Calculation of Dirichlet L-Functions

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Abstract. A method for calculating Dirichlet L-series is presented along with the theory of residue class characters and their automatic generation. Tables are given of zeros of L-series for moduli $\leqq 24$.

1. Introduction. In this paper, there is introduced a method for calculating the values of Dirichlet L-functions. The fundamental theorems on characters mod k are given in Section 2, and a numbering of these characters is defined. Formulas are found for the numbers of real and complex characters, both primitive and imprimitive. The automatic generation of a character with a given number is described. In Section 3, the method of asymptotic evaluation is discussed, and a description is given of a calculation of complex zeros of L-functions mod $k \leqq 24$ for $|t| \leqq 25$. A microfiche at the end of this issue gives these zeros and various other tables and also the FORTRAN programs.

2. Characters. Some of the material in this section is in the "folklore" of the subject and some material is a refinement of known results.

Let $\bar{a}$ be the residue class of a, where the modulus m will be clear from the context. If $c \in \bar{a}$, then $(c, m) = (a, m)$. We write $m = p_1^{\alpha_1} \cdots p_n^{\alpha_n}$, where $p_1, \cdots, p_n$ are distinct primes. By $M(m)$ we mean the group of residues $\bar{a}$ such that $(a, m) = 1$. We define a mapping $f: M(m) \to M(p_1^{\alpha_1}) \times \cdots \times M(p_n^{\alpha_n})$, where $\times$ means the usual Cartesian product, by $f(\bar{a}) = (\bar{a}_1, \cdots, \bar{a}_n)$, where $a \equiv a_j \pmod{p_j^{\alpha_j}}$. It is easy to see that this is a well-defined map, and, using the Chinese Remainder Theorem, that f is a multiplicative isomorphism of $M(m)$ and the group $M(p_1^{\alpha_1}) \times \cdots \times M(p_n^{\alpha_n})$. Next, for $i = 1, \cdots, n$, we define the map $f_i: M(p_i^{\alpha_i}) \to M(m)$, by $f_i(\bar{a}) = \bar{b}$ where $b \equiv a \pmod{p_i^{\alpha_i}}$, $b \equiv 1 \pmod{p_j^{\alpha_j}}$, $j = 1, \cdots, n, j \neq i$. One can easily verify that each of these maps is well-defined and is an into multiplicative isomorphism. Finally, we define the map $h : M(p_1^{\alpha_1}) \times \cdots \times M(p_n^{\alpha_n}) \to M(m)$ by

$$h((\bar{a}_1, \cdots, \bar{a}_n)) = f_1(\bar{a}_1) \cdots f_n(\bar{a}_n) , \tag{1}$$

and easily verify that $h(f(\bar{a})) = \bar{a}$.

A short calculation shows that if $i \neq j$, then the only common image of f_i and f_j is $\bar{1}$. Thus, the images under $f_1, \cdots, f_n$ of $M(p_1^{\alpha_1}), \cdots, M(p_n^{\alpha_n})$ are pairwise disjoint except for the common identity. We designate these images, which are obviously subgroups of $M(m)$, respectively by $G_1, \cdots, G_n$. Since every element of $M(m)$ is expressible as a product such as given in (1), and the cardinalities of $G_1, \cdots, G_n$ just multiply to the cardinality of $M(m)$, $M(m)$ is the internal direct product of $G_1, \cdots, G_n$.

Now, we write, for $k > 2$,

$$k = 2^\beta p_1^{\alpha_1} \cdots p_r^{\alpha_r}, \quad 2 < p_1 < \cdots < p_r, \quad r = 0 \text{ if } k = 2^\beta , \tag{2}$$

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and wish to determine a basis for $M(k)$ in a fixed unequivocal manner. For p an odd prime, we set $g_1 = g_1(p) =$ the least primitive root mod p. As shown in Landau [13, pp. 79–81], either g_1 or $g_1 + p$ is a primitive root mod p^α, $\alpha \geqq 1$, and we set $g = g(p) =$ the least number which is a primitive root for p^α, $\alpha \geqq 1$. Thus, for all odd primes p, $g_1 \leqq g \leqq g_1 + p$. For $p < 3000$, and possibly further, $g = g_1$. For $p = 2$, we set $g = 3$. For $p = 40487$, $g_1 = 5$, but $g = 10$; cf. [18].

Now, following the method of LeVeque [5, Vol. II, pp. 207–210], we can solve for the basis elements of $M(k)$, $B_1, \cdots, B_R$, where $R = r$ for $\beta \leqq 1$, $R = r + 1$ for $\beta = 2$, and $R = r + 2$ for $\beta \geqq 3$. For $\beta = 0$, we solve, for $i = 1, \cdots, r$, $B_i \equiv g(p_i) \pmod{p_i^{\alpha_i}}$ and $B_i \equiv 1 \pmod{p_j^{\alpha_j}}$ for $1 \leqq j \leqq r$, $j \neq i$. If $\beta = 1$, we adjoin to these conditions $B_i \equiv 1 \pmod 2$. For $\beta = 2$, we solve $B_1 \equiv 3 \pmod 4$, $B_1 \equiv 1 \pmod{p_j^{\alpha_j}}$, $1 \leqq j \leqq r$, and for $i = 1, \cdots, r$, we solve $B_{i+1} \equiv g(p_i) \pmod{p_i^{\alpha_i}}$, and $B_{i+1} \equiv 1 \pmod 4$, and $B_{i+1} \equiv 1 \pmod{p_j^{\alpha_j}}$, $1 \leqq j \leqq r$, $j \neq i$. Finally, for $\beta \geqq 3$, we solve $B_1 \equiv -1 \pmod{2^\beta}$ and $B_1 \equiv 1 \pmod{p_j^{\alpha_j}}$, $1 \leqq j \leqq r$; $B_2 \equiv 5 \pmod{2^\beta}$ and $B_2 \equiv 1 \pmod{p_j^{\alpha_j}}$, $1 \leqq j \leqq r$; and for $i = 1, \cdots, r$, we solve $B_{i+2} \equiv g(p_i) \pmod{p_i^{\alpha_i}}$, $B_{i+2} \equiv 1 \pmod{2^\beta}$ and $B_{i+2} \equiv 1 \pmod{p_j^{\alpha_j}}$, $1 \leqq j \leqq r$, $j \neq i$.

A technical procedure to solve such congruences may be found in Uspensky and Heaslet [6, pp. 189–191]. The process requires the computation of $a^{-1} \pmod m$, (where $(a, m) = 1$), which is most easily accomplished by expressing $\phi(m)$ in binary and calculating $a^{-1} \equiv a^{\phi(m)-2}$ by repeated squarings of $a \pmod m$.

