

Each (rational) prime  $p$  of the form  $8m + 1$  or  $8m + 3$  has a unique decomposition

$$p = a^2 + 2b^2 = (a + b(-2)^{1/2})(a - b(-2)^{1/2}).$$

This table gives  $a$  and  $b$  for the 4793 such primes  $p < 10^5$ , from

$$3 = 1^2 + 1^2 \cdot 2 \quad \text{to} \quad 99971 = 207^2 + 169^2 \cdot 2.$$

It is printed on 19+ sheets of computer paper, 250 primes per page, and was computed on an IBM 360, Mod 67. No details are given concerning the method or computer time.

The same table is contained (with much else) in Cunningham's valuable book [1] that is long out of print.

The present table is similar in character and scope to this Computing Center's earlier table of Gaussian primes. Our detailed and lengthy commentary [2] on that earlier table could have analogous remarks here, but we largely leave examination of such analogies to any interested reader. The largest complex prime in  $Z[(-2)^{1/2}]$  presently known [3] to the undersigned is the quite modest:

$$179991 + (-2)^{1/2}.$$

Clearly, larger complex primes here would not be difficult to find.

It may be of interest to add that while the primes  $p = a^2 + 2b^2$  constitute asymptotically one-half of the primes, and this does not differ from the primes

$$p = a^2 + nb^2$$

with  $n = -2, 1, 3, 4, 7$ , the number of composites  $c = a^2 + 2b^2$  is exceptionally large [4].

D. S.

1. A. J. C. CUNNINGHAM, *Quadratic Partitions*, Hodgson, London, 1904.

2. L. G. DIEHL & J. H. JORDAN, *A Table of Gaussian Primes*, UMT 19, *Math. Comp.*, v. 21, 1967, pp. 260-262.

3. DANIEL SHANKS, "On the conjecture of Hardy and Littlewood concerning the number of primes of the form  $n^2 + a$ ," *Math. Comp.*, v. 14, 1960, pp. 321-332. (We check their count by  $\pi(10^5) - \pi_2(10^5) - 1 = 4793$  from our Table 3.)

4. DANIEL SHANKS & LARRY P. SCHMID, "Variations on a theorem of Landau. Part I," *Math. Comp.*, v. 20, 1966, Sect. 6, pp. 560-561.

**31[9].**—BETH H. HANNON & WILLIAM L. MORRIS, *Tables of Arithmetical Functions Related to the Fibonacci Numbers*, Report ORNL-4261, Oak Ridge National Laboratory, Oak Ridge, Tenn., June 1968, iii + 57 pp., 28 cm.

Five arithmetical functions related to the Fibonacci numbers,  $u_n$ , are herein tabulated for all positive integer arguments  $m$  to 15600, inclusive.

The first of these, designated by  $\pi(m)$ , is called the *Pisano period* of  $m$ ; it is the least integer  $k$  such that  $u_k \equiv 0 \pmod{m}$  and  $u_{k+1} \equiv 1 \pmod{m}$ . Closely related to this function is the *restricted period* of  $m$ , here denoted by  $\alpha(m)$ , which is the least integer  $n$  such that  $u_n \equiv 0 \pmod{m}$ . This is generally called the "rank of apparition" or "entry point," and has been previously tabulated [1], [2] for all primes less than  $10^5$ . The quotient  $\beta(m) = \pi(m)/\alpha(m)$  is also tabulated in this report.

J. D. FULTON and the second of the present authors [3] have established the existence of two new arithmetical functions of the Fibonacci numbers, by virtue of a fixed-point theorem; namely, " $\pi(m) = m$  if and only if  $m = (24)5^{\lambda-1}$  for some integer  $\lambda > 1$ "; and an iteration theorem: "There exists a unique smallest positive integer  $\omega$  such that  $\pi^{\omega+1}(m) = \pi^\omega(m)$ , where  $\pi^{n+1}(m) = \pi^n\{\pi(m)\}$  for  $n \geq 1$ ." The authors call these functions  $\omega(m)$  and  $\lambda(m)$  such that  $\pi^\omega(m) = (24)5^{\lambda-1}$ , the *Fibonacci frequency* of  $m$  and the *Leonardo logarithm* of  $m$ , respectively.

The tables are arranged so that the five functional values for each of 300 consecutive arguments appear on each page. Equivalent Latin letters are used in the headings because of the resulting convenience in printing directly from the computer output tapes. The computation of the table was performed on an IBM 360/75 system in one hour.

These attractively printed tables supplement earlier tables, which have been restricted to tabular arguments that are primes. This restriction, however, is not serious with respect to the functions  $\alpha(m)$  and  $\pi(m)$ , since their values for composite  $m$  equal the least common multiples of the values corresponding to the constituent prime powers.

Important references not listed by the authors include a paper by Wall [4] and a book by Jarden [5], which has a very extensive bibliography, arranged chronologically.

J. W. W.

1. MARVIN WUNDERLICH, *Tables of Fibonacci Entry Points*, The Fibonacci Association, San Jose State College, San Jose, Calif., January 1965. (For a joint review of this and the following reference, see *Math. Comp.*, v. 20, 1966, pp. 618-619, RMT 87 and 88.)

2. DOUGLAS LIND, ROBERT A. MORRIS & LEONARD D. SHAPIRO, *Tables of Fibonacci Entry Points, Part Two*, The Fibonacci Association, San Jose State College, San Jose, Calif., September 1965.

3. JOHN D. FULTON & WILLIAM L. MORRIS, "On arithmetical functions related to the Fibonacci numbers," *Acta Arithmetica*. (To appear.)

4. D. D. WALL, "Fibonacci series modulo  $m$ ," *Amer. Math. Monthly*, v. 67, 1960, pp. 525-532.

5. DOV JARDEN, *Recurring Sequences*, second edition, Riveon Lematematika, Jerusalem, 1966. (See *Math. Comp.*, v. 23, 1969, pp. 212-213, RMT 9.)

32[9, 10, 11, 12].—R. F. CHURCHHOUSE & J. C. HERZ, Editors, *Computers in Mathematical Research*, North-Holland Publishing Co., Amsterdam, 1968, xi + 185 pp., 23 cm. Price \$9.00.

This book consists of fifteen papers and an extensive bibliography about the application of computers to mathematical research.

The papers are as follows:

"Machines and pure mathematics," by D. H. Lehmer discusses some of the opportunities for involving, not replacing, the pure mathematician with the computer. The author also submits a case for the construction of special purpose hardware for application to mathematical research.

"Congruences for modular forms," by A. O. L. Atkin describes in general terms the author's attempts to extend and generalize congruence properties of modular forms with the aid of a computer.

"Covering sets and systems of congruences," by R. F. Churchhouse describes the application of a computer to the problem of determining the number of distinct