conditions, where $L(u)$ and $F(u)$ are linear and nonlinear operators, respectively, involving $u$ and the derivatives of $u$ with respect to the space variables, and the order of the derivatives in $F$ is lower than of those in $L$. Both cases, of one and two space variables, are discussed. The basic idea throughout is to illustrate how expansions in series of eigenfunctions (the eigenfunctions being obtained from the linear problem) with time-varying coefficients can be used to discuss stability (or instability) and oscillatory phenomena in the solution of such boundary-value problems, as well as to obtain asymptotic forms for the solutions. The discussion centers around the case where the instability in the linear equation is determined by only one eigenvalue. There are also applications of the results to some particular problems that arise in fluid dynamics and a comparison of his results with those of other contributors. The author has certainly systematized this method to a point where in principle it can be applied to many problems, especially the scaling techniques necessary to determine relative magnitudes of the Fourier coefficients.

The book is fairly easy to read and should serve as a guide to further research in this area.

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**109[X].**—RICHARD COURANT & FRITZ JOHN, *Introduction to Calculus and Analysis*, Vol. I, John Wiley & Sons, Inc., New York, 1965, xxiii + 661 pp., 24 cm. Price $10.50.

We briefly report on this masterfully written volume on "calculus" for functions of a single variable, because of its treatment of selected topics in the field of numerical methods. The following subjects are presented maturely and in a clear, mathematically sound and practically useful manner:

i. interpolation by polynomials (including the case of coincident points of interpolation),
ii. approximation by algebraic and trigonometric polynomials,
iii. computation of integrals (Simpson's rule),
iv. calculus of errors,
v. solution of simultaneous (nonlinear) equations (Newton's method, false position method, iteration method),
vi. Bernoulli polynomials (numbers),
vii. Euler-Maclaurin summation formula.

Only topics iii–v are treated in the short separate chapter on numerical methods. The rest are interwoven through the main body of the book. Many other useful numerical methods are developed throughout the text, for example, the summation of series, and the evaluation of asymptotic series (not formally introduced).

I quote from the preface: "(The intention is) to lead the student directly to the heart of the subject and to prepare him for active application of his knowledge. It avoids the dogmatic style which conceals the motivation and the roots of the calculus in intuitive reality. To exhibit the interaction between mathematical analysis and its various applications and to emphasize the role of intuition remains an important aim of this new book. Somewhat strengthened precision does not, as we hope, interefere with this aim."

The authors succeed admirably in achieving this aim and we look forward eagerly to reading the second volume on the calculus for functions of several variables.

E. I.

**110[X, Z].**—J. A. ZONNEVELD, *Automatic Numerical Integration*, Mathematical Centre Tracts 8, Mathematisch Centrum Amsterdam, 1964, 110 pp., 24 cm.

This tract is concerned with the automatic integration of systems of ordinary differential equations with initial conditions. First order and second order equations are considered, including second order with first derivatives appearing, and without.

Equations which must be satisfied by the parameters in Runge-Kutta formulas are developed in a standard way, and formulas are obtained for all orders up to and including fifth order. Additional equations are developed for parameters which can be used to determine the accuracy of the method, and this leads to formulas for approximating the last term retained in the Taylor expansion of the true solution. (The increment in the Runge-Kutta formula approximates the *sum* of a certain number of terms in this expansion. The new formula approximates the last of these terms, and can be used to keep the error below a prescribed tolerance.)

The formula for the last term can be evaluated only at the cost of a slight increase in the number of function evaluations per step.

Formulas are given in each case for differential equations in which the independent variable appears explicitly, and also for equations in which it does not appear.

There is an interesting chapter on the choice of step-size, and on changing the variable of integration, including the use of the arc length for this variable.

Nine ALGOL 60 procedures are given, some for first order and some for second order equations. Two of them change the variable of integration automatically, and one uses the arc length.

Five numerical examples are presented to illustrate various possibilities. Two involve van der Pol's equation, one consists of 15 second order equations, and another contains a singularity. One is used to show how a "virtually foolproof" strategy can fail in special circumstances.

A bibliography of 24 items is included.

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**111[Z].**—S. H. HOLLINGDALE & G. C. TOOTILL, *Electronic Computers*, Penguin Books, Inc., Baltimore, Maryland, 1965, 335 pp., 19 cm. Price $1.65 (paperbound).

This delightful little book is within reach of everyone, both in price and in content, although some thought and patience will be required of the layman to realize the full rewards of a careful reading. It is an honest and apparently successful attempt at popularization of the "black arts" of computers.

Because the book was written in 1963 and 1964 the latest fashions in computing now sweeping the field, namely, time-sharing and its corollaries, are only mentioned in passing. We can hope for an early revision to bring the laity up to date