Using the above remarks, the following theorem is easily proved:

THEOREM 1. *If* $2 < k = 2^\beta p_1^{\alpha_1} \cdots p_r^{\alpha_r}$, *and* $r = 0$ *if* $k = 2^\beta$, *then* $M(k)$ *has a basis of* R *elements, all of even order* $\geqq 2$, *where*

$$
\begin{aligned}
&R = r \quad \text{if } \beta \leqq 1\,, \\
&R = r + 1 \quad \text{if } \beta = 2\,, \\
&R = r + 2 \quad \text{if } \beta \geqq 3\,.
\end{aligned} \tag{3}
$$

In Rotman [7, pp. 63–65] it is shown that any finite abelian group has a basis and that any two bases have the same cardinality.

The least positive k with R basis elements, for $R = 1, 2, \cdots$ is given by 3, 8, 24, 120, 840, 9240, 120120, $\cdots$.

Let h_i be the order of B_i. Recall that a character mod k is a nonzero multiplicative function on the residues mod k which is zero at residues not prime to k. It is shown in LeVeque [5, Vol. II, pp. 210–212] that a character is determined by its values at the basis elements, and the value at B_i can only be one of the h_ith roots of unity. We thus obtain

THEOREM 2. *There are exactly* $\phi(k)$ *characters* mod k.

Next we show:

THEOREM 3. *For* $k > 2$, *there are exactly* 2^R *real characters* mod k, *where* R *is the number of basis elements of* $M(k)$.

Proof. A character is real if and only if it is real at the basis elements. For each basis element, exactly two choices of the h_ith root of unity will be real, as h_i is even and $\geqq 2$.

Landau [14, p. 414] has shown that all characters mod k are real if and only if $k|24$.

Let $k > 2$ and let χ be a character mod k. We define $\beta_1, \cdots, \beta_R$ by

$$\chi(B_j) = \exp(2\pi i\beta_j/h_j), \qquad 0 \leqq \beta_j < h_j; \quad j = 1, \cdots, R. \tag{4}$$

Clearly, the R-tuple of nonnegative integers β_j determines and is determined by the character χ. Using this representation, it is now easy to see that the characters form a group isomorphic to $M(k)$ under the mapping $\chi \to B_1^{\beta_1} \cdot \cdots \cdot B_R^{\beta_R}$, (where $(\chi_1 \cdot \chi_2)(a)$ is defined as $\chi_1(a) \cdot \chi_2(a)$).

We now number the characters by defining

$$N = N(\chi) = \beta_1 + \beta_2 h_1 + \beta_3 h_1 h_2 + \cdots + \beta_R h_1 h_2 \cdots h_{R-1}. \tag{5}$$

It is clear that this is a Cantor numbering system and that the $\phi(k)$ characters will be numbered sequentially from 0 to $\phi(k) - 1$. We designate these characters by $\chi_0, \chi_1, \cdots$. The character corresponding to our χ_0 is the usual principal χ_0.

The characters corresponding to the R-tuples $(1, 0, \cdots, 0)$, $(0, 1, \cdots, 0)$, $\cdots$, $(0, \cdots, 0, 1)$ form a basis, and the corresponding N's are $1, h_1, h_1 \cdot h_2, \cdots, h_1 \cdot h_2 \cdots h_{R-1}$. It is easy to see from the basis representation that if $d \not\equiv 1 \pmod{k}$ and $(d, k) = 1$ then some character exists for which $\chi(\bar{d}) \neq 1$. It is clear that the characters can be considered as being defined over the integers.

Using the numbering defined, we can now introduce an unequivocal notation for the L-functions. If the modulus is fixed by the context, we can use $L(S, \chi_N)$. If not, we can use a new notation: $L(S, k, N) = L(S, \chi)$ where χ is character number N mod k.

To find the number N^* of the conjugate character with N given by (5), we set

$$\begin{aligned} \beta_j^* &= 0, \quad \text{if } \beta_j = 0, \\ &= h_j - \beta_j, \quad \text{if } \beta_j \neq 0, \end{aligned}$$

and use the β_j^*'s in (5) to form N^*. For a prime p, conjugate pairs are simply N and $p - N - 1$, for $N \geqq 1$.

Now we take up the important notion of primitivity. We say a character χ mod k is *imprimitive* if there is a proper divisor K of k such that if $a \equiv b \pmod{K}$ and $(a, k) = (b, k) = 1$ then $\chi(a) = \chi(b)$; otherwise, the character is called *primitive*. Such a number K is called a modulus of imprimitivity. The principal character for $k > 1$ is imprimitive, taking $K = 1$. Also, 1 is not a modulus of imprimitivity for nonprincipal characters. The study of the number of imprimitive characters is much simplified if we introduce the following notions. Let $m = p_1^{\alpha_1} \cdots p_n^{\alpha_n}$, and $f_1, \cdots, f_n, G_1, \cdots, G_n$ have the same meaning as above. Let $\chi^{(j)}$ be a character mod $p_j^{\alpha_j}$, $j = 1, \cdots, n$. We define, for $\bar{a} = \bar{e}_1 \cdot \cdots \cdot \bar{e}_n$, with $\bar{e}_j \in G_j$,

$$\begin{aligned} &\chi(\bar{a}) = \chi^{(1)}(f_1^{-1}(\bar{e}_1)) \cdot \chi^{(2)}(f_2^{-1}(\bar{e}_2)) \cdot \cdots \cdot \chi^{(n)}(f_n^{-1}(\bar{e}_n)), \\ &\chi(\bar{a}) = 0 \quad \text{if } (a, m) > 1, \end{aligned} \tag{6}$$

and call the resulting function the *exterior product* of $\chi^{(1)}, \cdots, \chi^{(n)}$.

THEOREM 4. *The exterior product of* $\chi^{(1)}, \cdots, \chi^{(n)}$ *is a character* mod m. *Every character* mod m *can be written uniquely as such an exterior product.*

