$\psi_n^s(x) = e^{-x/2}x^{s/2} L_n^s(x)$ is called a Laguerre function. In some circles use of the nomenclature Laguerre function may be misleading as this usually refers to $L_n^s(x)$ where n is an arbitrary parameter.

The following are tabulated:

$$L_n^{(s)}(x),\ \psi_n^{(s)}(x) \quad \text{for} \quad n = 2(1)7, \qquad s = 0(0.1)1.0,$$
$$x = 0(0.1)10.0(0.2)30.0,\ 6S.$$

Coefficients of the polynomials $L_n^{(s)}(x)$ for $n = 2(1)10$, $s = 0(0.05)1.0$, to 8S. (Note that these coefficients are not always exact.) Zeroes of the polynomials for $n = 2(1)10$, $s = 0(0.05)1.0$, 8S.

See the references [1]–[5] below and the sources they quote for tables relating to the abscissae and weights for numerical evaluation of $\int_0^\infty x^s e^{-x} f(x)dx$.

Y. L. L.

1. M. Abramowitz & I. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*, Applied Mathematics Series No. 55, U. S. Government Printing Office, 1964. (See *Math. Comp.*, v. 19, 1965, pp. 147–149.)

2. P. Concus, D. Cossat, G. Jaehnig & E. Melby, "Tables for the evaluation of $\int_0^\infty x^\beta e^{-x} f(x)dx$ by Gauss-Laguerre quadrature, *Math. Comp.*, v. 17, 1963, pp. 245–256.

3. P. Concus, "Additional tables for the evaluation of $\int_0^\infty x^\beta e^{-x} f(x)dx$ by Gauss-Laguerre quadrature," *Math. Comp.*, v. 18, 1964, p. 523.

4. T. S. Shao, T. C. Chen & R. M. Frank, "Tables of zeros and Gaussian weights of certain associated Laguerre polynomials and the related generalized Hermite polynomials," *Math. Comp.*, v. 18, 1964, pp. 598–616.

5. A. H. Stroud & D. Secrest, *Gaussian Quadrature Formulas*, Prentice-Hall, Englewood Cliffs, N. J., 1966. (See *Math. Comp*, v. 21, 1967, pp. 125-126, RMT **14.**)

36[L, X].—V. A. Ditkin, Editor, *Tables of Incomplete Cylindrical Functions*, Computing Center of the Academy of Sciences of the USSR, Moscow, 1966, xxix + 320 pp., 27 cm.

Consider the functions

$$(1) \qquad \frac{1}{2} J_\nu(\alpha, \rho) = \frac{\rho^\nu}{A_\nu} \int_0^\alpha \cos(\rho \cos u) \sin^{2\nu} u\, du,$$

$$(2) \qquad \frac{1}{2} H_\nu(\alpha, \rho) = \frac{\rho^\nu}{A_\nu} \int_0^\alpha \sin(\rho \cos u) \sin^{2\nu} u\, du,$$

$$(3) \qquad F_\nu^+(\alpha, \rho) = \frac{\rho^\nu}{A_\nu} \int_0^\alpha e^{\rho \cos u} \sin^{2\nu} u\, du,$$

$$(4) \qquad F_\nu^-(\alpha, \rho) = \frac{\rho^\nu}{A_\nu} \int_0^\alpha e^{-\rho \cos u} \sin^{2\nu} u\, du,$$
$$A_\nu = 2^\nu \Gamma(\nu + \tfrac{1}{2}) \Gamma(\tfrac{1}{2}).$$

Note that

$$J_\nu(\pi/2, \rho) = J_\nu(\rho), \qquad H_\nu(\pi/2, \rho) = H_\nu(\rho),$$
$$F_n^\pm(\pi/2, \rho) = \tfrac{1}{2}[I_n(\rho) \pm L_n(\rho)],$$

where $J_\nu(\rho)$ and $I_\nu(\rho)$ are the Bessel and modified Bessel functions of the first kind, respectively, and $H_\nu(\rho)$ and $L_\nu(\rho)$ are the Struve and modified Struve functions respectively.

The volume gives tables of (1) and (2) for

$$\nu = 0, 1, \qquad \rho = 0.1\,(0.1)5.0, \qquad \alpha = 0.01\,(0.01)1.57, \qquad \pi/2, \text{ 6D}.$$

Items (3) and (4) are also tabulated to 6S for the same ν, ρ and α as above. Reliefs of the functions are presented.

The introduction describes the method of computation and delineates numerous properties of the functions including series expansions, recurrence formulas and relations to other functions. Alternate methods of computation are not fully referenced. For example, the introduction shows how the tables may be used to evaluate the integrals

$$\int_0^\rho e^{-\alpha t} I_0(t)\,dt, \qquad \int_0^\rho e^{-\alpha t} K_0(t)\,dt,$$

$$\int_0^\rho e^{-i\alpha t} J_0(t)\,dt, \qquad \int_0^\rho e^{-i\alpha t} Y_0(t)\,dt,$$

but no mention is made to alternate schemes of computation which appear in the literature, e.g., the reviewer's *Integrals of Bessel Functions*, McGraw-Hill, New York, 1962. (See *Math. Comp.*, v. 17, 1962, 318–320.)

As the computations are for integer values of ν, we would have preferred tables of the functions

$$i_n(\alpha, \rho) = \int_0^\alpha e^{\rho \cos u} \cos nu\,du. \tag{5}$$

Thus, for example,

$$F_0^+(\alpha, \rho) = \frac{1}{\pi}\, i_0(\alpha, \rho), \qquad F_1^+(\alpha, \rho) = \frac{\rho}{2\pi}\,[i_2(\alpha, \rho) - i_0(\alpha, \rho)], \quad \text{etc.}$$

An efficient scheme to evaluate (5) would be to expand exp ($\rho \cos u$) in series of Bessel functions and termwise integrate to get

$$\begin{aligned} i_0(\alpha, \rho) &= \alpha I_0(\rho) + 2 \sum_{m=1}^{\infty} \frac{\sin m\alpha}{m} I_m(\rho), \\ i_n(\alpha, \rho) &= \frac{\sin n\alpha}{n} I_0(\rho) + 2\left(\frac{\alpha}{2} + \frac{\sin 2n\alpha}{4n}\right) I_n(\rho) \\ &\quad + 2 \sum_{m=1; m\neq n}^{\infty} (m^2 - n^2)^{-1} (m \sin m\alpha \cos n\alpha - n \sin n\alpha \cos m\alpha) I_m(\rho), \\ &\qquad n > 0. \end{aligned} \tag{6}$$

The evaluation of (6) is then quite easy on an electronic calculator, since the Bessel functions are readily computed by use of the backward recurrence formula.

Y. L. L.

37[L, X].—H. E. Hunter, D. B. Kirk, T. B. A. Senior & E. R. Wittenberg, *Tables of Prolate Spheroidal Functions for m* = 0, Vols. I & II, College of Engineering, The University of Michigan, under contract with Air Force Cambridge Research Laboratories, Bedford, Mass., April 1965, Report No. AFCRL-65-283(I), Vol. I, 69 + 218 unnumbered pp., Report No. AFCRL-65-283(II),