The title of the book suggests that the central theme of the conference in which these papers were presented was Queueing Theory and its Applications. However, a glance at the topics discussed discloses that the central theme has been stretched to include a wide variety of related topics.

The papers have been grouped into six sections, each containing papers presented at a particular session. These are: *Point Processes,* with papers "Processes Ponctuels dans les Modéles de Files d'Attente et de Stockage" by M. Girault, "A Self-Service System with Scheduled Arrivals" by A. Mercer and J. B. Parker, and "The Application of Stochastic Processes to Countdown Analysis" by David S. Stoller; *Mathematical Methods of Queueing Theory,* with papers "Pollaczek Method in Queueing Theory" by R. Syski, "Utilisation de la Théorie des Processus Semi-Markoviens dans l'Étude de Problèmes de Files d'Attente" by J. P. Lambotte and J. Teghem, and "The Custodian's Problem" by L. Kosten; *Nearly Saturated Queues and Transient Behavior,* with papers "Approximations for Queues in Heavy Traffic" by J. F. C. Kingman, "The Advantage of Precedence Operation with Overloaded or Fully Loaded Circuits" by E. P. G. Wright, and "Numerical Methods of Determining the Transient Behavior of Queues with Variable Arrival Rate" by E. L. Leese; *Inventories and Maintenance,* with papers "Files d'Attente et Stocks" by R. Debry and J. Teghem, and "A Mathematical Study of Unscheduled Open Hearth Furnace Repairs" by R. R. P. Jackson; *Statistical Problems,* with papers "Inference Statistique dans les Processus de Markov-Applications dans les Problémes de Files d'Attente" by Simone Huyberechts, "Estimation of the Dispersion Parameter of an (A,B) Process" by L. J. Govier and T. Lewis, and "Spectral Analysis of Time Series Generated by Simulation Models" by G. S. Fishman and P. J. Kiviat; *Case Histories and Simulation* with papers "Checking the Simulation of a Queueing Type Situation" by B. W. Conolly, "A Study of a Multi-Channel Queueing Process in a Communications Terminal" by G. Hatzikostandis and S. Howe, "Arrivèe à un bac d'un Traffic Composite" by B. Roy and J. Auberger, "Files d'Attente et Processus Markoviens à une Intersection de Traffic Comparison de Divers Modes de Contrôle" by P. Passau, and "Comparison Between Analogic and Digital Simulation of Telephone Gradings" by I. Capetti.

The book also includes a paper "The Application of Queueing Theory in Operations Research" by Philip Morse as Introduction, a paper "Ordering Disorderly Queues" by Thomas L. Saaty as Conclusion, and Final Remarks by R. Fortet.

Some of these papers have appeared before in journals in one form or the other (for instance see E. L. Leese and D. W. Boyd "Numerical Methods of Determining the Transient Behavior of Queues with Variable Arrival Rates," *J. Canad. Operations Res. Soc.,* v. 4, 1966, pp. 1–13). In the reviewer's opinion the major contribution of the book seems to lie in exposing to the English speaking world some of the outstanding work being carried out in France and Belgium.

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74[9].—ALBERT H. BEILER, *Consecutive Hypotenuses of Pythagorean Triangles,* ms. of 11 typewritten pp. (including table) deposited in the UMT file.

The author concerns himself with sequences of r integers:

$$z, z + 1, z + 2, \cdots, z + r - 1 \tag{1}$$

such that each of these r numbers can be the hypotenuse of a Pythagorean triangle, while $z - 1$ and $z + r$ cannot so be. Thus, he has sequences of *exactly* r Pythagorean triangles which have consecutive hypotenuses. In his Table 1 he lists the first such z if it does not exceed 10^5. There is such a z for $r = 2(1)22$ and for $r = 27$, but for no other r except 1. In his table he gives the complete sequence (1) with each number completely factored and expressed as

$$z + i = Q(m^2 + n^2), \qquad (n \neq m). \tag{2}$$

The number $z + i$ can be written as (2) if and only if it has at least one prime divisor $= 4k + 1$, this being the criterion for $z + i$ to be a Pythagorean hypotenuse.