Proof. By definition, $\chi(\bar{a}) = 0$ for $(a, m) > 1$. Since for $\bar{a} = 1$, $\bar{e}_1 = \bar{e}_2 = \cdots = \bar{e}_n = \bar{1}$, and $f_j(\bar{1}) = \bar{1}$ for $j = 1, \cdots, n$, we have $\chi(\bar{1}) = 1 \neq 0$. It remains to show that χ is multiplicative. For elements in $M(m)$, this follows from the multiplicativeness of $\chi^{(j)}$ and the isomorphic mapping property of f_j, for $j = 1, \cdots, n$. The other

cases are trivial. Finally, given a character χ mod m, define $\chi^{(j)}(\bar{a}) = \chi(f_j(\bar{a}))$ for $(a, p_j) = 1$, and $\chi^{(j)}(\bar{a}) = 0$ for $(a, p_j) > 1$, for $j = 1, \cdots, n$. It is easily seen that $\chi^{(j)}$ is a character mod $p_j^{\alpha_j}$. Also, if $\bar{a} = \bar{e}_1 \cdot \cdots \cdot \bar{e}_n$ with $\bar{e}_j \in G_j$,

$$\begin{aligned}\chi(\bar{a}) &= \chi(\bar{e}_1 \cdot \cdots \cdot \bar{e}_n) = \chi(\bar{e}_1) \cdot \cdots \cdot \chi(\bar{e}_n) \\ &= \chi(f_1(f_1^{-1}(\bar{e}_1))) \cdot \cdots \cdot \chi(f_n(f_n^{-1}(\bar{e}_n))) \\ &= \chi^{(1)}(f_1^{-1}(\bar{e}_1)) \cdot \cdots \cdot \chi^{(n)}(f_n^{-1}(\bar{e}_n)) ,\end{aligned}$$

which is the value of the exterior product of $\chi^{(1)}, \cdots, \chi^{(n)}$. Since the values of χ are determined at all elements of $M(m)$ by its values on $\bigcup G_j$, two distinct characters mod m must induce for some j two distinct $\chi^{(j)}$'s. Since the image under f_j of $M(p_j^{\alpha_j})$ is G_j, two distinct characters mod $p_j^{\alpha_j}$ will give rise to distinct exterior products.

THEOREM 5. *The exterior product is real if and only if the factors are real.*

Proof. Clear, since the values of χ are determined by the values on $\bigcup G_j$.

THEOREM 6. *The exterior product is primitive if and only if all the factors are primitive.*

Proof. If one of the factors, say $\chi^{(j)}$, is imprimitive, let K be the proper divisor. Let $K' = mK/p_j^{\alpha_j}$ and let $a \equiv b \pmod{K'}$ and $(a, m) = (b, m) = 1$. Then $a \equiv b \pmod{K}$ and $a \equiv b \pmod{p_i^{\alpha_i}}$ for $i \neq j$. Thus, $\chi^{(i)}(a) = \chi^{(i)}(b)$ for $i = j$ or not, so $\chi(a) = \chi(b)$ and K' is a modulus of imprimitivity for χ.

Now let χ have the modulus of imprimitivity $K < m$. Some prime, say p_j, appears in K to the αth power where $\alpha < \alpha_j$. Let $a \equiv b \pmod{p_j^{\alpha}}$ and $(a, p_j) = (b, p_j) = 1$. Let A be in $f_j(\bar{a})$ and B in $f_j(\bar{b})$. Then, $A \equiv a \pmod{p_j^{\alpha_j}}$, $B \equiv b \pmod{p_j^{\alpha_j}}$ and $A \equiv B \equiv 1 \pmod{p_i^{\alpha_i}}$, $i \neq j$. Hence $A \equiv B \pmod{K}$ so $\chi(A) = \chi(B)$. Thus $\chi^{(j)}(a) = \chi^{(j)}(b)$ and $\chi^{(j)}$ is imprimitive.

COROLLARY. *The number of primitive characters* mod m *is a multiplicative function of* m.

A slight extension of the arguments above can be used to show

THEOREM 7. *If χ is imprimitive, then there is a least modulus of imprimitivity, and all proper divisors of m which are multiples of this least modulus are also moduli of imprimitivity.*

For a complete and simple development of the theory of moduli of imprimitivity, see Spira [17].

We now count the primitive characters.

THEOREM 8. *If p is a prime, then the number of primitive characters* mod p^{α} *is* $p - 2$ *if* $\alpha = 1$ *and* $p^{\alpha-2}(p-1)^2$ *if* $\alpha \geq 2$.

Proof. The principal character is always imprimitive mod p^{α}, with modulus of imprimitivity $= 1$. If $\alpha = 1$, this is the only possible modulus, so the other characters are primitive as they are not identically 1. Since $\phi(p) = p - 1$, there are $p - 2$ primitive characters mod p. Now let $p > 2$ and g and χ_j be as defined above, so $\chi_j(g) = \exp(2\pi ij/\phi(p^{\alpha}))$ and $0 \leq j < p^{\alpha-1}(p-1)$. First let $j = sp$. For $s = 0$, we have the principal character, which is imprimitive. Thus, we take $1 \leq s < p^{\alpha-2}(p-1)$. Let $n_1 \equiv n_2 \pmod{p^{\alpha-1}}$ and $n_1 \not\equiv 0 \pmod{p}$. Let $n_1 \equiv g^u \pmod{p^{\alpha}}$ and $n_2 \equiv g^v \pmod{p^{\alpha}}$. Then, $g^u \equiv g^v \pmod{p^{\alpha-1}}$. Since g is a primitive root mod $p^{\alpha-1}$, $u \equiv v \pmod{p^{\alpha-2}(p-1)}$, or $u = v + cp^{\alpha-2}(p-1)$. Thus,

$$\begin{aligned}\chi_j(n_1) &= \chi_j(g^u) = \exp\,(u2\pi isp/(p^{\alpha-1}(p-1)))\\ &= \exp\,[(v + cp^{\alpha-2}(p-1))2\pi is/(p^{\alpha-2}(p-1))]\\ &= \chi_j(g^v)\cdot \exp\,(2\pi isc) = \chi_j(g^v) = \chi_j(n_2)\,.\end{aligned}$$

Thus, χ_j is imprimitive for $j = sp$.

Let now $(j, p) = 1$. Suppose p^γ were a modulus of imprimitivity for χ_j with $\gamma \geqq 1$. Let $a \equiv b \pmod{p^\beta}$ and $a \not\equiv 0 \not\equiv b \pmod p$. Then $a \equiv b \pmod p$. Let $a \equiv g^u \pmod{p^\alpha}$ and $b \equiv g^v \pmod{p^\alpha}$. Since $\chi_j(a) = \chi_j(b)$, we have

$$\exp\,(2\pi iju/(p^{\alpha-1}(p-1))) = \exp\,(2\pi ijv/(p^{\alpha-1}(p-1))),$$

so $uj = vj + 2\pi it$ for some t. Since $(p, j) = 1$, we have $u \equiv v \pmod{p^{\alpha-1}}$. Also $u \equiv v \pmod{p-1}$ as $g^u \equiv g^v \pmod p$. Thus $u \equiv v \pmod{p^{\alpha-1}(p-1)}$, so $a \equiv b \pmod{p^2}$. Thus χ_j is primitive, and indeed assumes distinct values on the residue classes mod p^α which have elements congruent to a single residue mod p.

Note that in this case of $p > 2$, for $\alpha = 1$, the real nonprincipal character is primitive, but for $\alpha > 1$, the real nonprincipal character is obtained for $j = p^{\alpha-1}(p-1)/2$, and is hence imprimitive.

