

D. A curiosity, following from the last point, is that [2, 959] has five solutions, [2, -959] none, and [8, -959] five; [2, -161] four solutions, [2, 161] none, and [8, 161] four. While one naturally suspects some algebraic relationship here, none was discovered by the reviewer—probably through insufficient diligence. The result is not general. Thus, [2, 194] has three solutions while both [8, 194] and [8, -194] have none.

D. S.

1. MOHAN LAL & JAMES DAWE, "Solutions of the Diophantine equation  $x^2 - Dy^4 = k$ ," *Math. Comp.*, v. 22, 1968, pp. 679-682.
2. RMT 89, *Math. Comp.*, v. 20, 1966, pp. 620-621.

76[10].—SAUNDERS MACLANE & GARRETT BIRKHOFF, *Algebra*, The Macmillan Co., New York, 1967, xix + 598 pp., 24 cm. Price \$11.95.

In spite of the similarity of the titles and the coincidence of the names (although not the sequence of names) of the authors, this is not a new edition of the *Survey of Modern Algebra* (Macmillan Co., 1953) but a new book.

The motivation for it is summarized in the first paragraph of the Preface: "Recent years have seen striking developments in the conceptual organization of mathematics. These developments use certain new concepts such as 'module,' 'category,' and 'morphism' which are algebraic in character and which indeed can be introduced naturally on the basis of elementary materials. The efficiency of these ideas suggests a fresh presentation of algebra."

As in the *Survey of Modern Algebra*, the concepts and basic facts of the theory of sets, integers, groups, rings, fields, matrices, and vector spaces are introduced and proved; in addition, modules, lattices, multilinear algebra and other topics have their own chapters and are treated either in greater detail or as new subjects. But most of these chapters are used now also for the purpose of introducing and illustrating the concepts of "category," "functor," and "universal element" which, in the penultimate chapter on *Categories and adjoint functors* become the main topic of the book. Functors on sets to sets are introduced on page 24. The definition of a universal element appears on page 26. Its description as "the most important concept in algebra" seems to refer to the present book rather than to algebra as a discipline (but this is indicated merely by the italicizing of the word "algebra").

Concrete categories are introduced on page 64. However, most of the theorems and proofs in the book can be read without knowledge of the theory of categories.

The book is extremely well organized and very well written. Examples illustrating a new concept are given immediately after its definition. Chapters and even sections are preceded by brief, summarizing statements. Theorems are followed frequently by elucidating comments. Proofs are chosen on the basis of transparency rather than brevity; for instance, the first Sylow theorem is proved in the traditional manner without using Wielandt's elegant combinatorial argument, and the Jordan-Hoelder theorem is proved without using the powerful but difficult lemma of Zassenhaus.

The omission of Galois Theory (which had a brief but important chapter in the "Survey") is deplored by the authors and is easily explained by the fact that the present book has 598 pages versus 472 of the "Survey" which also had a smaller format. However, this very fact indicates a serious difficulty arising in the teaching

of mathematics (or at least of algebra). Galois died 136 years ago. His theory of algebraic equations (or finite extensions of fields) is still a highly relevant and important part of algebra. But to get acquainted with it seems to require an increasing amount of studies. We know (Genesis 29, 30) that Jacob served for Rachel not only seven years but yet seven other years. Will it be the fate of our students to reach the goal of their studies only at the age granted to patriarchs?

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