conventional, u represents $F(\phi, k)$, the incomplete elliptic integral of the first kind in Legendre's form, and ϕ is the amplitude function, $am(u, k)$.

The integral $E(am(u), k)$ as here tabulated is a by-product of a concurrent calculation of the Jacobi zeta function, defined by the relation

$$Z(u, k) = E(am(u), k) - \frac{E(k)}{K(k)} u,$$

where $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kinds, respectively. The integral $K(k)$ and the ratio $E(k)/K(k)$ are also given to 10D for $k^2 = 0(0.01)0.99$.

These tables were calculated on an IBM 7094 computer system, using a subroutine based on the descending Landen transformation (also known as the Gauss transformation).

In addition to this information concerning the calculation of the tables, the authors include the definitions of the tabulated functions and a summary of their various properties.

This set of tables of the Jacobi elliptic functions is the most extensive compiled to date. A relatively inaccessible table prepared by the staff of the Project for Computation of Mathematical Tables [1] gave $sn\ u$, $cn\ u$, $dn\ u$ to 15D for $k^2 = 0(0.01)1$, $u/K = 0.01, 0.1(0.1)1$. The well-known tables of Milne-Thomson [2] give only 5D values of these functions, over a much more restricted set of values of the arguments than the tables under review. It may also be noted here that the 12D tables of Spenceley & Spenceley [3], on the other hand, are arranged with the modular angle and the ratio u/K as parameters.

J.W.W.

1. *Tables of Jacobi Elliptic Functions*, ms. prepared by the Project for Computation of Mathematical Tables, New York City; printed for limited distribution, Washington, D. C., 1942. (See *MTAC*, v. 1, 1943–1945, pp. 125–126, UMT **12;** *ibid.*, p. 425, RMT **207.**)
2. L. M. Milne-Thomson, *Jacobian Elliptic Function Tables*, Dover, New York, 1950. (See *MTAC*, v. 5, 1951, pp. 157–158, RMT **910.**)
3. G. W. Spenceley & R. M. Spenceley, *Smithsonian Elliptic Functions Tables* (*Smithsonian Miscellaneous Collections*, v. 109), Smithsonian Institution, Washington, D. C., 1947. (See *MTAC*, v. 3, 1948–1949, pp. 89–92, RMT **485.**)

26[I, L].—B. I. Korobochkin & Yu. A. Filippov, *Tablitsy modifitsirovannykh funktsiĭ Uittekera*, (*Tables of Modified Whittaker Functions*), Computing Center of the Academy of Sciences of the USSR, Moscow, 1965, xvi + 322 pp., 27 cm. Price 2.66 rubles.

These tables, in the well-known series under the general editorship of V. A. Ditkin, were produced by collaboration between the Computing Center of the Academy of Sciences of the USSR and the Computing Center of the Latvian State University. The functions tabulated are connected with solutions of the hypergeometric equation

$$zy'' + (\gamma - z)y' - \beta y = 0,$$

where we follow the authors in using β, rather than the more usual α, for the first (or numerator) parameter. Much of the introductory text relates to the case in which γ is any positive integer k, but the tables relate entirely to the case $\gamma = k = 2$, to which we shall confine ourselves.

The functions which (with their first derivatives) are actually tabulated are $\Psi_1(\beta, x)$ and $\Psi_2(\beta, x)$. They are connected with confluent hypergeometric functions $Y_{01}(\beta, 2, z)$ and $Y_{02}(\beta, 2, z)$ by the equations

$$\Psi_1(\beta, x) = xe^{-x}[Y_{01}(\beta, 2, 2x)K_1(x) + Y_{02}(\beta, 2, 2x)I_1(x)],$$

$$\Psi_2(\beta, x) = xe^{-x}[-Y_{01}(\beta, 2, 2x)K_0(x) + Y_{02}(\beta, 2, 2x)I_0(x)],$$

where I and K denote the usual Bessel functions, or by the equivalent equations

$$Y_{01}(\beta, 2, 2x) = e^{x}[\Psi_1(\beta, x)I_0(x) - \Psi_2(\beta, x)I_1(x)],$$

$$Y_{02}(\beta, 2, 2x) = e^{x}[\Psi_1(\beta, x)K_0(x) + \Psi_2(\beta, x)K_1(x)].$$

Here $Y_{01}(\beta, 2, z)$ is what is usually denoted by ${}_1F_1(\beta; 2; z)$, $F(\beta, 2, z)$, $M(\beta, 2, z)$ or $\Phi(\beta, 2; z)$, while $Y_{02}(\beta, 2, z)$ is a logarithmic second solution. As the reviewer had considerable difficulty in making quite sure of its meaning, it will be well to state explicitly what $Y_{02}(\beta, 2, z)$ denotes.

