If D decimals are sought, it is clear that the number of operations involved in this method is $O(D^2)$, as is also a binomial series calculation, since the latter converges nearly geometrically. A comparison in speed between these methods would depend on the relative operation-times for subtraction and division. In the second method one would use a large solution of either of the so-called Pell equations

$$x^2 - 2y^2 = \pm 1,$$

and then expand

$$\sqrt{2} = (x/y)(1 \mp x^{-2})^{1/2}.$$

For example, on a binary machine, the evaluation of

$$\sqrt{2} = \frac{941664}{665857}\left[1 + \frac{2^{-9}}{865948329}\right]^{1/2}$$

should be quite fast.

We might note that there is nothing in this data to give encouragement to the stated opinion of J. E. Maxfield to the effect that $\sqrt{2}$ is probably not normal (in the decimal system). On the contrary, the apparent equidistribution mentioned above would suggest that $\sqrt{2}$ is, at least, simply normal. Similarly, the more recent stated opinion of I. J. Good that $\sqrt{2}$ is perhaps not normal in the base 2 has contrary evidence in [1], since it is shown there that $\sqrt{2}$ has apparent equidistribution not only in decimal but also in octal.

D.S.

1. Kōki Takahashi & Masaaki Sibuya, "Statistics of the digits of $\sqrt{n}$," *Jōhō Shori (Information Processing)*, v. 6, 1965, pp. 221–223. (Japanese) (See also the next review here.)

18[B, K.]—Kōki Takahashi & Masaaki Sibuya, *The Decimal and Octal Digits of $\sqrt{n}$*, The Institute of Statistical Mathematics, Tokyo, August 1966, ms. of iii + 83 pp. deposited in the UMT file.

As stated in the Foreword, the iteration $x_{k+1} = x_k(1.5 - 0.5nx_k^2)$ was used by the authors in the underlying electronic calculations on an HIPAC-103 system.

The numerical output, on standard computer sheets, is arranged in two sections. The first, designated Part I, includes approximations to $\sqrt{2}$ and $\sqrt{3}$ extending to 14000D and to 15360 octal places. The cumulative frequencies of individual digits are given for successive blocks of 100 decimal digits and 128 octal digits, respectively. The frequencies in each of these blocks are separately tabulated for only the first half of the range of digits calculated. The corresponding χ^2 values are given to 3D for the cumulative distributions and to 1D for the others.

In Part II we find similar information for the square roots of the integers 5, 6, 7, 8, and 10. Here, however, the approximations are carried to about one-half the extent of those in Part I. Specifically, the square roots of 5, 6, and 10 are given to 7000D and to 7680 octal digits, whereas those of 7 and 8 appear to 6900D and to 7552 octal digits.

The decimal approximations are conveniently displayed in groups of 10 digits, with 10 such groups in each line, and spaces between successive sets of five lines. A total of 5000D can thereby be shown on each computer sheet. The octal representations are presented in groups of eight digits, with eight groups to a line. Fifty lines are shown on each page, with spaces between successive sets of five lines, as before; thus, a total of 3200 octal digits are accommodated on each sheet.

In a related paper [1] the authors have presented similar statistical information concerning these square roots. While this statistical information is believed correct, the unpublished printed-out values of the roots computed at that time contained several erroneous digits because of a programming error. Discrepancies between those decimal approximations obtained for $\sqrt{2}$ and $\sqrt{3}$ and the values previously published by Uhler [2, 3] were erroneously attributed by the authors to purported errors in Uhler's calculations. The corrected values appearing in the present manuscript are believed by the authors to be free from error. In partial confirmation of this, the reviewer has found complete agreement of the value of $\sqrt{2}$ herein with the unpublished approximation [4] of Lal, which extends to 19600D.

J. W. W.

1. Kōki Takahashi & Masaaki Sibuya, "Statistics of the digits of $\sqrt{n}$," *Jōhō Shori (Information Processing)*, v. 6, 1965, pp. 221–223. (Japanese)
2. H. S. Uhler, "Many-figure approximations to $\sqrt{2}$, and distribution of digits in $\sqrt{2}$ and $1/\sqrt{2}$," *Proc. Nat. Acad. Sci. U. S. A.*, v. 37, 1951, pp. 63–67.
3. H. S. Uhler, "Approximations exceeding 1300 decimals for $\sqrt{3}$, $1/\sqrt{3}$, $\sin(\pi/3)$ and distribution of digits in them," *ibid.*, pp. 443–447.
4. M. Lal, *Expansion of $\sqrt{2}$ to 19600 Decimals*, ms. deposited in the UMT file. (See *Math. Comp.*, v. 21, 1967, pp. 258–259, RMT 17.)

19[F].—L. G. Diehl & J. H. Jordan, *A Table of Gaussian Primes*, Washington State University, Pullman, Washington, a strip of computer paper deposited in the UMT file.

If p, a prime, is of the form $4m + 1$ then $p = A^2 + B^2$. As is known

$$\pm A \pm Bi \quad \text{and} \quad \pm B \pm Ai$$

are then Gaussian primes. This table lists A and B for each $p = 4m + 1 \leqq 90997$.

Whereas such a table is not readily available, a somewhat larger table for $p < 10^5$ was published by A. J. C. Cunningham long ago [1].

The present table was computed in 15 minutes on an IBM 709 at Washington State University. No details are given as to how this was done. It may be of interest to survey briefly known methods that have been used for these and related problems.

Four methods of theoretical interest are reviewed by Davenport [2]. The simplest conceptually is that of Gauss. If $p = 4m + 1$, set

$$A \equiv (2m)!/2(m!)^2, \qquad B \equiv (2m)!\,A \pmod p.$$

It is clear that this is quite inefficient arithmetically speaking. Related to this is Jacobsthal's method. Let

$$S(a) = \sum_{n=1}^{p-1} \left(\frac{n(n^2 - a)}{p}\right)$$

where the quantity summed is the Legendre symbol. Then if R is any quadratic residue, say $R = 1$, and N is any nonresidue, set

$$A = \tfrac{1}{2}\,|\,S(R)\,|, \qquad B = \tfrac{1}{2}\,|\,S(N)\,|.$$

Recently Chowla [3] has given an attractive proof of Jacobsthal's method, a consequent simple proof of Gauss's method, and the relation of these Jacobsthal sums to the Riemann Hypothesis.