We abbreviate his Table 1 by listing merely the first $z < 10^5$ for each possible r. An industrious and interested reader could then reconstruct the sets of triangles.

r	z	r	z	r	z
2	25	3	39	4	50
5	218	6	775	7	949
8	673	9	403	10	1597
11	2190	12	2820	13	6050
14	8577	15	12423	16	27325
17	34075	18	52724	19	37088
20	74649	21	68150	22	43795
23	——	24	——	25	——
26	——	27	87594		

Thus, for $r = 4$, the triangles are

$$(14, 48, 50), \quad (24, 45, 51), \quad (20, 48, 52), \quad (28, 45, 53).$$

For any r, the paper here shows how one could compute a set of *at least* (although not necessarily *exactly*) r triangles, but this method will usually not yield the smallest such z.

There is no discussion of the asymptotic behavior as $z \to \infty$. The whole tone of the paper could suggest to the reader that such long strings of r triangles are rare. Actually, as $z \to \infty$ the opposite is true, since the numbers that *cannot* be written as (2) have zero density. This is clear, since such numbers can have as prime factors only 2 and the primes $4k - 1$. By a theorem due to Landau, cf. [1], the number of such (nonhypotenuse) numbers $\leqq x$ is asymptotic to

$$\frac{Ax}{\sqrt{(\log x)}}$$

for some computable constant A. While such numbers are common for small x, they take on a local density that (very gradually) goes to zero.

For the finite analogue of consecutive integers whose prime factors are only those taken from a given finite set, see Lehmer's analysis [2].

D. S.

1. Daniel Shanks & Larry P. Schmid, "Variations on a theorem of Landau, Part I," *Math. Comp.*, v. 20, 1966, pp. 551–569. See also [2] of this paper.
2. D. H. Lehmer, "On a problem of Størmer," *Illinois J. Math.*, v. 8, 1964, pp. 57–79. Reviewed in RMT 67, *Math. Comp.*, v. 18, 1964, p. 510.

75[9].—Mohan Lal & James Dawe, *Solutions of the Diophantine Equation* $x^2 - Dy^4 = k$, Memorial University of Newfoundland, October 1967, v + 122 pp. Deposited in the UMT file.

Bound in a gorgeous (cloudy-blue) loose-leaf binder are tables of solutions of

$$x^2 - Dy^4 = k$$

with $D = 2(1)43$, excluding the squares: $D = 4, 9, 16, 25, 36$, with $|k| \leqq 999$, and with $y \leqq 200{,}000$. All solutions for $D = 2$ are listed first, with $|k|$ in ascending order, then those with $D = 3$, etc. The *imprimitive* solutions are those with

$$x^2 = a^2b^2\,, \qquad D = a^2d\,, \qquad k = a^2K\,,$$

or with

$$x^2 = z^2a^4\,, \qquad y^4 = a^4b^4\,, \qquad k = a^4K\,.$$

These are marked with an asterisk. There are 2672 primitive solutions with k positive and 2150 primitive solutions with k negative. The tables are prefaced by their note [1] appearing elsewhere in this issue.

Here are some observations obtained by casual examination of the tables.

A. Although solutions were sought with $y \leqq 200{,}000$, there are, in fact, none here with $y > 6227$. This "largest" solution is

$$54836879^2 - 2\cdot 6227^4 = 959\,.$$

This strengthens, some, the Result #1 in [1], and suggests that for most pairs $[D, k]$, at least, the sets of solutions (x, y) here are *complete*. The authors cautiously refrain from drawing this inference.

B. The maximum number (six) of primitive solutions, occurs for $[D, k] = [3, 913]$ and $[19, 657]$. Specifically, for the first, $(x, y) = (31, 2), (34, 3), (41, 4), (626, 19), (51241, 172)$, and $(1292969, 864)$, while for the second we have $(26, 1), (31, 2), (159, 6), (354, 9), (2306, 23)$, and $(53706, 111)$.

C. Whenever D is not of the form $u^2 + v^2$, we cannot have solutions for both $[D, k]$ and $[D, -k]$, since that would imply the impossible equation

$$x_1^2 + x_2^2 = D(y_1^4 + y_2^4)\,.$$

Thus, from the two cases above, there are *no* solutions for $[3, -913]$ or $[19, -657]$. But, *more generally*, it is noted that whenever $[D, k]$ has six, five, or four primitive solutions, then $[D, -k]$ has no solution here, whether or not $D = u^2 + v^2$. This is very similar to our observation [2] concerning

$$y^3 - x^2 = \pm k\,.$$

For some cases of three solutions, such as $[D, k] = [5, -44]$, or $[5, -209]$, one does find a solution for $[D, -k]$. No such three-and-one set is found, however, if $D = 2$.