7[P, Q, S, X, Z].—R. E. BELLMAN & R. E. KALABA, *Quasilinearization & Nonlinear Boundary-Value Problems*, Vol. 3, American Elsevier Publishing Co., New York, ix + 206 pp., 24 cm. Price $8.50.

In this book a number of nonlinear boundary value problems are discussed, using analytic and numerical methods. Various applications—like a control problem, an inverse problem in radiation transfer theory and even in cardiology—are considered, but clearly the methods dominate the applications to these problems. The methods which the authors apply mainly consist of approximating the solution of the nonlinear problems by solving successively a sequence of linear problems as in the case of the well-known Newton-Raphson method. This approach is explained in very simple terms in the initial chapter and it is observed that for a number of problems the monotonicity of the approximating sequence can be insured and used for convergence proofs. The linear equations which one is led to are then treated by various numerical methods. The authors emphasize that this approach—which they call "quasilinearization"—is not the same as the Newton-Raphson method, but in all cases treated, the methods seem to be identical and the subtle distinction remained unclear to the reviewer. The main part of the book is restricted to boundary value problems for ordinary differential equations and a brief chapter is devoted to partial differential equations.

The book does not require a strong background in mathematics or numerical analysis. It is written in a fluent and informal style. However, while simple concepts and ideas are explained very clearly, the uninformed reader will be stopped by terms like dynamic programming, invariant imbedding techniques, which are not at all explained but frequently used in a casual manner. For example, on p. 52, one finds a derivation of the Hamilton-Jacobi equations for a simple example, however, reference is made to dynamic programming, but not to Hamilton-Jacobi. This is certainly misleading.

To sum up: This books contains an informal—not so informative—approach to some nonlinear boundary value problems with a variety of applications which are discussed briefly and supplied with computer programs.

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8[P, S, X].—LOUIS A. PIPES, *Operational Methods in Nonlinear Mechanics*, Dover Publications, New York, 1965, viii + 99 pp. 22 cm. Price $1.50.

Perturbation and iteration procedures are frequently employed to obtain approximate solutions of nonlinear vibration problems. Thus the nonlinear problem is replaced by a sequence of linear problems. In this book, the Laplace transform method is applied to solve the resulting linear problems. The author claims that this method reduces the algebraic labor involved in obtaining solutions. It is applied to a variety of problems most of which arise in circuit theory and are of nonautonomous type.

The purposes of the book are best described by quoting from the Introduction: "... the object of ... this book (is) to present some useful methods for the solution of certain classes of important nonlinear technical problems in a manner available

and understandable to the engineer and physicist interested primarily in applications. To do this, it appeared particularly desirable to present the mathematical techniques by applying them to definite examples of physical interest even at the expense of mathematical generality and elegance."

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9[P, X].—WENDELL E. GROVE, *Brief Numerical Methods*, Prentice-Hall, Englewood Cliffs, N. J., 1966, ix + 117 pp., 24 cm. Price $6.75.

This short book of 114 pages is an introduction to some elementary numerical methods. The emphasis is on presenting the numerical method and how to apply it. Although there is essentially no mathematical analysis, the methods discussed are sufficiently motivated.

The level of the book is sophomore-junior. It is probably more appropriate for engineering students than mathematics majors. It is computer oriented. The following list of chapter titles indicates the scope of the book: Iterative solution of algebraic and transcendental equations; Complex roots; Simultaneous equations; Interpolation techniques; Curve fitting; Numerical integration; Solution of differential equations; A simple boundary value problem.

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10[P, X, Z].—N. E. KOBRINSKII & B. A. TRAKHTENBROT, *Introduction to the Theory of Finite Automata*, translated from the Russian, North-Holland Publishing Co., Amsterdam, 1965, x + 337 pp., 23 cm. Price $8.40.

This book presents automata theory as applied mathematics and so is quite distinct in its treatment from much work on "abstract" automata in the United States.

The book begins with a self-contained lucid account of elementary logic. Real-time devices for processing digital data are introduced and shown to be associated with a definite class of mathematical operators. The physical characteristics of vacuum tubes (valves!), diodes, transistors, and ferromagnetic elements, are briefly discussed and their use in constructing flip-flops and in realizing basic logical operators is indicated. The problems of analyzing (going from a physical circuit to the operator it realizes) and synthesizing (going from a mathematical operator to a circuit realizing it) are discussed in detail. A final chapter describes the work of Shannon and Lupanov on asymptotic estimates for nets realizing a given operator.

The book is very well written and the English translation reads quite smoothly. It is an important contribution to the developing literature of automata theory.

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