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I certify that the statements made by me above are correct and complete.—Gordon L. Walker

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The editorial committee would welcome readers' comments about this microfiche feature. Please send comments to Professor Eugene Isaacson, MATHEMATICS OF COMPUTATION, Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, New York 10012.

# Mathematics of Computation

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Joyce Weil, Tadepalli S. Murty and Desiraju B. Rao, Zeros  
of  $J_n(\lambda)Y_n(\eta\lambda) - J_n(\eta\lambda)Y_n(\lambda)$  and  $J'_n(\lambda)Y'_n(\eta\lambda) - J'_n(\eta\lambda)Y'_n(\lambda)$

The first ten positive zeros of the two functions specified in the title are tabulated to seven significant figures (4D to 6D) for  $n = 0(1)10$  and  $\eta = 0(0.5)0.95$ . Details of the underlying computational procedure have been published [1] by the authors. The zeros of the second function were found in the same manner as those of the first, after use was made of the relation  $Z'_n(x) = \frac{n}{x} Z_n(x) - Z_{n+1}(x)$ , where  $Z_n$  represents either  $J_n$  or  $Y_n$ .

J. W. W.

1. Joyce Weil, Tadepalli S. Murty and Desiraju B. Rao; "Zeros of  $J_n(\lambda)Y_n(\eta\lambda) - J_n(\eta\lambda)Y_n(\lambda)$ ." Math. Comp., v. 21, 1967, pp. 722-7.

$$J_n(\lambda)Y_n(\gamma\lambda) - J_n(\gamma\lambda)Y_n(\lambda) = 0$$

$\eta = 0$	1	2	3	4	5	6	7	8	9	10
1 2.404825	3.011736	5.135622	6.380161	7.548342	8.771463	9.316109	11.08637	12.2259	13.35430	14.47550
2 5.520078	7.015586	8.417244	9.161123	11.06470	12.33860	13.58929	14.82126	16.03777	17.24122	18.43346
3 9.653727	10.17346	11.61934	13.01320	14.37253	15.70017	17.00382	19.28758	19.55453	20.80704	22.04698
4 11.79153	13.32369	14.79375	16.22346	17.61536	18.48015	20.32078	21.64154	22.94517	24.23388	25.50345
5 14.93091	16.47063	17.95341	19.40141	20.82693	22.21740	23.58678	24.93492	26.26681	27.58374	28.88377
6 18.07106	19.61585	21.11639	22.59273	24.01902	25.43034	26.82015	28.19118	29.54566	30.80537	32.21185
7 21.21163	22.76058	24.27017	25.74116	27.19908	28.62661	30.03372	31.42279	32.79580	34.15437	35.49990
8 24.35247	25.90367	27.42037	28.70135	30.37100	31.81171	33.23304	34.63708	36.02561	37.40010	38.76180
9 27.47347	29.36682	30.56920	32.06485	33.54711	34.98878	36.42202	37.83871	39.24344	40.62955	42.33419
10 30.63460	32.16968	33.71622	35.21467	36.69900	38.15956	39.60323	41.03077	42.44388	43.84382	45.23157

$\eta = 0$	1	2	3	4	5	6	7	8	9	.10
1 9.66399	10.860326	5.136070	6.140165	7.548340	8.771476	9.316103	11.08637	12.2259	13.35430	14.47550
2 6.422392	7.104545	8.420219	9.761085	11.06470	12.33860	13.58928	14.82126	16.03777	17.24121	18.43346
3 9.766746	10.34729	11.62995	13.01549	14.37253	15.70017	17.00382	18.28759	19.55454	20.80704	22.04698
4 13.09748	13.60045	14.82058	16.22451	17.61599	18.98014	20.32079	21.64154	22.94516	24.23387	25.50345
5 16.42191	16.96334	18.00875	19.41228	20.82703	22.21779	23.58608	24.93492	26.26680	27.58374	28.88377
6 19.74227	20.13468	21.20166	22.58737	24.01932	25.43035	26.82015	28.13118	29.54566	30.80538	32.21186
7 23.05985	23.41213	24.40268	25.76163	27.19986	28.62665	30.03373	31.42279	32.79579	34.15438	35.49991
8 26.37543	26.69451	27.61315	28.73294	30.37278	31.81180	33.23304	34.63709	36.32561	37.40003	38.76180
9 29.68952	29.98076	30.83324	32.1619	33.54076	34.98879	36.42203	37.83872	39.24344	40.62954	42.00418
10 33.00246	33.27010	34.06223	35.24354	36.70584	38.16033	39.60326	41.03076	42.44388	43.84380	45.23157

$\eta = 0$	1	2	3	4	5	6	7	8	9	.10
1 3.313942	3.940940	5.142341	6.340455	7.588326	8.771416	9.316103	11.08637	12.2259	13.35430	14.47550
2 6.857582	7.3305174	8.4571407	9.764106	11.06498	12.33861	13.58928	14.82126	16.03777	17.24121	18.43346
3 10.311741	10.74831	11.73854	12.92979	14.37375	15.70024	17.00382	18.28759	19.55454	20.80704	22.04698
4 13.88642	14.18863	15.04407	16.26113	17.62120	18.98058	20.32062	21.64154	22.94516	24.23387	25.50345
5 17.38962	17.64330	18.38338	19.51281	20.84345	22.21967	23.56625	24.93494	26.26680	27.58374	28.88376
6 20.88939	21.10737	21.75310	22.77148	24.06632	25.43634	26.42062	27.19124	29.54566	30.80538	32.21186
7 24.38694	24.57746	25.14716	26.07145	27.24567	28.64274	29.03588	31.42402	32.79581	34.15438	35.49991
8 27.88297	28.355219	29.555975	30.529425	31.84771	33.23902	34.63785	36.02570	37.40010	38.76180	
9 31.31795	31.527944	31.98703	32.74463	33.70537	35.06020	36.43636	37.84091	39.24071	40.62857	42.30418
10 34.87213	35.01001	35.42507	36.11705	37.04663	38.63378	39.28647	41.03630	42.44469	43.84382	45.23157