1. Daniel Shanks & Larry P. Schmid, "Variations on a theorem of Landau, Part I," *Math. Comp.*, v. 20, 1966, pp. 551–569. See also [2] of this paper.
2. D. H. Lehmer, "On a problem of Størmer," *Illinois J. Math.*, v. 8, 1964, pp. 57–79. Reviewed in RMT 67, *Math. Comp.*, v. 18, 1964, p. 510.

75[9].—Mohan Lal & James Dawe, *Solutions of the Diophantine Equation* $x^2 - Dy^4 = k$, Memorial University of Newfoundland, October 1967, v + 122 pp. Deposited in the UMT file.

Bound in a gorgeous (cloudy-blue) loose-leaf binder are tables of solutions of

$$x^2 - Dy^4 = k$$

with $D = 2(1)43$, excluding the squares: $D = 4, 9, 16, 25, 36$, with $|k| \leqq 999$, and with $y \leqq 200{,}000$. All solutions for $D = 2$ are listed first, with $|k|$ in ascending order, then those with $D = 3$, etc. The *imprimitive* solutions are those with

$$x^2 = a^2b^2, \qquad D = a^2d, \qquad k = a^2K,$$

or with

$$x^2 = z^2a^4, \qquad y^4 = a^4b^4, \qquad k = a^4K.$$

These are marked with an asterisk. There are 2672 primitive solutions with k positive and 2150 primitive solutions with k negative. The tables are prefaced by their note [1] appearing elsewhere in this issue.

Here are some observations obtained by casual examination of the tables.

A. Although solutions were sought with $y \leqq 200{,}000$, there are, in fact, none here with $y > 6227$. This "largest" solution is

$$54836879^2 - 2 \cdot 6227^4 = 959.$$

This strengthens, some, the Result #1 in [1], and suggests that for most pairs $[D, k]$, at least, the sets of solutions (x, y) here are *complete*. The authors cautiously refrain from drawing this inference.

B. The maximum number (six) of primitive solutions, occurs for $[D, k] = [3, 913]$ and $[19, 657]$. Specifically, for the first, $(x, y) = (31, 2), (34, 3), (41, 4), (626, 19), (51241, 172)$, and $(1292969, 864)$, while for the second we have $(26, 1), (31, 2), (159, 6), (354, 9), (2306, 23)$, and $(53706, 111)$.

C. Whenever D is not of the form $u^2 + v^2$, we cannot have solutions for both $[D, k]$ and $[D, -k]$, since that would imply the impossible equation

$$x_1^2 + x_2^2 = D(y_1^4 + y_2^4).$$

Thus, from the two cases above, there are *no* solutions for $[3, -913]$ or $[19, -657]$. But, *more generally*, it is noted that whenever $[D, k]$ has six, five, or four primitive solutions, then $[D, -k]$ has no solution here, whether or not $D = u^2 + v^2$. This is very similar to our observation [2] concerning

$$y^3 - x^2 = \pm k.$$

For some cases of three solutions, such as $[D, k] = [5, -44]$, or $[5, -209]$, one does find a solution for $[D, -k]$. No such three-and-one set is found, however, if $D = 2$.

D. A curiosity, following from the last point, is that [2, 959] has five solutions, [2, −959] none, and [8, −959] five; [2, −161] four solutions, [2, 161] none, and [8, 161] four. While one naturally suspects some algebraic relationship here, none was discovered by the reviewer—probably through insufficient diligence. The result is not general. Thus, [2, 194] has three solutions while both [8, 194] and [8, −194] have none.

D. S.

1. MOHAN LAL & JAMES DAWE, "Solutions of the Diophantine equation $x^2 - Dy^4 = k$," *Math. Comp.*, v. 22, 1968, pp. 679–682.
2. RMT **89**, *Math. Comp.*, v. 20, 1966, pp. 620–621.

76[10].—SAUNDERS MACLANE & GARRETT BIRKHOFF, *Algebra*, The Macmillan Co., New York, 1967, xix + 598 pp., 24 cm. Price $11.95.

In spite of the similarity of the titles and the coincidence of the names (although not the sequence of names) of the authors, this is not a new edition of the *Survey of Modern Algebra* (Macmillan Co., 1953) but a new book.

The motivation for it is summarized in the first paragraph of the Preface: "Recent years have seen striking developments in the conceptual organization of mathematics. These developments use certain new concepts such as 'module,' 'category,' and 'morphism' which are algebraic in character and which indeed can be introduced naturally on the basis of elementary materials. The efficiency of these ideas suggests a fresh presentation of algebra."

As in the *Survey of Modern Algebra*, the concepts and basic facts of the theory of sets, integers, groups, rings, fields, matrices, and vector spaces are introduced and proved; in addition, modules, lattices, multilinear algebra and other topics have their own chapters and are treated either in greater detail or as new subjects. But most of these chapters are used now also for the purpose of introducing and illustrating the concepts of "category," "functor," and "universal element" which, in the penultimate chapter on *Categories and adjoint functors* become the main topic of the book. Functors on sets to sets are introduced on page 24. The definition of a universal element appears on page 26. Its description as "the most important concept in algebra" seems to refer to the present book rather than to algebra as a discipline (but this is indicated merely by the italicizing of the word "algebra").

Concrete categories are introduced on page 64. However, most of the theorems and proofs in the book can be read without knowledge of the theory of categories.

The book is extremely well organized and very well written. Examples illustrating a new concept are given immediately after its definition. Chapters and even sections are preceded by brief, summarizing statements. Theorems are followed frequently by elucidating comments. Proofs are chosen on the basis of transparency rather than brevity; for instance, the first Sylow theorem is proved in the traditional manner without using Wielandt's elegant combinatorial argument, and the Jordan-Hoelder theorem is proved without using the powerful but difficult lemma of Zassenhaus.

The omission of Galois Theory (which had a brief but important chapter in the "Survey") is deplored by the authors and is easily explained by the fact that the present book has 598 pages versus 472 of the "Survey" which also had a smaller format. However, this very fact indicates a serious difficulty arising in the teaching