Finally, let $p = 2$. For $\alpha = 2$, the result is obtained by calculation, and in this case, the real nonprincipal character is also primitive. Let $\alpha \geqq 3$. Then $M(2^\alpha)$ has a basis $-\bar 1$, $\bar 5$, of respective orders 2 and $2^{\alpha-2}$. Thus, if χ is a character mod 2^α, then $\chi(\bar 5) = \exp\,(2\pi it/2^{\alpha-1})$, where $0 \leqq t < 2^{\alpha-2}$. We will show χ is primitive if and only if t is odd. Let $t = 2s$. If $s = 0$, the character is principal and hence imprimitive. Now let $n_1 \equiv n_2 \pmod{2^{\alpha-1}}$ and let $n_1 \equiv \epsilon_1 5^u \pmod{2^\alpha}$, $n_2 \equiv \epsilon_2 5^v \pmod{2^\alpha}$, where ϵ_1 and ϵ_2 are real and of absolute value 1. Hence $\epsilon_1\epsilon_2 5^{u-v} \equiv 1 \pmod{2^{\alpha-1}}$, so $\epsilon_1\epsilon_2 = 1$, as the powers of 5 do not represent -1 mod $2^{\alpha-1}$ if $\alpha \geqq 3$. Thus, $u \equiv v \pmod{2^{\alpha-3}}$. Now

$$\begin{aligned}\chi(n_1) &= \chi(\epsilon_1 5^u) = \chi(\epsilon_1)\exp\,(2\pi i2su/2^{\alpha-1})\\ &= \chi(\epsilon_2)\exp\,(2\pi i(2s)(v + b2^{\alpha-3})/2^{\alpha-2})\\ &= \chi(\epsilon_2)\chi(5^v)\exp\,(2\pi i(2s)\cdot b) = \chi(n_2)\,,\end{aligned}$$

so indeed χ is imprimitive. A similar argument to the one above for $p > 2$ shows that if $j = 2s + 1$, then χ is primitive.

COROLLARY 1. *If* $k \equiv 2 \pmod 4$, *then there are no primitive characters* mod k.

COROLLARY 2. *The number of primitive characters* mod k is $\sum_{d|n} \mu(n)\phi(n/d)$.

Using the facts given in the proof above, it is easy to machine generate the N's corresponding to primitive characters.

For the primitive character mod 2^α to be real, we need $2\pi i(2s+1)/2^{\alpha-2} = l\pi i$, or $l\cdot 2^{\alpha-3} = 2s + 1$. Thus $\alpha \leqq 3$, and these cases are easily settled. From the above remarks on real primitive characters, and from Theorems 5 and 6, we obtain

THEOREM 9. *The number of real primitive characters* mod $2^\beta \cdot t$, *where* $(t, 2) = 1$, *is* 0 *if* $\beta = 1$ *or* $\beta > 3$ *or* t *not squarefree*, 1 *if* $\beta = 0$ *or* 2 *and* t *squarefree and* 2 *if* $\beta = 3$ *and* t *squarefree.*

The numbers N for real primitive characters are given by

$$N = \tfrac{1}{2}(h_1 + h_1h_2 + \cdots + h_1h_2\cdots h_R)$$

if $\beta = 0$ or 2, t squarefree, and this N and $N - 1$ if $\beta = 3$ and t squarefree. The notation of Rosser [4] of putting a star after the modulus when $\beta = 3$ refers to character N, and the unstarred modulus refers to character $N - 1$. Either of these may have $\chi(-1) = +1$ or -1.

For machine notation, the values of a character χ mod k can be represented by integers. If $\chi(n) = 0$, we use 0; if $\chi(n) = \exp(2\pi it/\phi(k))$, we use t, with $1 \leqq t \leqq \phi(k)$. In the calculation of a character χ_N, the basis is first determined by the solution of linear congruences, using an internal table of primitive roots mod p^α, or a generator for such primitive roots. The parameter N is then decoded into the β_j's and the character is computed, using the orders of the basis elements as parameters in the loops. It is convenient to separate out the translation of an R-tuple of exponents of the basis elements into the corresponding residue mod k. Testing is best done by generating the $\phi(k)$ characters, checking that they are distinct, and testing each character to see if it is multiplicative and not identically 0. Primitivity is also easily checked.

N. G. Čudakov [9] has given a development of the theory of characters based on different methods.

3. An Asymptotic Formula for $L(s, \chi)$. Davies and Haselgrove [1] give an asymptotic formula for $L(s, \chi)$ which requires the computation of coefficients. We give a different formula which merely requires the character, but which has the disadvantage that it loses accuracy near $s = 1$.

The computing times for the two methods are roughly equal. L. Schoenfeld [12] has generalized the Davies-Haselgrove method. Another formula is given in Davies [2].

We have, for $\sigma > 1$

$$L(s, \chi) = \sum_{n=1}^{\infty} \chi(n) n^{-s} = k^{-s} \sum_{j=1}^{k} \chi(j) \left[\sum_{n=0}^{\infty} (n + j/k)^{-s} \right], \tag{7}$$

since there is absolute convergence for $\sigma > 1$. By a slight change in the proof of the Euler-Maclaurin formula for $\zeta(s)$, we have

$$\begin{aligned} \sum_{n=0}^{\infty} (n + j/k)^{-s} = {} & \sum_{n=0}^{N-1} (n + j/k)^{-s} + \frac{1}{2} (N + j/k)^{-s} + (N + j/k)^{1-s}/(s - 1) \\ & + \sum_{\nu=1}^{m} [B_{2\nu}/(2\nu)!] \left(\prod_{r=0}^{2\nu-2} (s + r) \right) (N + j/k)^{1-s-2\nu} + \text{error}, \end{aligned} \tag{8}$$

and this expression provides an analytic continuation. Combining (7) and (8), we obtain

$$\begin{aligned} L(s, \chi) = \sum_{j=1}^{k} \chi(j) \Bigg\{ & \sum_{n=0}^{N-1} (kn + j)^{-s} + \frac{1}{2} (kN + j)^{-s} + (kN + j)^{1-s}/(k(s - 1)) \\ & + \sum_{\nu=1}^{m} [B_{2\nu}/(2\nu)!] \left(\prod_{r=0}^{2\nu-2} (s + r) \right) (kN + j)^{1-s-2\nu} \cdot k^{2\nu-1} \Bigg\} \\ & + \text{error}. \end{aligned} \tag{9}$$

