theorems in function algebras. For these reasons the book represents an important contribution to the mathematical literature. But it also merits additional kudos: the author is noted for (among other things) the clarity of his mathematical exposition and the present book continues in this trend. It is a pleasure to read!

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16[2.05, 7, 12].—John F. Hart, E. W. Cheney, Charles L. Lawson, Hans J. Maehly, Charles K. Mesztenyi, John R. Rice, Henry G. Thacher, Jr. & Christoph Witzgall, *Computer Approximations*, John Wiley & Sons, Inc., New York, 1968, x + 343 pp., 23 cm. Price $17.50.

This encyclopedic work represents the culmination of the combined efforts of the authors and other contributors to provide an extensive set of useful approximations for computer subroutines, along the lines of the pioneer work of Hastings [1]. The stated aim of the book is "to acquaint the user with methods for designing function subroutines (also called mathematical function routines), and in the case of the most commonly needed functions, to provide him with the necessary tables to do so efficiently."

The book divides into two parts: the first four chapters deal with the problems of computation and approximation of functions in general; the last two chapters (following one devoted to a description and use of the tables) consist, respectively, of summaries of the relevant mathematical and computational properties of the elementary and selected higher transcendental functions and tables of coefficients for least maximum error approximations of appropriate forms for accuracies extending to 25 significant figures.

The wealth of material may be more readily inferred from the following summarization of the contents of the individual chapters and appendices.

The introduction to Chapter 1 (The Design of a Function Subroutine) emphasizes that the efficient computation of a function no longer is simply mathematical in the classical sense, but requires a thorough knowledge of the manner of operation and potentialities of the computing equipment involved. The principal matters considered in this chapter are certain general considerations in preparing a function subroutine, the main types of function subroutine, special programming techniques, subroutine errors, and final steps in preparing a subroutine.

Chapter 2 (General Methods of Computing Functions) summarizes the various techniques for evaluating a function; these include infinite expansions (series, continued fractions, infinite products), recurrence and difference relations, iterative techniques, integral representations, differential equations, polynomial and rational approximations, and transformations for the acceleration of convergence.

Chapter 3 (Least Maximum Approximations) contains a discussion of the characteristic properties of least maximum approximations, together with a description of the second algorithm of Remez for the determination of the optimum polynomial approximation of given degree to a specified function in the sense of Chebyshev. Also considered are nearly least maximum approximations resulting from the truncation of Chebyshev expansions, several algorithms for determining best rational approximations, and segmented approximation.

Chapter 4 (The Choice and Application of Approximations) deals with various ways in which to increase the efficiency of procedures for approximating a function. These include the reduction of the range over which the approximation is performed, utilizing such properties as periodicity, addition formulas, symmetry, and recurrence relations. Illustrative comparisons of approximations of comparable accuracy to the same function are made with respect to the relative number of arithmetic and logical operations required in their computer evaluation. The significant problem of improving the conditioning of approximations is discussed with the aid of two examples. Various ways of evaluating a polynomial, such as Horner's (nested) form, the product form, the orthogonal polynomial form, Newton's form, and streamlined forms (Pan, Motzkin-Belaga), are described and considered with respect to ill-conditionedness. The evaluation of rational functions by means of associated continued fractions and J-fractions is also considered in some detail. This chapter closes with a brief presentation of methods and algorithms for transforming one polynomial or rational form into an equivalent form.

Chapter 5 (Description and Use of the Tables) serves as an introduction to the concluding chapters with their appendices. A description is given of the tabular material, its preparation, checking, and use.

The specific functions discussed in Chapter 6 (Function Notes) include the square root, cube root, exponential and hyperbolic functions, the logarithm function, direct and inverse trigonometric functions, the gamma function and its logarithm, the error function, the Bessel functions $J_n(x)$ and $Y_n(x)$, and the complete elliptic integrals of the first and second kinds. For each function the notes include definition and analytic behavior, fundamental formulas, error propagation, design of subroutines, checking, relevant constants, a priori computation, and index tables (for guidance in using the tables of coefficients).

