know-how of the computer laboratory; by clouds, theoretical topics that underlie numerical integration. The prerequisite for this book is a course in advanced calculus. It would also be helpful, though not strictly necessary, if the reader had an introductory course in numerical analysis so that he will be familiar with the motivation and the goals of computing. There are several places where some mathematics beyond calculus is used, but they are relatively few. The book is not wholly self-contained; nor has it been possible to include proofs for all statements made. Where these gaps occur, references to other texts or to original articles have been given."

The authors succeed by presenting the subject from a mature point of view and in delightfully readable prose. The mathematics is as sophisticated as the subject matter demands (and on rare occasions, complex variable theory and functional analysis are appealed to). The modest size of this comprehensive monograph reflects the care with which the authors have selected their material. This book will prove to be invaluable to anyone interested in using and/or studying numerical integration and sets a standard for the Blaisdell series which it is going to be hard to maintain. A listing of the chapter headings and appendices will serve to outline the wide scope of the book.

1, Introduction; 2, Approximate Integration over a Finite Interval; 3, Approximate Integration over Infinite Intervals; 4, Error Analysis; 5, Approximate Integration in Two or More Dimensions; 6, Automatic Integration;

Appendix 1, "On the Practical Evaluation of Integrals" by Milton Abramowitz;
Appendix 2, Some FORTRAN Programs;
Appendix 3, Bibliography of ALGOL Procedures;
Appendix 4, Bibliography of Tables;
Appendix 5, Bibliography of Books and Articles.

E. I.

44[2.10, 7].—E. N. Dekanosidze, *Tabitsy korneĭ i vesovykh mnozhiteleĭ obobshchennyĭ polinomov Lagerra* (*Tables of the zeros and weight factors of the generalized Laguerre polynomials*), Computing Center, Acad. Sci. USSR, Moscow, 1966, xx + 306 pp., 27 cm. Price 2.33 rubles.

The main table (Table I, pp. 1–301) consists of 15S approximations (in floating-point form) to the zeros $x_i^{(n;\,\alpha)}$ of the generalized Laguerre polynomials, defined by the Rodrigues formula

$$L_n^{(\alpha)}(x) = \frac{x^{\alpha}e^{x}}{n!}\frac{d^n}{dx^n}(x^{\alpha+n}e^{-x}),$$

and to the associated weight factors $A_i^{(n;\,\alpha)}$ and $B_i^{(n;\,\alpha)}$ $(= A_i^{(n;\,\alpha)} \exp[x_i^{(n;\,\alpha)}])$ occurring in the Gaussian quadrature formulas

$$\int_0^\infty e^{-x}x^\alpha f(x)dx \approx \sum_{i=1}^n A_i^{(n;\,\alpha)} f(x_i^{(n;\,\alpha)}),$$

$$\int_0^\infty x^\alpha f(x)dx \approx \sum_{i=1}^n B_i^{(n;\,\alpha)} f(x_i^{(n;\,\alpha)}).$$

The tabular ranges of the parameters are $i = 1(1)n$, $n = 2(1)16$, $\alpha = 0(-0.01)-0.99$. The values for $n = 1$ are omitted; however, in the introduction the author gives the formulas $x_1^{(1;\ \alpha)} = \alpha + 1$, $A_1^{(1;\ \alpha)} = \Gamma(1 + \alpha)$, $B_1^{(1;\ \alpha)} = \Gamma(1 + \alpha)e^{1+\alpha}$, with the remark that values of the weight factors in this case can be obtained by use of well-known tables.

Table I constitutes an excellent supplement to the closely related tables of Rabinowitz & Weiss [1], Aizenshtat, Krylov & Metleskiĭ [2], Concus, Cassatt, Jaehnig & Melby [3], Concus [4], and Shao, Chen & Frank [5].

