2. KOKI TAKAHASHI & MASAAKI SIBUYA, *The Decimal and Octal Digits of* $\sqrt{n}$, reviewed in *Math. Comp.*, v. 21, 1967, pp. 259–260, UMT 18.
3. M. LAL, *Expansion of* $\sqrt{3}$ *to* 19600 *Decimals*, reviewed in *Math. Comp.*, v. 21, 1967, p. 731, UMT 84.
4. M. LAL, *First* 39000 *Decimal Digits of* $\sqrt{2}$, reviewed in *Math. Comp.*, v. 22, 1968, p. 226, UMT 12.

23 [9].—MOHAN LAL & JAMES DAWE, *Tables of Solutions of the Diophantine Equation* $x^2 + y^2 + z^2 = k^2$, Memorial University of Newfoundland, St. John's, Newfoundland, Canada, February 1967, xiii + 60 pp., 28 cm. Price $7.50.

Table 3 of this attractively printed and bound volume lists all integral solutions of

$$x^2 + y^2 + z^2 = k^2 \qquad (0 < x \leqq y \leqq z) \tag{1}$$

for $k = 3(2)381$. The brief introduction points out that F. L. Miksa published such a table to $k = 207$, but neglects to mention that he also extended this himself to $k = 325$ [1]. The present extension does not, therefore, constitute a large increase in the upper limit for k, but since the number of solutions of (1) is roughly proportional to k, the number of listed solutions is increased over [1] by a somewhat larger factor.

The imprimitive solutions—those where x, y, z, and k all have a common divisor >1—are marked with an asterisk. (In [1] this was done only for $k > 207$.)

Table 1 lists the number of solutions for each k, and Table 2 lists the number of primitive solutions. (In [1], this data was not given.) The introduction makes no reference to theoretical treatments of the numbers in Tables 1, 2, cf. [2], nor are any empirical observations made concerning these numbers. It is quite convincing, however, from a brief examination of these results, and without reference to the theory, that if k is a prime of the form $8n \pm 1$ or $8n \pm 5$, then there are exactly n solutions, all of which, of course, are primitive.

The introduction points out that none of the listed solutions of (1) are of the form

$$x^4 + y^4 + z^4 = k^4\,, \tag{2}$$

and this proves that (2) has no solutions for $k < 20$. But M. Ward had already proved that result for $k \leqq 10^4$, and recently [3] this was extended to $k \leqq 22 \cdot 10^4$.

D. S.

1. FRANCIS L. MIKSA, "A table of integral solutions of $A^2 + B^2 + C^2 = R^2$, etc.," UMT 82, *MTAC*, v. 9, 1955, p. 197.
2. LEONARD EUGENE DICKSON, *History of the Theory of Numbers*, Volume II, Chapter VII, Stechert, New York, 1934.
3. L. J. LANDER, T. R. PARKIN & J. L. SELFRIDGE, "A survey of equal sums of like powers," *Math. Comp.*, v. 21, 1967, p. 446.

24 [12, 13.35].—J. HARTMANIS & R. E. STEARNS, *Algebraic Structure Theory of Sequential Machines*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1966, viii + 211 pp., 23 cm. Price $9.00.

This excellent little book brings together in one place most known results on the algebraic structure theory of sequential machines. By a structure theory for sequential machines the authors mean "an organized body of techniques and results which deal with the problems of how sequential machines can be realized from sets of smaller component machines, how these component machines have to be interconnected, and how 'information' flows in and between these machines when they operate."

The book is written so as to be understandable by people of either an engineering or mathematical background, and it is essentially self-contained. The necessary mathematical concepts are introduced in Chapter 0; the fundamental concepts of sequential machines are given in Chapter 1. Chapter 2 introduces the substitution property, as applied to partitions of the state set, and shows how it relates to two-machine serial and parallel decompositions and to state reduction. Partition pairs and pair algebra are presented in Chapter 3 and are applied to the state assignment problem. A general concept of a network of machines is also given in this chapter. Chapter 4 is concerned with the relationship between the lattice of the partitions with the substitution property and "loop-free" decompositions of a machine (i.e., with decompositions in which the component machines are not connected in a circle). This reviewer found these results to be a particularly fine example of the structural approach. Multiple coding of states, or "state splitting" is investigated in Chapter 5. The notion of set systems (or "overlapping partitions") is introduced in this section and serves as a primary tool for the analysis in this and subsequent chapters. It seems to this reviewer that by introducing the concept of a set system earlier in the development, the authors could have made their treatment somewhat more concise and elegant. Chapter 6 is concerned with the mathematical treatment of the intuitive notion of feedback and its role in machine decomposition. A study is also made of the propagation of state-transition and input errors. Chapter 7 consists of a partial treatment of the results of Krohn and Rhodes [1] on the relationship between the semigroup and the decompositions of a sequential machine. The treatment follows that of H. Zeiger [2], but the more "involved and uninformative" proofs are omitted.