The advantage of programming the first sum from 1 to k is that the routine can be tested using check values of $\zeta(s)$ by setting $\chi(j) = 1$ for $j = 1, \cdots, k$. Calculating

formulas very similar to those in Spira [10] can be used. A formula for $L'(s, \chi)$ similar to one in Spira [10] can also be easily found. For values near $s = 1$, one could use the functional equation:

$$(10)\qquad \begin{aligned} L(s, \chi) &= 2^s k^{-s} \epsilon \pi^{s-1} \Gamma(1 - s) \begin{matrix}\sin \\ \cos\end{matrix} (\tfrac{1}{2}\pi s) L(1 - s, \bar{\chi}) , \\ \chi(-1) &= \begin{matrix}+1 \\ -1\end{matrix}, \quad \chi \text{ primitive} , \end{aligned}$$

where

$$(11)\qquad \epsilon = \sum_{a=1}^{k} \chi(a) \begin{matrix}\cos \\ \sin\end{matrix} \left(\frac{2\pi a}{k}\right), \quad \chi(-1) = \begin{matrix}+1 \\ -1\end{matrix} .$$

For characters with $\chi(-1) = 1$, we have $L(0, \chi) = 0$. For real primitive characters, with $\chi(-1) = -1$, we have $L(0, \chi) = h$, the class number, for $k > 4$. This follows from the functional equation (10), the class number formula (Davenport [11, pp. 37–51]), and the fact that $\epsilon = \sqrt{k}$ (Landau [13, p. 174, Satz 215]). For $k = 3$ and 4 we obtain 1/3 and 1/2 respectively at 0.

The keyhole integral of Davies-Haselgrove [1, (2.1)] provides a continuation of $L(s, \chi)$ to the entire plane. Putting $s = 0$ in that formula, it follows from Schoenfeld [12], that for any primitive $\chi \pmod k$

$$(12)\qquad L(0, \chi) = (-1/k) \sum_{j=1}^{k-1} j\chi(j) ,$$

and using this equation, further check values can be obtained, e.g., mod 13, $L(0, \chi_1) = 1 + i$.

Zeros were calculated for $k \leqq 24$ and $|t| \leqq 25$. An integration was performed to verify the number of zeros obtained, and the number of sign changes of $Z(t, \chi)$ was also counted (Davies-Haselgrove [1]). All zeros were on $\sigma = 1/2$.

A comparison was made with the zeros calculated by Davies and Haselgrove in [1] and in the manuscript table [16]. The Davies-Haselgrove numbering of characters agrees with the numbering introduced above.

The character opposite Z_{15} in [1, p. 127] is imprimitive with resolving modulus $K = 5$. However, the corresponding table in [16] for the signed modulus of the L-function along $\frac{1}{2} + it$ has changes of sign near the true zeros.

In the [16] tables of zeros of L-series mod 5, 7, 11 and 19, and of the real primitive character L-series (given as factors of Dedekind zeta functions) mod 3, 4, 5 and 20, the following errors were found:

Mod 5. Character 3. $|L'|$ for their zero number 4 should have terminal digits 642.

Mod 11. Character 5. Zero number 1 should be $\frac{1}{2} + i$ 2.477244, with $|L'| =$ 1.41292. They have erroneously inserted the first zero from Character 6.

Mod 11. Character 6. The numbering of the zeros is off by 1. The first zero should be $\frac{1}{2} + i$ 2.696004 with $|L'| = 1.34773$.

Mod 19. Character 1. Missed first zero, $\frac{1}{2} + i$ 2.392764 with $|L'| = 1.98624$. Numbering of zeros off by 1.

Mod 19. Character 9. Missed first zero, $\frac{1}{2} + i$ 1.516084 with $|L'| = 1.35929$. Numbering of zeros off by 1.

$\zeta \cdot L_{20}$. Zero 16 should be $\frac{1}{2} + i$ 24.90661.

There were also numerous one and two unit terminal digit errors. The introduction does not describe some of the tables, and the tables of zeros for the real primitive character L-functions mod 5 and mod 7 were missing. The reproduction has some unreadable pages and some duplicate pages.

The paper [1] contains many errors, which are described in Schoenfeld [12].

Once having the lower zeros of all L-series mod k, it is easy to calculate $E(k)$ = the largest $t > 0$ such that $L(s, \chi) \neq 0$ for $0 < \sigma < 1$, $0 \leqq |t| < E(k)$, over all characters χ mod k. As it happens for $k \leqq 24$, $k \not\equiv 2 \pmod 4$, the character for which $L(\sigma + iE(k), \chi) = 0$ is primitive. The zeros missed in the Davies-Haselgrove calculation did not affect the value of $E(k)$.

Shanks and Wrench [15] calculated values of

$$L_a(s) = \sum_{n=1}^{\infty} \left(\frac{-a}{2n+1}\right)(2n+1)^{-s}$$

at integer points. These are indeed L-series, and, for example, in our notation, $L_2(s) = L(s, 8, 3)$.

We now list the tables on the microfiche.

Table I. Basis for $M(k)$, $k = 1(1)200$.

Table II. Characters mod k, $k = 1(1)24$.

Table III. Real and complex primitive character N's, $k = 1(1)100$.

Table IV. $L(0, \chi)$, $L'(0, \chi)$ for primitive characters mod k, $k \leqq 24$, 15D.

Table V. Re ϵ, Im ϵ, Arg ϵ for primitive characters mod k, $k \leqq 24$, 15D.

Table VI. Zeros ρ_n of $L(s, \chi)$ and $L'(\rho_n)$ for $0 < \sigma < 1$, $0 \leqq t \leqq 25$, for primitive characters mod k, $k \leqq 24$, 17S.

Table VII. $E(k)$, $k = 1(1)24$, $k \not\equiv 2 \pmod 4$, 5D, and N for which attained.

In addition, there is on the microfiche a description and listing of FORTRAN programs for calculating L-functions mod k for $k \leqq 2048$.

The calculations were carried out at the Michigan State University Computing Center. Further extensive computations are being carried out on a study of real roots of real L-series (Rosser [3], [4]), and on class numbers of cyclotomic fields. This paper was prepared with partial support from NSF grant GP-8957.

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16. *Royal Society of London, Mathematical Tables Committee. Royal Society Depository for Unpublished Mathematical Tables.* Table 80. (Copy in Pennsylvania State University Library.)
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TABLE V. ϵ and Arg ϵ, ϵ from the functional equation for an L-series mod K, K = 3(1)24 for a primitive character with number N. We have $\epsilon = \mathrm{Re}\epsilon + i\mathrm{Im}\epsilon$, $-\pi < \mathrm{Arg}\ \epsilon \leq \pi$. Values are given only for the smaller N from a pair of conjugate characters. 2 pp.

TABLE VI. Zeros of L-series mod K, K = 3(1)24, for primitive characters. N = the character number. All zeros $\frac{1}{2} + i\gamma_n$ are given with $0 \leq \gamma_n \leq 25$. The zeros are listed under ZERO, and also given is $L'(\frac{1}{2} + i\gamma_n, \chi)$ = RE DERIV + i(IM DERIV). 17S. The ampersand indicates a non-negative exponent of ten, while a minus sign in a similar position indicates a negative exponent of ten. 36 pp.

