

$Y_p(\alpha)J_q(k\alpha) = 0$ is derived in Appendix 2. Two supplementary tables are included therein. The first table consists of floating-point 14S approximations to the first 20 coefficients in the asymptotic expansion of the phase angle of the Hankel function $H_p^{(1)}(x) = J_p(x) + iY_p(x)$ when $p = 0$ and 1. The second table gives floating-point 15S values of the coefficients of the first 15 partial quotients in the continued-fraction expansion of $H_0^{(1)}(x)$ and $H_1^{(1)}(x)$. This expansion was used by the authors in their evaluation of the Bessel functions $J_p(x)$, $Y_p(x)$ ($p = 0, 1$) for x exceeding 5; otherwise the standard power series were used.

An insert sheet clarifies a number of illegibly printed tabular entries and corrects one erroneous table title (on p. 79).

These extensive tables constitute a significant contribution to the relatively limited tabular literature relating to this class of transcendental equations.

J. W. W.

1. B. P. BOGERT, "Some roots of an equation involving Bessel functions," *J. Math. and Phys.*, v. 30, 1951, pp. 102–105.
2. M. ABRAMOWITZ & I. A. STEGUN, Editors, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards, Applied Mathematics Series No. 55, Washington, D. C., 1964.
3. *Math. Comp.*, v. 20, 1966, pp. 469–470, MTE 393.
4. H. F. BAUER, "Tables of zeros of cross product Bessel functions $J_p'(k\xi)Y_p'(k\xi) - J_p'(k\xi)Y_p(\xi) = 0$," *Math. Comp.*, v. 18, 1964, pp. 128–135.

65[L].—HENRY E. FETTIS & JAMES C. CASLIN, *Jacobian Elliptic Functions for Complex Arguments*, ms. of 75 computer sheets deposited in the UMT file.

These tables of the Jacobian elliptic functions $\operatorname{sn}(u + iv)$, $\operatorname{cn}(u + iv)$, and $\operatorname{dn}(u + iv)$ consist of 5D values of these functions for the ranges $u/K = 0(0.1)1$, $v/K' = 0(0.1)1$, and $\sin^{-1}k = 5^\circ(5^\circ)80^\circ(1^\circ)89^\circ$, where K and K' represent the complete elliptic integral of the first kind for modulus k and complementary modulus k' , respectively.

These tabular data resulted from a test run of an IBM 1620 subroutine prepared by the authors.

Entries corresponding to a given function and a prescribed value of $\sin^{-1}k$ are arranged on a single page of computer output. No provision has been made for interpolation in the tables. Beneath the heading of each page appears a 7D approximation to the Jacobi nome, $q = \exp(-\pi K'/K)$, for the corresponding value of k .

These new tables supplement both in precision and in range the published tables of Henderson [1].

J. W. W.

1. F. M. HENDERSON, *Elliptic Functions with Complex Arguments*, The University of Michigan Press, Ann Arbor, 1960. [See *Math. Comp.*, v. 15, 1961, pp. 95–96, RMT 18.]

66[L].—M. I. ZHURINA & L. N. KARMAZINA, *Tables and Formulae for the Spherical Functions $P_{-1/2+i\tau}^m(z)$* , Pergamon Press, New York, 1966, vii + 107 pp., 26 cm. Price \$3.50.

This is an English translation of the Russian edition previously reviewed in these annals (*Math. Comp.*, v. 18, pp. 521–522, 1964, item b). The former reviewer noted a major error in the table for arc cosh x at $x = 11$ where final 689 should read 699. This error is retained in the English translation. The previous reviewer

also noted that the bibliography had 43 items. The number in the present edition is 44. We should like to add that the bibliography is quite extensive though not complete. In the applications one often needs integrals involving $P_{m-1/2+ir}^m(z)$ where the integration may be with respect to τ or z . In this connection and for additional references to applications, one should consult F. Oberhettinger and T. P. Higgins, *Tables of Lebedev, Mehler and Generalised Mehler Transforms*, Math. Note No. 246, October, 1961, Boeing Scientific Research Laboratories, Seattle, Washington, (*Math. Comp.*, v. 17, 1963, p. 95) the references given there, and J. Wimp, "A class of integral transforms," *Proc. Edinburgh Math. Soc.*, (2), v. 14, 1964, pp. 33-40.

Y. L. L.

67[L].—C. W. CLENSHAW & SUSAN M. PICKEN, *Chebyshev Series for Bessel Functions of Fractional Order*, Mathematical Tables, Vol. 8, National Physical Laboratory, London. Her Majesty's Stationary Office, 1966, iii + 54 pp., 28 cm. Price 17s. 6d.

These short tables are a noteworthy addition to the NPL Mathematical Tables Series started in 1957.

The main body of this volume (Tables 1-28) lists the Chebyshev coefficients for the Bessel functions of real and imaginary argument for the following arguments and orders:

For $J_\nu(x)$, $Y_\nu(x)$, $I_\nu(x)$:

$$x \leq 8, \nu = 0, \frac{1}{4}, \pm \frac{1}{3}, \pm \frac{1}{2}, \pm \frac{2}{3}, \pm \frac{3}{4}, 1,$$

$$x \geq 8, \nu = 0, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, 1.$$

For $K_\nu(x)$:

$$x \leq 8, \nu = 0, 1,$$

$$x \geq 8, \nu = 0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, 1.$$

The next 14 tables give for the same range of ν , in the range $x \leq 8$, Chebyshev coefficients such that $J_\nu(x)$ and $I_\nu(x)$ can be computed from a single auxiliary function and in the range $x \geq 8$, the Chebyshev series expansion for two auxiliary functions which permit the computation of $J_\nu(x)$, $Y_\nu(x)$, $I_\nu(x)$, and $K_\nu(x)$.

The last table is a double-series expansion to permit the calculation of $J_\nu(x)$ and $I_\nu(x)$ for any value of ν in the range $-1 \leq \nu \leq 1$ when $x \leq 8$. For all tables the coefficients are given to a high degree of accuracy, usually 20 decimal places.

In order to use the coefficients tabulated in this report one should be familiar with the discussion of the properties of Chebyshev series and with the methods for their computation and manipulation found in Volume 5 of this series, *Chebyshev Series for Mathematical Functions* (1962) by Clenshaw. It would have been extremely useful if the pertinent formulas on summation by recurrence and on the transformation of argument necessary for even series, from Section 5 of Volume 5, were included in the present volume.

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