As a rule, the book is well written and is well supplied with motivation and examples. The book contains a fair number of typographic errors, but no particularly serious ones were encountered. There is, however, an error in the statement (and thus the proof) of Lemma 7.7, which was brought to my attention by Ann Penton. In brief, the function λ^*, defined at the bottom of p. 203, has a range which properly contains, rather than is, the set ψ. Penton [3] suggests that this difficulty can be overcome by introducing the notion of a *cover* of the set S of states, that is a collection of subsets of S whose union is S. Extending the concepts of machine, the substitution property, and the Max operation, to covers, we may replace 7.7 with the following statement:

Given a machine $M = (S, I, \delta)$, a set system $\phi > 0$ with substitution property on S and a cover ϕ' for S such that Max $\phi' = \phi$ and a machine $M_{\phi'} = (S', I, \delta')$ for ϕ' such that under a mapping $\lambda' : S \to \phi'$ we have

$$\lambda'(\delta(s', a)) = \delta(\lambda'(s'), a)$$

for all $s' \in S'$ and $a \in I$, then

1. there exists a set system $\psi < \phi$ with substitution property and a cover ψ' for S such that Max $\psi' = \psi$, and

2. there exists a P-R machine $M_{\phi/\psi}$ such that the group part of the semigroup of $M_{\phi/\psi}$ is a factor group of a subgroup of M, and

3. the serial connection of $M_{\phi'}$ and $M_{\phi/\psi}$ is a machine $M_{\psi'}$ for cover ψ' with function λ^* mapping the states of $M_{\psi'}$ *onto* the set ψ'.

(Thus $M_{\psi'}$ replaces M_ψ and, ψ' replaces ψ in the definition of λ^*.)

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1. K. B. Krohn & J. L. Rhodes, *Algebraic Theory of Machines*, Proc. Sympos. Math. Theor. Automata, New York, April 25–26, 1962, *Microwave Research Institute Symposium Series*, Vol. XII, Polytechnic Press, Brooklyn, New York, 1963.
2. H. P. Zeiger, *Loop-free Synthesis of Finite State Machines*, Ph.D. Thesis, Massachusetts Institute of Technology, Cambridge, 1964.
3. Ann Penton, *Algebraic Study of Sequential Machine Decomposition*, Master's Thesis, Wesleyan University, Middletown, Conn., 1967.

25 [12].—Frank Bates & Mary L. Douglas, *Programming Language/One*, Prentice-Hall, Inc., Englewood Cliffs., N. J., 1967, viii + 375 pp., 25 cm. Price $5.95.

This book is a welcome addition to the literature of PL/I. It is written in a clear and concise style covering a wide field. In spite of the uncertainties about how the PL/I language will finally be implemented (witness the frequently changing specifications used by the manufacturer), this book manages to convey an idea of the power of the PL/I language and to develop in the reader a facility for writing clear, efficient PL/I code.

The examples are easily understood and to the point. Several of the problems provide good practice in the fine art of debugging. The answers seem to be correct and complete.

A very fortunate feature of this book is that technical points (e.g., the inaccuracy caused by representing a decimal fraction in a base other than ten) are reserved until the end of the appropriate chapter, where they appear in sets of notes. This is commendable, since it provides the reader with useful technical information without disturbing the flow of the more basic material.

The book contains several useful appendices, including PL/I character sets, keywords and abbreviations, built-in functions, conditions and format specifications. These tables, combined with a thorough index, make this book valuable as a reference work for the experienced programmer as well as useful as an introductory text to the subject.

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26 [12].—Charles R. Bauer, Anthony P. Peluso & William S. Worley, Jr., *IItran/360: Self-Instructional Manual and Text*, Addison-Wesley Publishing Co., Reading, Mass., 1967, xi + 212 pp., 28 cm. Price $4.95.

This book is an introduction to a new breed of computer languages called IITRAN, an acronym for Illinois Institute of Technology Translator, a language