TABLE VII. E(K), K = 1(1)24, K $\not\equiv$ 2 (mod 4), 5D. E(K) is the height of the largest rectangle, $0 < \sigma < 1$, $0 \leq t < E(K)$ which is free of zeros of L-series mod K, including ones with imprimitive characters. In the table, N is such that $L(\frac{1}{2} + iE(k), \chi_N) = 0$. 1 page.

TABLE I. BASIS FOR M(K)

K	PHI	R	B1	H1	B2	H2	B3	H3	B4	H4
121	110	1	2	110						
122	60	1	63	60						
123	80	2	83	2	88	40				
124	60	2	63	2	65	30				
125	100	1	2	100						
126	36	2	29	6	73	6				
127	126	1	3	126						
128	64	2	127	2	5	32				
129	84	2	44	2	46	42				
130	48	2	27	4	41	12				
131	130	1	2	130						
132	40	3	67	2	89	2	13	10		
133	108	2	115	6	78	18				
134	66	1	69	66						
135	72	2	56	18	82	4				
136	64	3	103	2	65	2	105	16		
137	136	1	3	136						
138	44	2	47	2	97	22				
139	138	1	2	138						
140	48	3	71	2	57	4	101	6		
141	92	2	95	2	52	46				
142	70	1	7	70						
143	120	2	79	10	67	12				
144	48	3	127	2	37	4	65	6		
145	112	2	117	4	31	28				
146	72	1	5	72						
147	84	2	50	2	52	42				
148	72	2	75	2	113	36				
149	148	1	2	148						
150	40	2	101	2	127	20				
151	150	1	6	150						
152	72	3	39	2	77	2	97	18		
153	96	2	137	6	37	16				
154	60	2	45	6	57	10				
155	120	2	32	4	96	30				
156	48	3	79	2	53	2	145	12		
157	156	1	5	156						
158	78	1	3	78						
159	104	2	107	2	55	52				
160	64	3	31	2	101	8	97	4		

TABLE I. BASIS FOR M(K)

K	PHI	R	B1	M1	B2	M2	B3	M3	B4	M4
161	132	2	24	6	120	22				
162	54	1	83	54						
163	162	1	2	162						
164	80	2	83	2	129	40				
165	80	3	56	2	67	4	46	10		
166	82	1	85	82						
167	166	1	5	166						
168	48	4	127	2	85	2	113	2	73	6
169	156	1	2	156						
170	64	2	137	4	71	16				
171	108	2	20	6	154	18				
172	84	2	87	2	89	42				
173	172	1	2	172						
174	56	2	59	2	31	28				
175	120	2	127	20	101	6				
176	80	3	111	2	133	4	145	10		
177	116	2	119	2	61	58				
178	88	1	3	88						
179	178	1	2	178						
180	48	3	91	2	101	6	37	4		
181	180	1	2	180						
182	72	2	157	6	15	12				
183	120	2	62	2	124	60				
184	88	3	47	2	93	2	97	22		
185	144	2	112	4	76	36				
186	60	2	125	2	127	30				
187	160	2	35	10	122	16				
188	92	2	95	2	5	46				
189	108	2	29	18	136	6				
190	72	2	77	4	21	18				
191	190	1	19	190						
192	64	3	127	2	133	16	65	2		
193	192	1	5	192						
194	96	1	5	96						
195	96	3	131	2	157	4	106	12		
196	84	2	99	2	101	42				
197	196	1	2	196						
198	60	2	155	6	145	10				
199	198	1	3	198						
200	80	3	151	2	101	2	177	20		

TABLE I. BASIS FOR M(K)

K	PHI	R	B1	M1	B2	M2	B3	M3	B4	M4
1	1									
2	1									
3	2	1	2	2						
4	2	1	3	2						
5	4	1	2	4						
6	2	1	5	2						
7	6	1	3	6						
8	4	2	7	2	5	2				
9	6	1	2	6						
10	4	1	7	4						
11	10	1	2	10						
12	4	2	7	2	5	2				
13	12	1	2	12						
14	6	1	3	6						
15	8	2	11	2	7	4				
16	8	2	15	2	5	4				
17	16	1	3	16						
18	6	1	11	6						
19	18	1	2	18						
20	8	2	11	2	17	4				
21	12	2	8	2	10	6				
22	10	1	13	10						
23	22	1	5	22						
24	8	3	7	2	13	2	17	2		
25	20	1	2	20						
26	12	1	15	12						
27	18	1	2	18						
28	12	2	15	2	17	6				
29	28	1	2	28						
30	8	2	11	2	7	4				
31	30	1	3	30						
32	16	2	31	2	5	8				
33	20	2	23	2	13	10				
34	16	1	3	16						
35	24	2	22	4	31	6				
36	12	2	19	2	29	6				
37	36	1	2	36						
38	18	1	21	18						
39	24	2	14	2	28	12				
40	16	3	31	2	21	2	17			

TABLE 1. BASIS FOR M(K)

K	PHI	R	B1	M1	B2	M2	B3	M3	B4	M4
41	40	1	6	40						
42	12	2	29	2	31	6				
43	42	1	3	42						
44	20	2	23	2	13	10				
45	24	2	11	6	37	4				
46	22	1	5	22						
47	46	1	5	46						
48	16	3	31	2	37	4	17	2		
49	42	1	3	42						
50	20	1	27	20						
51	32	2	35	2	37	16				
52	24	2	27	2	41	12				
53	52	1	2	52						
54	18	1	29	18						
55	40	2	12	4	46	10				
56	24	3	15	2	29	2	17	6		
57	36	2	20	2	40	18				
58	28	1	31	28						
59	58	1	2	58						
60	16	3	31	2	41	2	37	4		
61	60	1	2	60						
62	30	1	3	30						
63	36	2	29	6	10	6				
64	32	2	63	2	5	16				
65	48	2	27	4	41	12				
66	20	2	23	2	13	10				
67	66	1	2	66						
68	32	2	35	2	37	16				
69	44	2	47	2	28	22				
70	24	2	57	4	31	6				
71	70	1	7	70						
72	24	3	55	2	37	2	65	6		
73	72	1	5	72						
74	36	1	39	36						
75	40	2	26	2	52	20				
76	36	2	39	2	21	18				
77	60	2	45	6	57	10				
78	24	2	53	2	67	12				
79	78	1	3	78						
80	32	3	31	2	21	4	17	4		

TABLE I. BASIS FOR M(K)