Three appendices, following Chapter 6, are entitled, respectively, Conversion Algorithms, Bibliography of Approximations, and Decimal and Octal Constants. The four algorithms listed (in FORTRAN language) are concerned with the conversion of rational functions to and from equivalent continued fraction forms. The bibliography of approximations includes an approximation list arranged in the categories set forth in the FMRC *Index* [2] and a list of 62 sources of the approximations, which includes the Russian handbook of Lyusternik et al. [3]. The list of constants gives approximations both to 35D and 40 octal places of 74 basic numbers required in the function subroutines under consideration.

These appendices are followed by a list of 115 references to the literature on the approximation of functions. It seems appropriate here to mention the recent supplementary bibliography compiled by Lawson [4].

Chapter 7 (Tables of Coefficients), which concludes the book, gives in a space of 151 pages the coefficients, up to 25S, of least maximum error approximations of appropriate forms to the functions discussed in Chapter 6.

A work of this magnitude almost invariably has a number of typographical and other kinds of errors. Those found by this reviewer, which are listed elsewhere in this issue, do not seriously detract from the value of this excellent book.

This definitive collection of computer approximations, which represents the result of a systematic approach to new methods and new approximations, should be an indispensable reference work for all those engaged in scientific computation.

J. W. W.

1. Cecil Hastings, Jr., Jeanne T. Hayward & James P. Wong, Jr., *Approximations for Digital Computers*, Princeton Univ. Press, Princeton, N. J., 1955. (See *MTAC*, v. 9, 1955, pp. 121–123, RMT **56.**)

2. A. Fletcher, J. C. P. Miller, L. Rosenhead & L. J. Comrie, *An Index of Mathematical Tables*, second edition, Addison-Wesley Publishing Co., Inc., Reading, Mass., 1962. (See *Math. Comp.*, v. 17, 1963, pp. 302–303, RMT **33.**)

3. L. A. Lyusternik, O. A. Chervonenkis & A. R. Yanpol'skii, *Handbook for Computing Elementary Functions*, Pergamon Press, New York, 1965. (See *Math. Comp.*, v. 20, 1966, pp. 452–453, RMT **64.**)

4. Charles L. Lawson, *Bibliography of Recent Publications in Approximation Theory with Emphasis on Computer Applications*, Technical Memorandum No. 201, Jet Propulsion Laboratory, Pasadena, Calif., 16 August 1968.

17[2.10].—Arne P. Olson, "Gaussian quadratures for $\int_1^\infty \exp(-x)f(x)dx/x^m$ and $\int_1^\infty g(x)dx/x^m$," tables appearing in the microfiche section of this issue.

Abscissas and weights of N-point Gaussian quadrature formulas for the integrals in the title are given in Table II to 16 significant figures for $N = 2(1)10$, $M = 0(1)10$. Table I contains 24S values of the moments $a_{n,l} = \int_1^\infty \exp(-t)dt/t^{n-l}$ for $n - l = 10(-1)-19$. As a possible application of Table II, the author mentions the evaluation of the series

$$\sum_{k=0}^{\infty} E_m(t + kh) = \int_1^\infty \frac{\exp(-xt)dx}{[1 - \exp(-hx)]x^m} \quad (E_m \text{ the exponential integral}),$$

which occurs in the calculation of neutron collision rates in infinite slab geometry.

W. G.

18[2.45, 12].—Donald E. Knuth, *The Art of Programming*, Vol. I: *Fundamental Algorithms*, Addison-Wesley Publishing Co., Reading, Mass., 1968, xxi + 634 pp., 25 cm. Price $19.50.

Most people think of mathematics as being a very complicated subject. On the contrary, if we define the complexity of a problem as the amount of information required to describe the problem and its solution, it is apparent that mathematics can deal only with simple problems. Further, if we consider the universe of all problems, almost all of them are too complex for solution by mathematical methods, which rely heavily on abstraction and conceptualization, which themselves reflect the limitations of the brain. Thus, while it is reasonable to expect a mathematical solution to the four-color problem, it is not reasonable to expect a mathematical solution to the problem of language translation, which can be stated only in terms of massive amounts of information from grammars and dictionaries, and whose solution will probably require an algorithm of comparable size. The importance of the computer is that it permits the consideration of such irreducibly complex problems.

It is not surprising therefore, that what looks like the most authoritative work on computer programming is entitled "The Art of Computer Programming". Programming, which is concerned with algorithms for solving complex problems, is itself a complex problem, and we would be mistaken if we expected it to have a neat