In terms of the functional notation used by Miller [1],

$$Y_{02}(\beta, 2, z) = -M(\beta, 2, z)\,[\ln z + C + \psi(\beta) - \psi(2)] - N(\beta, 2, z) - S(\beta, 2, z),$$

where C is Euler's constant $(0.5772 \cdots)$, $\psi(x)$ is the logarithmic derivative of the gamma function (not the factorial function), satisfying

$$\psi(x + 1) - \psi(x) = 1/x,$$

$N(\beta, 2, z)$ is what results on substituting $\gamma = 2$ in

$$N(\beta, \gamma, z) = \left(\frac{1}{\beta} - \frac{1}{\gamma} - 1\right)\frac{\beta}{\gamma}\frac{z}{1!} + \left(\frac{1}{\beta} + \frac{1}{\beta + 1} - \frac{1}{\gamma} - \frac{1}{\gamma + 1} - 1 - \frac{1}{2}\right)\frac{\beta(\beta + 1)}{\gamma(\gamma + 1)}\frac{z^2}{2!} + \cdots,$$

while $S(\beta, \gamma, z)$ reduces for the particular value $\gamma = 2$ to the single term

$$S(\beta, 2, z) = 1/(\beta - 1)z.$$

In the functional notation used by Erdélyi and collaborators [2, p. 261],

$$Y_{02}(\beta, 2, z) = -\Gamma(\beta - 1)\Psi(\beta, 2; z),$$

and in the functional notation used by Miss Slater [3, p. 8]

$$Y_{02}(\beta, 2, z) = -\Gamma(\beta - 1)U(\beta; 2; z).$$

The reviewer's confirmation of the above statement would have been easier if the power series for $\Psi_1(\beta, x)$ and $\Psi_2(\beta, x)$ given by the authors on page x had not been wrong. It would follow from equations (23) and (24) that $a_1(\beta)$ and $b_1(\beta)$ are coefficients of $\frac{1}{2}x$ in the expansions of Ψ_1 and Ψ_2 respectively, but the explicit expressions for $a_1(\beta)$ and $b_1(\beta)$ given in equations (28) and (27) are actually coefficients of x, as may be verified from the tables.

Table I (pp. 2–103) lists 8S values of $\Psi_1(\beta, x)$, $\Psi_2(\beta, x)$ and their first x-derivatives, along with second differences of all four functions with respect to both β and x, for $\beta = 3(0.02)4$, $x = 0(0.05)2.50$.

Table II (pp. 106–269) lists 8S values of the same four functions, again with second differences in both arguments, for $\beta = 3(0.02)4$, $x = 2.5(0.1)10$.

Table III (pp. 272–313) lists 8S values of the products of the same four functions by $A(\beta, x) = (2x)^{3/2-\beta}\,\Gamma(\beta)$, with second differences in both arguments, for $\beta = 3(0.05)4$, $x = 10(0.1)15$. In the sub-title on p. 271, for ρ read β.

Table IV (pp. 316–318) lists 9S values of $A(\beta, x)$, without differences, for $\beta = 3(0.05)4$, $x = 10(0.1)15$.

There is also (pp. 320–321) an 8D table of Everett interpolation coefficients, without differences, at interval 0.001.

ALAN FLETCHER

1. J. C. P. MILLER, "Note on the general solution of the confluent hypergeometric equation," *MTAC.*, v. 11, 1957, pp. 97–99.
2. A. ERDÉLYI, W. MAGNUS, F. OBERHETTINGER & F. G. TRICOMI, *Higher Transcendental Functions*, Vol. 1, McGraw-Hill, New York, 1953.
3. LUCY J. SLATER, *Confluent Hypergeometric Functions*, Cambridge University Press, Cambridge, 1960.

27[I, M].—J. E. KILPATRICK, SHIGETOSHI KATSURA & YUJI INOUE, *Tables of Integrals of Products of Bessel Functions*, Rice University, Houston, Texas and Tôhoku University, Sendai, Japan, 1966, ms. of 55 typewritten sheets deposited in the UMT file.

This unpublished report tabulates the integral

$$A \int_0^\infty t^\alpha J_{3/2+n_1}(at) J_{3/2+n_2}(bt) J_{3/2+n_3}(ct) f(t)\, dt$$

for the following cases: (1) $A = 4(2\pi)^{1/2}$, $\alpha = -5/2$, $f(t) = 1$, $a = b = c = 1$' and n_i are nonnegative integers $\leqq 20$ such that $n_1 + n_2 + n_3$ is even; (2) $A = 2\pi$, $\alpha = -4$, $f(t) = J_{3/2+n_4}(t)$, $a = b = c = 1$, and n_i are nonnegative integers $\leqq 10$ such that $n_1 + n_2 + n_3 + n_4$ is even; (3) same as the case (1) except that a, b, and c equal 1 or 2, and $n_i \leqq 16$.

Although the tabulated data are given to 16S (in floating-point form), they are generally not that accurate. A short table of the estimated accuracy (6 to 14S), which depends on the maximum value of the integers n_i, is given on p. 3. For some entries the exact value of the integral, as a rational number or as a rational multiple of $\sqrt{2}$, is also given (pp. 10, 27, and 55).

The integrals were evaluated by transforming them into Mellin-Barnes integrals and then applying the calculus of residues. As a by-product of these calculations the authors include a 16S table of ln $[(-1)^s/(-s)!]$ for $s = -25(1)0$ and of ln $\Gamma(s)$ for $s = -24.5(1)0.5(0.5)45$. A spot check revealed that several entries are accurate to only 14S.

Integrals of the type evaluated in this report have also been considered by this reviewer [1].

Y. L. L.

1. Y. L. LUKE, *Integrals of Bessel Functions*, McGraw-Hill, New York, 1962, pp. 331–332.

28[K, L].—L. S. BARK, L. N. BOL'S͡HEV, P. I. KUZNET͡SOV & A. P. C͡HERENKOV, *Tabli͡tsy raspredeleni͡ia Rele͡ia-Raĭsa* (*Tables of the Rayleigh-Rice Distribution*),