inventory problems. The emphasis on Green's functions is partly a matter of terminology, as the author so labels all transition measures (which indeed are Green's functions of the space-time process). The book is more highly recommended to the reader engaged in sophisticated applications than to the serious beginner in stochastic processes.

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64[L].—Henry E. Fettis & James C. Caslin, *An Extended Table of Zeros of Cross Products of Bessel Functions*, Report ARL 66-0023, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, February 1966, v + 126 pp., 28 cm. [Copies obtainable from the Defense Documentation Center, Cameron Station, Alexandria, Virginia.]

This useful report presents 10D tables of the first five roots of the equations: (a) $J_0(\alpha)Y_0(k\alpha) - Y_0(\alpha)J_0(k\alpha) = 0$, (b) $J_1(\alpha)Y_1(k\alpha) - Y_1(\alpha)J_1(k\alpha) = 0$, (c) $J_0(\alpha)Y_1(k\alpha) - Y_0(\alpha)J_1(k\alpha) = 0$.

In particular, Table 1a gives such roots of Eq. (a) for $k = 0.01(0.01)0.99$, while Table 1b gives the corresponding normalized roots $\gamma_n = (1 - k)\alpha_n/(n\pi)$, which are better adapted to interpolation, as originally observed by Bogert [1].

The same information for Eq. (b) is given in Tables 2a and 2b. In Tables 3a and 4a we find the corresponding roots of Eq. (c) for the respective ranges $k = 0.01(0.01)0.99$ and $k = 1.01(0.01)20$; the corresponding normalized roots $\gamma_n = |1 - k|\alpha_n/[(n - \frac{1}{2})\pi]$ appear in Tables 3b and 4b. The last two tables (5a and 5b) give the roots of Eq. (c) and their normalized equivalents for $k^{-1} = 0.001(0.001)0.050$.

As the authors note, because of symmetry it suffices for Eq. (a) and Eq. (b) to tabulate the roots corresponding to $0 < k < 1$.

The values of the roots γ_n were calculated by the method of false position on an IBM 7094 system, subject to the requirement that the corresponding values of the left member of the appropriate equation not exceed 10^{-16} numerically. These values of γ_n were then converted to the corresponding values of α_n, and both sets of data were then rounded to 10D.

Previously published tables of this kind have been very limited in scope and precision; one of the most extensive of these appears in a compilation (to 5D and 8D) on page 415 of the NBS *Handbook* [2]. The present authors have announced [3] a number of errors therein as a result of their more extensive calculations.

This reviewer has compared entries in Table 2a with the corresponding 5D approximations appearing in the table of roots of $\Delta_0(\xi) = 0$ in a recent paper by Bauer [4]. The accuracy of at least 4D claimed by Bauer is now confirmed.

In a brief introduction the authors show how such equations involving Bessel functions arise in certain boundary-value problems. This is elaborated upon in Appendix 1, which shows the relation of the tables to the solution of a problem in heat conduction involving three sets of boundary conditions.

An asymptotic series for the higher roots of the equation $J_p(\alpha)Y_q(k\alpha) -$

$Y_p(\alpha)J_q(k\alpha) = 0$ is derived in Appendix 2. Two supplementary tables are included therein. The first table consists of floating-point 14S approximations to the first 20 coefficients in the asymptotic expansion of the phase angle of the Hankel function $H_p^{(1)}(x) = J_p(x) + iY_p(x)$ when $p = 0$ and 1. The second table gives floating-point 15S values of the coefficients of the first 15 partial quotients in the continued-fraction expansion of $H_0^{(1)}(x)$ and $H_1^{(1)}(x)$. This expansion was used by the authors in their evaluation of the Bessel functions $J_p(x)$, $Y_p(x)(p = 0, 1)$ for x exceeding 5; otherwise the standard power series were used.

An insert sheet clarifies a number of illegibly printed tabular entries and corrects one erroneous table title (on p. 79).

These extensive tables constitute a significant contribution to the relatively limited tabular literature relating to this class of transcendental equations.

J. W. W.

1. B. P. Bogert, "Some roots of an equation involving Bessel functions," *J. Math. and Phys.*, v. 30, 1951, pp. 102–105.
2. M. Abramowitz & I. A. Stegun, Editors, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards, Applied Mathematics Series No. 55, Washington, D. C., 1964.
3. *Math. Comp.*, v. 20, 1966, pp. 469–470, MTE **393**.
4. H. F. Bauer, "Tables of zeros of cross product Bessel functions $J_p'(\xi)Y_p'(k\xi) - J_p'(k\xi)Y_p'(\xi) = 0$," *Math. Comp.*, v. 18, 1964, pp. 128–135.

65[L].—Henry E. Fettis & James C. Caslin, *Jacobian Elliptic Functions for Complex Arguments*, ms. of 75 computer sheets deposited in the UMT file.

These tables of the Jacobian elliptic functions $\mathrm{sn}(u + iv)$, $\mathrm{cn}(u + iv)$, and $\mathrm{dn}(u + iv)$ consist of 5D values of these functions for the ranges $u/K = 0(0.1)1$, $v/K' = 0(0.1)1$, and $\sin^{-1}k = 5°(5°)80°(1°)89°$, where K and K' represent the complete elliptic integral of the first kind for modulus k and complementary modulus k', respectively.

These tabular data resulted from a test run of an IBM 1620 subroutine prepared by the authors.

Entries corresponding to a given function and a prescribed value of $\sin^{-1}k$ are arranged on a single page of computer output. No provision has been made for interpolation in the tables. Beneath the heading of each page appears a 7D approximation to the Jacobi nome, $q = \exp(-\pi K'/K)$, for the corresponding value of k.

These new tables supplement both in precision and in range the published tables of Henderson [1].

J. W. W.

1. F. M. Henderson, *Elliptic Functions with Complex Arguments*, The University of Michigan Press, Ann Arbor, 1960. [See *Math. Comp.*, v. 15, 1961, pp. 95–96, RMT **18**.]

66[L].—M. I. Zhurina & L. N. Karmazina, *Tables and Formulae for the Spherical Functions* $P^m_{-1/2+i\tau}(z)$, Pergamon Press, New York, 1966, vii + 107 pp., 26 cm. Price $3.50.

This is an English translation of the Russian edition previously reviewed in these annals (*Math. Comp.*, v. 18, pp. 521–522, 1964, item b). The former reviewer noted a major error in the table for arc cosh x at $x = 11$ where final 689 should read 699. This error is retained in the English translation. The previous reviewer