21 [9, 11].—M. F. JONES, *Isoperimetric Right-Triangles*, Memorial University of Newfoundland, St. John's, Newfoundland, Canada, April 1967. Computer output deposited in the UMT file.

Let $\pi_n(P)$ be the number of integers $p \leqq P$ such that exactly n different primitive Pythagorean triangles exist having the same perimeter p. Let $T(P)$ be the total number of primitive triangles with perimeter $\leqq P$. Clearly,

$$T(P) = \sum_{n=1}^{\infty} n\pi_n(P) .$$

For example,

$$\pi_2(1715) = 0 , \qquad \pi_2(1716) = 1 ,$$

since

$$(748, 195, 773) \quad \text{and} \quad (364, 627, 725)$$

constitutes the pair of isoperimetric primitive triangles with the smallest perimeter.

In [1], we find $T(P)$ for $P = 10^3(10^3)120 \cdot 10^3$. Later, [2], the value $T(120{,}000) = 8430$ was corrected to 8432, and all 8432 triangles were listed. In [3], there are listed $175 \leqq \pi_3(10^6)$ triples and $7 = \pi_4(10^6)$ quadruples of isoperimetric triangles of perimeter $\leqq 10^6$. Subsequently, [4], in connection with the massive complete listing of $T(500{,}000) - T(120{,}000) = 26{,}683$ triangles, ten other triples with $p < 10^6$ were found, and added to the UMT **107** of [3].

In Table Errata E-419 of this issue, still six more such triples are listed. It is asserted that this is now complete, so that we have $\pi_3(10^6) = 191$ exactly.

In the present table we find, first,

$$\pi_n(P) \quad \text{for} \quad n = 2(1)5, \qquad P = 5 \cdot 10^4(5 \cdot 10^4)25 \cdot 10^5 .$$

From this table we excerpt the following:

	n			
$P \cdot 10^{-5}$	2	3	4	5
5	1751	65	1	–
10	3819	191	7	–
15	6021	311	13	–
20	8323	433	27	4
25	10690	549	47	5

All triples, quadruples, and quintuples with $p \leqq 2{,}533{,}500$ are listed. Listings of such multiplets are continued to $p \leqq 5{,}060{,}250$, but are *not complete* here because of the computation method. Similarly, the $\pi_n(P)$ listed for $P > 25 \cdot 10^5$ are merely lower bounds. The criterion for listing multiplets is that at least one triangle of the multiplet, with sides $a^2 \pm b^2$ and $2ab$, has both generators a and b less than 1126.

The functions $T(P)$ and $\pi_1(P)$ are not discussed, and $\pi_n(P) = 0$ over this range for $n > 5$.

D. S.

1. A. S. ANEMA, UMT **106**, *MTAC*, v. 4, 1950, p. 224.
2. A. S. ANEMA, UMT **111**, *MTAC*, v. 5, 1951, p. 28.
3. A. S. ANEMA & F. L. MIKSA, UMT **107**, *MTAC*, v. 4, 1950, p. 224.
4. F. L. MIKSA, UMT **133**, *MTAC*, v. 5, 1951, p. 232.

22 [9].—M. F. JONES, 22900D *Approximation to the Square Roots of the Primes less than 100*, Memorial University of Newfoundland, St. John's, Newfoundland, Canada, June 1967. Copy of computer printout deposited in the UMT file.

There are given here values of $\sqrt{p}$ accurate to 22900D for each prime p less than 100. These values were computed on an IBM 1620 by "twelve stages" of Newton's method starting with 25D approximations. (Since ten iterations should suffice, one presumes that the first stage here is merely the initial approximation, and that the twelfth was performed to check the eleventh.) A few of the high-accuracy digits in $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$ here were compared with those of other recent calculations [1], [2], [3], [4] and no discrepancy was found.

The decimal-digit distribution over the *entire* range of 22900D is also given, together with corresponding values of χ^2. No counts are given for smaller blocks. For $p = 17, 19, 67, 37$ one finds

$$\chi^2 = 17.74, 17.32, 16.43, \text{and } 15.41\,,$$

respectively, and the author concludes: "On the basis of this test, it can be said with a 95% confidence that the tested digits of $\sqrt{17}$ and $\sqrt{19}$ are not random and further that $\sqrt{67}$ and $\sqrt{37}$ come very close to the rejection region."

Nonstatisticians often find the χ^2 statistic as elusive as nonphysicists find entropy; dubious conclusions similar to the foregoing are even found in published papers; and while the reviewer is himself a nonstatistician, he feels called upon to comment. The χ^2-statistic for a random sequence, according to the theory, should be *distributed* around a mean nearly equal to the number of degrees of freedom, here equal to 9, according to a prescribed distribution if a sufficient number of samples of χ^2 are computed. Now, 95% of such values (and this is the figure that the author alludes to) should have $\chi^2 < 16.9$. But that is merely another way of saying that *one time out of twenty* the χ^2 will be larger. If, with *one trial only*, one obtains a χ^2 somewhat greater than 16.9, this is hardly something to be alarmed at, since nothing is shown to indicate that this trial was not that "one time." That the author is being unduly concerned about the *large* χ^2 found for $\sqrt{(17)}$ is also shown by his lack of concern for certain small values. Thus, 5% of the time (only) the χ^2 should be <3.3, but the χ^2 for the $\sqrt{5}$ here is 3.05, and therefore the $\sqrt{5}$ is equally "nonrandom"—that is, not at all—as the $\sqrt{(17)}$ is. Actually, all experience has shown that similar conclusions as those here, for example, von Neumann's concern about the low χ^2 for the first 2000 digits of e, are generally rectified when a larger sample of χ^2 values is computed.

D. S.

1. M. LAL, *Expansion of* $\sqrt{2}$ *to* 19600 *Decimals*, reviewed in *Math. Comp.*, v. 21, 1967, pp. 258–259, UMT **17**.