1923–1924!) is also given. The second article, by V. V. Novozhilov, is considerably longer, and essentially constitutes a pedagogical monograph, written to appeal to the Russian economic manager (presumably as nervous about these new-fangled inventions of the ivory-tower types as are U. S. managers) of elementary linear programming. Profitability (in percent per year) is emphasized as an investment criterion, this criterion is shown to be equivalent to various criteria of desirability: maximum growth rate, maximum return on total investment, etc. A certain amount of effort is devoted to establishing the compatibility (in a suitably 'higher' sense) of the new rational methods with the obligatory Marxist ideological base.

Two additional articles, by L. V. Kantorovich, develop the linear programming method in more detail, bringing out the mathematical bases of the method in terms of the dual problem of linear programming—called by Kantorovich the method of resolving multipliers. A variety of elegant small applications to machine shop scheduling, plywood cutting, etc. are discussed in detail, and some indication of numerical methods given. The first of these two papers (published 1939) is in fact one of the pioneering works on linear programming.

A good bibliography by A. A. Korbut surveys the development of linear programming, both in the USSR and abroad.

A short article by Oscar Lange discusses models of economic growth in the context of input-output analysis; these models are regarded as multi-sector developments of primitive two- and three-sector ones which may be construed out of Marx.

An article by A. L. Lure derives useful practical algorithms for the solution of rail-transport optimization problems by elementary linear programming and graph theoretical means.

Apparent in the whole volume is the convergence of Soviet planning economics with American single-firm efficiency economics. As Novozhilov puts it, quoting a Russian proverb: "Every vegetable has its season."

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49[X].—F. CESCHINO & J. KUNTZMANN, *Numerical Solution of Initial Value Problems*, translated by D. Boyanovitch, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1966, xvi + 318 pp., 24 cm. Price $10.50.

In Chapter 1 the authors introduce notational conventions, terminology (canonical forms, resolved forms, equilibrated resolved forms, etc.), brief statements of existence theorems (assuming functions are differentiable), and brief comments on numerical procedures.

Chapters 2–5 cover approximately 100 pages and are devoted mainly to deriving various single-step formulas, from Euler's method, through Runge-Kutta methods, to variants on these methods, such as one due to Blaess, and the implicit Runge-Kutta methods. Some Runge-Kutta formulas of order 5 and 6 are given, as well as a proof that formulas of order 5 cannot be obtained with only five function evaluations.

In Chapter 6 (Adams Method and Analogues) some special multistep formulas are derived, including explicit and implicit Adams formulas, and those due to Cowell, Nystrom, and Milne.

Chapter 7 (Different Multistep Formulas) is devoted mainly to a discussion of stability, without attempting to prove any of the basic theorems due to Dahlquist, even though both authors have made interesting contributions to this area in at least one of their earlier papers. Brief mention is also made of special formulas, such as those "which appeal to higher order derivatives," and also some "of the Obrechkoff type."

Chapter 8 (Application of the Runge-Kutta Principle to the Multistep Methods) considers very briefly the idea of combining Runge-Kutta and multistep ideas into composite formulas.

Chapter 9 (Theoretical Considerations) consists mainly of remarks about the characteristic roots of linear homogeneous systems with constant coefficients, the "propagation matrix" for the variational equations, and the use of something called a "coaxial" in investigating the errors associated with various methods.

A final chapter (Practical Considerations) is concerned with a variety of topics, such as different ways of estimating local truncation errors, choosing the step-size, and changing the step-size.

Numerical results for relatively simple problems are used frequently, in almost every chapter, to illustrate the methods being discussed.

There is a bibliography of nearly 600 items, including a few for 1962. (The text from which the translation has been made was copyrighted in 1963.)

This book provides a large number of formulas for the numerical integration of ordinary differential equations. Unfortunately, despite the claim in the preface that the authors have tried to group the methods around a central idea, the result is a hodge-podge of formulas, facts, near-facts, and opinions. The treatment is sometimes incomplete, and often superficial.

The presentation is frequently rather vague or misleading. For example, when "methods of approximate solution" are first introduced on p. 11, it is stated that "These methods do not give the general integral but only a well-determined integral. These methods can furnish, instead of the exact solution which we do not know or do not wish to write down, an approximate solution in the sense of numerical calculus. We thus understand that this solution is defined in a finite interval by a procedure actually executable and that we possess certain information on the error by which it is affected." Does this help the reader to understand the nature of numerical methods?

To illustrate some of the carelessness with which this book is written, consider the way in which the method of Euler-Cauchy is introduced, after the tangent method has been described, and subsequently improved. On p. 40, it is stated that "Another manner of improving the tangent method consists of noting that, on a small arc, the slope of the chord is obviously the arithmetic mean of the slopes of the tangents at the end points."

The English is not good, and the fact that the book is a translation is frequently obvious. But the main difficulties with the presentation must have existed in the original version.

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50[X].—I. G. PETROVSKI, *Ordinary Differential Equations*, translated by Richard A. Silverman, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1966, x + 232 pp., 24 cm. Price $7.95.

The work under review covers a variety of topics in the field of ordinary differential equations, and also contains a brief supplement on first order partial differential equations. The scope and level of the material is such that, in terms of an American university curriculum, it fits a senior level or first year graduate level one semester course for mathematics majors.

In principle no prior acquaintance with the field of differential equations is required. In the first two chapters the basic ideas are introduced. The discussion of the special solvable cases is unusually careful and detailed. The chapter devoted to existence and uniqueness theorems is among the best in the book. Arzéla's theorem is proved and used to prove Peano's existence theorem. Uniqueness questions are then discussed via Osgood's uniqueness theorem. The method of successive approximations is then discussed as an application of the fixed point theorem for contracting mappings. The classical Cauchy theorem regarding analytic cases is also discussed. The same chapter covers in detail the continuous dependence of solutions on initial data and parameters.

The chapters covering linear systems are adequate. The sections devoted to the canonical form of linear systems with constant coefficients are unnecessarily cumbersome. A simple statement of the Jordan form for square matrices and its application to linear systems would have been adequate. Lyapunov's second method is introduced to discuss some stability questions. The proof of the theorem on p. 151 regarding asymptotic stability is faulty. The function V must also be assumed to have an "infinitesimal upper bound" to guarantee asymptotic stability (see Massera, *Ann. of Math.* **50** (1949), 118-126.)

The last chapter is devoted to a number of topological questions; limit cycles are discussed briefly. A proof of the Brouwer fixed point theorem is given and some nice applications of that theorem are provided. The supplement is devoted to those aspects of first order partial differential equations that can be discussed in terms of ordinary differential equations. A brief and good introduction to generalized solutions is provided.

As has been indicated, those topics covered in the book are done well. Unfortunately there are many other topics that are completely omitted. For example, there is no mention of classical stability theory, linear systems with periodic coefficients, perturbation theory, boundary value problems (Sturm-Liouville problems), Green's functions, and equations with singularities, especially Fuchsian singularities (Legendre polynomials, Bessel functions etc.). There are many aspects of nonlinear