REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

61[2.05, 2.20, 2.35, 3, 4, 5].—F. JOHN, *Lectures on Advanced Numerical Analysis*, Gordon and Breach, New York, 1967, xiv + 179 pp., 24 cm. Price $9.50 cloth bound, $6.25 paper bound.

This improved version of Fritz John's New York University lecture notes, entitled *Advanced Numerical Methods* and first issued in 1956, has been long awaited.

From the treatment of norms of vectors and matrices to the study of methods in linear algebra, in approximation theory, in finding roots of systems of equations, in solving ordinary and partial differential equations, the author leaves his characteristic imprint. Here is the work of a master analyst who spent an intensive and short productive period in the illumination of a wide variety of topics in numerical analysis. The book is a classic that invites mathematicians to learn what numerical analysis is about. The reviewer is only one of the generations that have benefitted from this fine work.

E. I.

62[3, 10, 13.35].—A. KAUFMANN, *Graphs, Dynamic Programming, and Finite Games*, translated from French by Henry C. Sneyd, Academic Press, New York, 1967, xvii + 484 pp., 24 cm. Price $14.50.

This volume is an attempt to introduce at both an elementary level and a mathematically rigorous level three areas of modern mathematics—graph theory, dynamic programming, and game theory. To do this, the author divides the presentation into two parts, each part having three chapters, one for each topic. The first part has elementary applications of each theory, which are worked out in detail. The second part is a detailed mathematical treatment of each area.

Typical of the first part of the book are worked examples, such as critical paths, minimal trees, network flows and textual emendation problems, which are solved step-by-step in the graph chapter, with extensive diagrams and numerical calculations. In these elementary chapters concepts are introduced quickly and effectively with a minimum of definitional complexity and immediate appeal to easily understood examples.

The second part of the work is an attempt to treat in less than 100 pages per topic the mathematical theory of each of the three areas. The graph theory chapter in this part of the book paraphrases much of the material in Berge [1], but in addition contains detailed descriptions of an algorithm of the author's for finding Hamilton paths, and a section on the graph isomorphism problem. The game theory chapter follows McKinsey [2] in its proof of the fundamental theorem of game theory on the existence of optimal strategies for rectangular games. Various examples are worked (also many examples are worked in the elementary chapter on game theory) and the correspondence to linear programming is treated along with brief presentations of other topics. The dynamic programming chapter has material which appears in more detail in Kaufmann and Cruon [3] in another volume of the same series. Of

special interest is the author's use of the zeta-transform to study Markovian processes.

In attempting to treat so much material in the advanced portion of the book, the author is not able to maintain the same clarity found in the first part of the book. Some topics are explained thoroughly in the advanced part, but others are poorly developed and confusing. I count the first half a great success as a lucid introduction to these topics. However, the second half is more useful when augmented by readings from other sources. A minor irritation is an abundance of typographical errors. Its use as a text is possible for many types of courses; it has a useful bibliography and index but no exercises. It is a wide-ranging book with sections comprehensible at many levels of mathematical sophistication.

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1. CLAUDE BERGE, *The Theory of Graphs and Its Applications*, Methuen, London, 1962.
2. J. C. C. MCKINSEY, *An Introduction to the Theory of Games*, McGraw-Hill, New York, 1952.
3. A. KAUFMANN & R. CRUON, *Dynamic Programming*, Academic Press, New York, 1967.

63[4].—WERNER GLASMACHER & DIETMAR SOMMER, *Implizite Runge-Kutta-Formeln*, Westdeutscher Verlag, Köln, 1966, 178 pp., 24 cm. Price DM 44.00.

Implicit Runge-Kutta methods based on Gauss-Legendre quadrature formulae were introduced by Ceschino and Kuntzmann [1] and by the reviewer [2]. Methods based on Lobatto and on Radau quadrature formulae were introduced by the reviewer [3]. These methods have the property that if m is the number of stages, then the order is $2m$ for the Gauss case, $2m - 2$ for the Lobatto case, and $2m - 1$ for the two types of the Radau case. A disadvantage of the methods for integrating differential equations in practice is their implicit nature.

The only previous tables of the coefficients of the methods are those of the reviewer [4], which give coefficients to 20D for methods of the four types with orders not exceeding 20. The present tables give coefficients to 24S for the four methods up to $m = 20$. In addition to the tables, full descriptions of the evaluation methods are given, including an Algol programme and flow charts.

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1. F. CESCHINO & J. KUNTZMANN, *Problèmes Différentiels de Conditions Initiales*, Dunod, Paris, 1963.
2. J. C. BUTCHER, "Implicit Runge-Kutta processes," *Math. Comp.*, v. 18, 1964, pp. 50–64.
3. J. C. BUTCHER, "Integration processes based on Radau quadrature formulas," *Math. Comp.*, v. 18, 1964, pp. 233–244.
4. J. C. BUTCHER, *Tables of Coefficients for Implicit Runge-Kutta Processes*, ms. of 9 sheets deposited in the UMT file [see *Math. Comp.*, v. 19, 1965, p. 348, RMT 56].

64[4].—MINORU URABE, *Nonlinear Autonomous Oscillations, Analytical Theory*, Academic Press, New York, 1967, xi + 330 pp., 24 cm. Price $16.00.

This monograph contains results in the theory of nonlinear autonomous oscillations, most of them based on the author's research. The main topic is the analytical