16[A].—RUDOLPH ONDREJKA, *The First 100 Exact Double Factorials*, ms. of 12 handwritten sheets (undated) deposited in the UMT file.

These unpublished tables consist of two parts: the first consists of the exact values of $(2n - 1)!! = 1 \cdot 3 \cdot 5 \cdots (2n - 1)$ for $n = 1(1)100$; the second consists of the exact values of $(2n)!! = 2 \cdot 4 \cdot 6 \cdots 2n$ for the same range of n. These data were computed by the author on a desk calculator "many years ago" and were subsequently misplaced. Each tabular entry after the first was calculated from its predecessor, and an overall check consisted of forming the product of the last entry in each table and comparing that result with $200!$.

The only tables of this kind of comparable size appear to be unpublished ones [1] calculated by J. C. P. Miller on the EDSAC in 1955. His tables cover the same range for the even double factorial and a larger range for the odd double factorial; however, the increment in the argument n is 10 to $n = 100$, and it is 50 beyond that to $n = 250$.

The most extensive published tables of exact values of such numbers are those of Potin [2] and Hayashi [3], which extend to only $n = 25$.

Consequently, the present manuscript tables supply valuable numerical information that has been hitherto unavailable.

J.W.W.

1. A. FLETCHER, J. C. P. MILLER, L. ROSENHEAD & L. J. COMRIE, *An Index of Mathematical Tables*, Vol. I, 2nd ed., Addison-Wesley, Reading, Mass., 1962, p. 53.
2. L. POTIN, *Formules et Tables Numériques*, Gauthier-Villars, Paris, 1925.
3. K. HAYASHI, *Fünfstellige Funktionentafeln*, Springer, Berlin, 1930.

17[A, F].—M. LAL, *Expansion of* $\sqrt{2}$ *to* 19600 *Decimals*, Memorial University of Newfoundland, St. John's, Newfoundland, Canada, ms. of 4 typewritten pp. + 2 tables deposited in the UMT file.

The main table here has an attractively printed value of $\sqrt{2}$ correct to 19600 decimals. This is somewhat more accurate than the recent computation to 14000 decimals [1]. As in [1], we also have here a table of the distribution of the decimal digits and a chi-square analysis of their presumed, and apparent, equidistribution.

This computation required 14.2 hours on an IBM 1620, and the check $(\sqrt{2})^2 = 2$ required 4 hours. Since this computer is a small decimal machine, the author elected to compute $\sqrt{2}$ *one digit at a time*. We may indicate his method as follows:

Let $A_k = [10^k\sqrt{2}]$ and $A_{k+1} = 10A_k + a_{k+1}$, so that a_k is the kth digit. Let $B_0 = 1$ and

$$B_{k+1} = 100B_k - \sum_{n=1}^{a_{k+1}} (20A_k + 2n - 1)$$

where a_{k+1} is the largest value of n which leaves the difference positive. If there is no such n, then $a_{k+1} = 0$. Thus we have

$$A_0 = 1 \qquad B_0 = 1$$
$$A_1 = 14 \qquad B_1 = 4$$
$$A_2 = 141 \qquad B_2 = 119, \quad \text{etc.}$$

If D decimals are sought, it is clear that the number of operations involved in this method is $O(D^2)$, as is also a binomial series calculation, since the latter converges nearly geometrically. A comparison in speed between these methods would depend on the relative operation-times for subtraction and division. In the second method one would use a large solution of either of the so-called Pell equations

$$x^2 - 2y^2 = \pm 1,$$

and then expand

$$\sqrt{2} = (x/y)(1 \mp x^{-2})^{1/2}.$$

For example, on a binary machine, the evaluation of

$$\sqrt{2} = \frac{941664}{665857}\left[1 + \frac{2^{-9}}{865948329}\right]^{1/2}$$

should be quite fast.

We might note that there is nothing in this data to give encouragement to the stated opinion of J. E. Maxfield to the effect that $\sqrt{2}$ is probably not normal (in the decimal system). On the contrary, the apparent equidistribution mentioned above would suggest that $\sqrt{2}$ is, at least, simply normal. Similarly, the more recent stated opinion of I. J. Good that $\sqrt{2}$ is perhaps not normal in the base 2 has contrary evidence in [1], since it is shown there that $\sqrt{2}$ has apparent equidistribution not only in decimal but also in octal.

D.S.

1. Kōki Takahashi & Masaaki Sibuya, "Statistics of the digits of $\sqrt{n}$," *Jōhō Shori (Information Processing)*, v. 6, 1965, pp. 221–223. (Japanese) (See also the next review here.)

18[B, K.]—Kōki Takahashi & Masaaki Sibuya, *The Decimal and Octal Digits of $\sqrt{n}$*, The Institute of Statistical Mathematics, Tokyo, August 1966, ms. of iii + 83 pp. deposited in the UMT file.

As stated in the Foreword, the iteration $x_{k+1} = x_k(1.5 - 0.5nx_k^2)$ was used by the authors in the underlying electronic calculations on an HIPAC-103 system.

The numerical output, on standard computer sheets, is arranged in two sections. The first, designated Part I, includes approximations to $\sqrt{2}$ and $\sqrt{3}$ extending to 14000D and to 15360 octal places. The cumulative frequencies of individual digits are given for successive blocks of 100 decimal digits and 128 octal digits, respectively. The frequencies in each of these blocks are separately tabulated for only the first half of the range of digits calculated. The corresponding χ^2 values are given to 3D for the cumulative distributions and to 1D for the others.

In Part II we find similar information for the square roots of the integers 5, 6, 7, 8, and 10. Here, however, the approximations are carried to about one-half the extent of those in Part I. Specifically, the square roots of 5, 6, and 10 are given to 7000D and to 7680 octal digits, whereas those of 7 and 8 appear to 6900D and to 7552 octal digits.

The decimal approximations are conveniently displayed in groups of 10 digits, with 10 such groups in each line, and spaces between successive sets of five lines. A total of 5000D can thereby be shown on each computer sheet. The octal representations are presented in groups of eight digits, with eight groups to a line. Fifty