K	PHI	R	B1	M1	B2	M2	B3	M3	B4	M4
81	54	1	2	54						
82	40	1	47	40						
83	82	1	2	82						
84	24	3	43	2	29	2	73	6		
85	64	2	52	4	71	16				
86	42	1	3	42						
87	56	2	59	2	31	28				
88	40	3	23	2	45	2	57	10		
89	88	1	3	88						
90	24	2	11	6	37	4				
91	72	2	66	6	15	12				
92	44	2	47	2	5	22				
93	60	2	32	2	34	30				
94	46	1	5	46						
95	72	2	77	4	21	18				
96	32	3	31	2	37	8	65	2		
97	96	1	5	96						
98	42	1	3	42						
99	60	2	56	6	46	10				
100	40	2	51	2	77	20				
101	100	1	2	100						
102	32	2	35	2	37	16				
103	102	1	5	102						
104	48	3	79	2	53	2	41	12		
105	48	3	71	2	22	4	31	6		
106	52	1	55	52						
107	106	1	2	106						
108	36	2	55	2	29	18				
109	108	1	6	108						
110	40	2	67	4	101	10				
111	72	2	38	2	76	36				
112	48	3	15	2	85	4	17	6		
113	112	1	3	112						
114	86	2	77	2	97	18				
115	88	2	47	4	51	22				
116	56	2	59	2	89	28				
117	72	2	92	6	28	12				
118	58	1	61	58						
119	96	2	52	6	71	16				
120	32	4	31	2	61	2	41	2	97	4

TABLE II. CHARACTERS MOD K

K = 1

N	A	H	R	T	CHARACTER
0	1	1	1	R	1

K = 2

N	A	H	R	T	CHARACTER
0	1	1	1	R	1 C

K = 3

N	A	H	R	T	CHARACTER
0	1	1	1	R	2 2 C
1	2	2	0	R	2 1 C

K = 4

N	A	H	R	T	CHARACTER
0	1	1	1	R	2 0 2 0
1	3	2	0	R	2 0 1 0

K = 5

N	A	H	R	T	CHARACTER
0	1	1	1	R	4 4 4 4 C
1	2	4	0	C	4 1 3 2 C
2	4	2	0	R	4 2 2 4 C
3	3	4	0	C	4 3 1 2 C

N=CHAR NO. A=RELATED RES MOD K OF ORDER=H. R=RESOL MODULUS. T=REAL OR COMPLEX

TABLE II. CHARACTERS MOD K

K = 6

N	A	H	R	T	CHARACTER					
0	1	1	1	R	2	0	C	C	2	0
1	5	2	3	R	2	0	0	0	1	0

K = 7

N	A	H	R	T	CHARACTER						
0	1	1	1	R	6	6	6	6	6	6	0
1	3	6	0	C	6	2	1	4	5	3	C
2	2	3	0	C	6	4	2	2	4	6	0
3	6	2	0	R	6	6	3	6	3	3	0
4	4	3	0	C	6	2	4	4	2	6	C
5	5	6	0	C	6	4	5	2	1	3	C

K = 8

N	A	H	R	T	CHARACTER							
0	1	1	1	R	4	C	4	C	4	0	4	0
1	7	2	4	R	4	0	2	C	4	C	2	C
2	5	2	0	R	4	C	2	0	2	C	4	0
3	3	2	0	R	4	C	4	C	2	C	2	0

K = 9

N	A	H	R	T	CHARACTER								
0	1	1	1	R	6	6	0	6	6	C	6	6	C
1	2	6	0	C	6	1	C	2	5	0	4	3	0
2	4	3	0	C	6	2	C	4	4	C	2	6	C
3	8	2	3	R	6	3	C	6	3	0	6	3	C
4	7	3	0	C	6	4	C	2	2	0	4	6	0
5	5	6	0	C	6	5	C	4	1	0	2	3	C

N=CHAR NO. A=RELATED RES MOD K OF ORDER H. R=RESOL MODULUS. T=REAL OR COMPLEX

TABLE II. CHARACTERS MOD K

K = 10

N	A	M	R	T	CHARACTER									
0	1	1	1	R	4	C	4	C	C	0	4	C	4	C
1	7	4	5	C	4	0	3	C	C	C	1	0	2	0
2	9	2	5	R	4	C	2	C	C	C	2	0	4	0
3	3	4	5	C	4	C	1	C	C	C	3	C	2	0

K = 11

N	A	M	R	T	CHARACTER									
0	1	1	1	R	10	10	10	10	10	10	10	10	10	C
1	2	10	0	C	10	1	8	2	4	9	7	3	6	5	0
2	4	5	0	C	10	2	6	4	8	8	4	6	2	10	C
3	8	10	0	C	10	3	4	6	2	7	1	9	8	5	0
4	5	5	0	C	10	4	2	8	6	6	8	2	4	10	0
5	10	2	0	R	10	5	10	10	10	5	5	5	10	5	C
6	9	5	0	C	10	6	8	2	4	4	2	8	6	10	C
7	7	10	0	C	10	7	6	4	8	3	9	1	2	5	0
8	3	5	0	C	10	8	4	6	2	2	6	4	8	10	C
9	6	10	0	C	10	9	2	8	6	1	3	7	4	5	C

K = 12

N	A	M	R	T	CHARACTER											
0	1	1	1	R	4	C	C	C	4	0	4	C	0	0	4	0
1	7	2	4	R	4	0	C	0	4	0	2	C	C	C	2	0
2	5	2	3	R	4	0	0	0	2	0	4	0	0	0	2	0
3	11	2	0	R	4	C	C	C	2	0	2	0	0	0	4	0

N=CHAR NO. A=RELATED RES MOD K OF CROER=H. R=RESOL MODULUS. T=REAL OR COMPLEX

TABLE II. CHARACTERS MOD K

K = 13

N	A	H	R	T	CHARACTER												
0	1	1	1	R	12	12	12	12	12	12	12	12	12	12	12	12	0
1	2	12	0	C	12	1	4	2	9	5	11	3	8	10	7	6	0
2	4	6	0	C	12	2	8	4	6	10	10	6	4	8	2	12	0
3	8	4	0	C	12	3	12	6	3	3	9	9	12	6	9	6	0
4	3	3	0	C	12	4	4	8	12	8	8	12	8	4	4	12	0
5	6	12	0	C	12	5	8	10	9	1	7	3	4	2	11	6	0
6	12	2	0	R	12	6	12	12	8	6	6	6	12	12	6	12	0
7	11	12	0	C	12	7	4	2	3	11	5	9	8	10	1	6	0
8	9	3	0	C	12	8	8	4	12	4	4	12	4	8	8	12	0
9	5	4	0	C	12	9	12	6	9	9	3	3	12	6	3	6	0
10	10	6	0	C	12	10	4	8	6	2	2	6	8	4	10	12	0
11	7	12	0	C	12	11	8	10	3	7	1	9	4	2	5	6	0