Table II (pp. 303–306) includes 15S approxmiations to the zeros $x_{\pm i}^{(2n)}$, $x_{\pm i}^{(2n+1)}$ of the Hermite polynomials, which, as the author notes in the introduction, are related to the zeros of the generalized Laguerre polynomials by means of the equations

$$x_{\pm i}^{(2n)} = \pm[x_i^{(n;\ -0.5)}]^{1/2}, \qquad x_{\pm i}^{(2n+1)} = \pm[x_i^{(n;\ 0.5)}]^{1/2}, \qquad (1 \leqq i \leqq n) .$$

Also included in this table are 15S values of the weight factors associated with the Hermite quadrature formulas

$$\int_{-\infty}^{\infty} e^{-x^2} f(x)dx \approx C_0^{(2n+1)} f(0) + \sum_{i=1}^{n} C_i^{(2n+1)}[f(-x_i^{(2n+1)}) + f(x_i^{(2n+1)})] ,$$

$$\int_{-\infty}^{\infty} f(x)dx \approx D_0^{(2n+1)} f(0) + \sum_{i=1}^{n} D_i^{(2n+1)}[f(-x_i^{(2n+1)}) + f(x_i^{(2n+1)})] ,$$

for $i = 0(1)n$, $n = 2(1)16$. Exact values of the zeros and weight factors corresponding to $n = 0, 1$ are given in the introduction (p. viii). Equations relating the tabulated Hermite integration coefficients to the weight factors $A_i^{(n;\ 0.5)}$ and $B_i^{(n;\ 0.5)}$ appear on p. vii.

The 19-page introduction also includes mathematical details (including omnibus checks) of the calculation of these tables by double-precision arithmetic on the BESM2, as well as illustrative numerical examples of Newton interpolation in Table I and of two applications of that table. The remainder term for generalized Gauss-Laguerre quadrature is given explicitly and was evaluated to 2S when $\alpha = 0$ and -0.995 for the case $f(x) = x^{2n+j}$, $n = 2(1)16$, $j = 0(1)18$. However, only those computed values of this remainder are tabulated (p. xv) which are less than 0.33 when $n < 4$, and which are less than 0.2 when $n \geqq 4$.

Appended to the introduction is a list of 12 references, which includes all the published tables cited in this review.

This reviewer has repeated the author's comparison of relevant sections of his tables with corresponding entries in the tables appearing in [1], [3], and [5]. None of the observed discrepancies exceeds a unit in the least significant tabulated figure; however, a study of these discrepancies suggests that most of the entries in the present tables have been left unrounded.

J. W. W.

1. Philip Rabinowitz & George Weiss, "Tables of abscissas and weights for numerical evaluation of integrals of the form $\int_0^\infty e^{-x}x^n f(x)dx$," *MTAC.*, v. 13, 1959, pp. 285–294.

2. V. S. Aizenshtat, V. I. Krylov & A. S. Metleskiĭ, *Tablitsy dlia chislennogo preobrazovaniia Laplasa i vychisleniia integralov vida* $\int_0^\infty x^s e^{-x} f(x)dx$ (*Tables for Calculating Laplace Transforms and Integrals of the form* $\int_0^\infty x^s e^{-x} f(x)dx$), Izdat. Akad. Nauk SSSR, Minsk, 1962. (See *Math. Comp.*, v. 17, 1963, p. 93, RMT 9.)

3. P. Concus, D. Cassatt, G. Jaehnig & E. Melby, "Tables for the evaluation of $\int_0^\infty x^\beta e^{-x} f(x)dx$ by Gauss-Laguerre quadrature," *Math. Comp.*, v. 17, 1963, pp. 245–256.

4. P. Concus, *Additional Tables for the Evaluation of* $\int_0^\infty x^\beta e^{-x} f(x)dx$ *by Gauss-Laguerre Quadrature*, ms. in UMT file. (See *Math. Comp.*, v. 18, 1964, p. 523, RMT 81.)

5. T. S. Shao, T. C. Chen & R. M. Frank, "Tables of zeros and Gaussian weights of certain associated Laguerre polynomials and the related generalized Hermite polynomials," *Math. Comp.*, v. 18, 1964, pp. 598–616.