

The functions $T(P)$ and $\pi_1(P)$ are not discussed, and $\pi_n(P) = 0$ over this range for $n > 5$.

D. S.

1. A. S. ANEMA, UMT 106, *MTAC*, v. 4, 1950, p. 224.
2. A. S. ANEMA, UMT 111, *MTAC*, v. 5, 1951, p. 28.
3. A. S. ANEMA & F. L. MIKSA, UMT 107, *MTAC*, v. 4, 1950, p. 224.
4. F. L. MIKSA, UMT 133, *MTAC*, v. 5, 1951, p. 232.

22 [9].—M. F. JONES, 22900D *Approximation to the Square Roots of the Primes less than 100*, Memorial University of Newfoundland, St. John's, Newfoundland, Canada, June 1967. Copy of computer printout deposited in the UMT file.

There are given here values of \sqrt{p} accurate to 22900D for each prime p less than 100. These values were computed on an IBM 1620 by "twelve stages" of Newton's method starting with 25D approximations. (Since ten iterations should suffice, one presumes that the first stage here is merely the initial approximation, and that the twelfth was performed to check the eleventh.) A few of the high-accuracy digits in $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$ here were compared with those of other recent calculations [1], [2], [3], [4] and no discrepancy was found.

The decimal-digit distribution over the *entire* range of 22900D is also given, together with corresponding values of χ^2 . No counts are given for smaller blocks. For $p = 17, 19, 67, 37$ one finds

$$\chi^2 = 17.74, 17.32, 16.43, \text{ and } 15.41,$$

respectively, and the author concludes: "On the basis of this test, it can be said with a 95% confidence that the tested digits of $\sqrt{17}$ and $\sqrt{19}$ are not random and further that $\sqrt{67}$ and $\sqrt{37}$ come very close to the rejection region."

Nonstatisticians often find the χ^2 statistic as elusive as nonphysicists find entropy; dubious conclusions similar to the foregoing are even found in published papers; and while the reviewer is himself a nonstatistician, he feels called upon to comment. The χ^2 -statistic for a random sequence, according to the theory, should be *distributed* around a mean nearly equal to the number of degrees of freedom, here equal to 9, according to a prescribed distribution if a sufficient number of samples of χ^2 are computed. Now, 95% of such values (and this is the figure that the author alludes to) should have $\chi^2 < 16.9$. But that is merely another way of saying that *one time out of twenty* the χ^2 will be larger. If, with *one trial only*, one obtains a χ^2 somewhat greater than 16.9, this is hardly something to be alarmed at, since nothing is shown to indicate that this trial was not that "one time." That the author is being unduly concerned about the *large* χ^2 found for $\sqrt{17}$ is also shown by his lack of concern for certain small values. Thus, 5% of the time (only) the χ^2 should be < 3.3 , but the χ^2 for the $\sqrt{5}$ here is 3.05, and therefore the $\sqrt{5}$ is equally "nonrandom"—that is, not at all—as the $\sqrt{17}$ is. Actually, all experience has shown that similar conclusions as those here, for example, von Neumann's concern about the low χ^2 for the first 2000 digits of e , are generally rectified when a larger sample of χ^2 values is computed.

D. S.

1. M. LAL, *Expansion of $\sqrt{2}$ to 19600 Decimals*, reviewed in *Math. Comp.*, v. 21, 1967, pp. 258–259, UMT 17.

2. KOKI TAKAHASHI & MASAOKI SIBUYA, *The Decimal and Octal Digits of \sqrt{n}* , reviewed in *Math. Comp.*, v. 21, 1967, pp. 259–260, UMT 18.

3. M. LAL, *Expansion of $\sqrt{3}$ to 19600 Decimals*, reviewed in *Math. Comp.*, v. 21, 1967, p. 731, UMT 84.

4. M. LAL, *First 39000 Decimal Digits of $\sqrt{2}$* , reviewed in *Math. Comp.*, v. 22, 1968, p. 226, UMT 12.

23 [9].—MOHAN LAL & JAMES DAWE, *Tables of Solutions of the Diophantine Equation $x^2 + y^2 + z^2 = k^2$* , Memorial University of Newfoundland, St. John's, Newfoundland, Canada, February 1967, xiii + 60 pp., 28 cm. Price \$7.50.

Table 3 of this attractively printed and bound volume lists all integral solutions of

$$(1) \quad x^2 + y^2 + z^2 = k^2 \quad (0 < x \leq y \leq z)$$

for $k = 3(2)381$. The brief introduction points out that F. L. Miksa published such a table to $k = 207$, but neglects to mention that he also extended this himself to $k = 325$ [1]. The present extension does not, therefore, constitute a large increase in the upper limit for k , but since the number of solutions of (1) is roughly proportional to k , the number of listed solutions is increased over [1] by a somewhat larger factor.

The imprimitive solutions—those where x , y , z , and k all have a common divisor > 1 —are marked with an asterisk. (In [1] this was done only for $k > 207$.)

Table 1 lists the number of solutions for each k , and Table 2 lists the number of primitive solutions. (In [1], this data was not given.) The introduction makes no reference to theoretical treatments of the numbers in Tables 1, 2, cf. [2], nor are any empirical observations made concerning these numbers. It is quite convincing, however, from a brief examination of these results, and without reference to the theory, that if k is a prime of the form $8n \pm 1$ or $8n \pm 5$, then there are exactly n solutions, all of which, of course, are primitive.

The introduction points out that none of the listed solutions of (1) are of the form

$$(2) \quad x^4 + y^4 + z^4 = k^4,$$

and this proves that (2) has no solutions for $k < 20$. But M. Ward had already proved that result for $k \leq 10^4$, and recently [3] this was extended to $k \leq 22 \cdot 10^4$.

D. S.

1. FRANCIS L. MIKSA, "A table of integral solutions of $A^2 + B^2 + C^2 = R^2$, etc.," UMT 82, *MTAC*, v. 9, 1955, p. 197.

2. LEONARD EUGENE DICKSON, *History of the Theory of Numbers*, Volume II, Chapter VII, Stechert, New York, 1934.

3. L. J. LANDER, T. R. PARKIN & J. L. SELFRIDGE, "A survey of equal sums of like powers," *Math. Comp.*, v. 21, 1967, p. 446.

24 [12, 13.35].—J. HARTMANIS & R. E. STEARNS, *Algebraic Structure Theory of Sequential Machines*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1966, viii + 211 pp., 23 cm. Price \$9.00.

This excellent little book brings together in one place most known results on the algebraic structure theory of sequential machines. By a structure theory for sequential machines the authors mean "an organized body of techniques and results which deal with the problems of how sequential machines can be realized from