The rest are interwoven through the main body of the book. Many other useful numerical methods are developed throughout the text, for example, the summation of series, and the evaluation of asymptotic series (not formally introduced).

I quote from the preface: "(The intention is) to lead the student directly to the heart of the subject and to prepare him for active application of his knowledge. It avoids the dogmatic style which conceals the motivation and the roots of the calculus in intuitive reality. To exhibit the interaction between mathematical analysis and its various applications and to emphasize the role of intuition remains an important aim of this new book. Somewhat strengthened precision does not, as we hope, interfere with this aim."