K = 14

N	A	H	R	T	CHARACTER												
0	1	1	1	R	6	0	6	0	6	0	0	0	6	0	6	0	6	0
1	3	6	7	C	6	0	1	0	5	0	0	0	2	0	4	0	3	0
2	9	3	7	C	6	0	2	0	4	0	0	0	4	0	2	0	6	0
3	13	2	7	R	6	0	3	0	3	0	0	0	6	0	6	0	3	0
4	11	3	7	C	6	0	4	0	2	0	0	0	2	0	4	0	6	0
5	5	6	7	C	6	0	5	0	1	0	0	0	4	0	2	0	3	0

K = 15

N	A	H	R	T	CHARACTER													
0	1	1	1	R	8	8	0	8	0	0	8	8	0	0	8	0	8	0
1	11	2	3	R	8	4	0	8	0	0	8	4	0	0	4	0	8	4	0
2	7	4	5	C	8	2	0	4	0	0	2	6	0	0	8	0	6	4	0
3	2	4	0	C	8	6	0	4	0	0	2	2	0	0	4	0	6	8	0
4	4	2	5	R	8	4	0	8	0	0	4	4	0	0	8	0	4	8	0
5	14	2	0	R	8	8	0	8	0	0	4	8	0	0	4	0	4	4	0
6	13	4	5	C	8	6	0	4	0	0	6	2	0	0	8	0	2	4	0
7	8	4	0	C	8	2	0	4	0	0	6	6	0	0	4	0	2	8	0

N=CHAR NO. A=RELATED RES MOD K OF ORDER=H. R=RESOL MODULUS. T=REAL OR COMPLEX

TABLE II. CHARACTERS MOD K

K = 16

N	A	H	R	T	CHARACTER															
0	1	1	1	R	8	C	8	C	8	C	8	0	8	0	8	0	8	0	8	0
1	15	2	4	R	8	C	4	C	8	C	4	C	8	0	4	C	8	0	4	0
2	5	4	0	C	8	0	6	0	2	0	4	C	4	C	2	C	6	0	8	0
3	11	4	C	C	8	C	2	C	2	0	8	0	4	0	6	0	6	0	4	0
4	9	2	8	R	8	0	4	C	4	C	8	C	8	C	4	C	4	0	8	0
5	7	2	8	R	8	0	8	0	4	0	4	0	8	C	8	C	4	0	4	0
6	13	4	0	C	8	C	2	C	8	C	4	C	4	0	6	0	2	0	8	0
7	3	4	0	C	8	C	8	C	8	C	8	C	4	C	2	C	2	0	4	0

K = 17

N	A	H	R	T	CHARACTER															
0	1	1	1	R	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
					C															
1	3	16	0	C	16	14	1	12	5	15	11	10	2	3	7	13	4	9	6	8
					0															
2	9	8	0	C	16	12	2	8	10	14	6	4	4	6	14	10	8	2	12	16
					0															
3	10	16	0	C	16	10	3	4	15	13	1	14	6	9	5	7	12	11	2	8
					0															
4	13	4	0	C	16	8	4	16	4	12	12	8	8	12	12	4	16	4	8	16
					0															
5	5	16	0	C	16	6	5	12	5	11	7	2	10	15	3	1	4	13	14	8
					0															
6	15	8	0	C	16	4	6	8	14	10	2	12	12	2	10	14	8	6	4	16
					0															
7	11	16	0	C	16	2	7	4	3	9	13	6	14	5	1	11	12	15	10	8
					0															
8	16	2	0	R	16	16	8	16	8	8	8	16	16	8	8	8	16	8	16	16
					0															
9	14	16	C	C	16	14	9	12	13	7	3	10	2	11	15	5	4	1	6	8
					0															
10	8	8	0	C	16	12	10	8	2	6	14	4	4	14	6	2	8	10	12	16
					0															

N=CHAR NO. A=RELATED RES MOD K OF ORDER H. R=RESOL MODULUS. T=REAL OR COMPLEX

TABLE 18. CHARACTERS MOD K

K = 17 (CONTINUED)

N	A	H	R	T	CHARACTER														
11	7	16	0	C	16 0	10	11	4	7	5	9	14	6	1	13	15	12	3	2	8
12	4	4	0	C	16 0	8	12	16	12	4	4	8	8	4	4	12	16	12	8	16
13	12	16	0	C	16 0	6	13	12	1	3	15	2	10	7	11	9	4	5	14	8
14	2	8	0	C	16 0	4	14	8	6	2	10	12	12	10	2	6	8	14	4	16
15	6	16	0	C	16 0	2	15	4	11	1	5	6	14	13	9	3	12	7	10	8

K = 18

N	A	H	R	T	CHARACTER														
0	1	1	1	R	6 6	C 0	C	0	6	0	6	0	0	0	6	0	6	0	0	0
1	11	6	9	C	6 3	C C	0	0	5	C	4	0	C	0	1	0	2	0	0	0
2	13	3	9	C	6 6	0 0	C	C	4	C	2	C	C	C	2	0	4	0	0	0
3	17	2	3	R	6 3	C 0	C	0	3	C	6	C	0	0	3	0	6	0	0	0
4	7	3	9	C	6 6	C C	0	0	2	C	4	C	0	0	4	C	2	0	0	0
5	5	6	9	C	6/ 3	C 0	C	C	1	C	2	C	C	C	5	C	4	0	0	0

N=CHAR NO. A=RELATED RES MOD K OF ORDER=H. R=RESOL MODULUS. T=REAL OR COMPLEX

TABLE II. CHARACTERS MOD K

K = 19

N	A	F	R	T	CHARACTER															
0	1	1	1	R	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18
					18	18	C													
1	2	18	0	C	18	1	13	2	16	14	6	3	8	17	12	15	5	7	11	4
					10	9	C													
2	4	9	0	C	18	2	8	4	14	10	12	6	16	16	6	12	10	14	4	8
					2	18	C													
3	8	6	0	C	18	3	3	6	12	6	18	9	6	15	18	9	15	3	15	12
					12	9	C													
4	16	9	0	C	18	4	16	8	10	2	6	12	14	14	12	6	2	10	8	16
					4	18	C													
5	13	18	0	C	18	5	11	10	8	16	12	15	4	13	6	3	7	17	1	2
					14	9	C													
6	7	3	0	C	18	6	6	12	6	12	18	18	12	12	18	18	12	6	12	6
					6	18	C													
7	14	18	C	C	18	7	1	14	4	8	6	3	2	11	12	15	17	13	5	10
					16	9	C													
8	9	9	0	C	18	8	14	16	2	4	12	6	10	10	6	12	4	2	16	14
					8	18	C													
9	18	2	0	R	18	9	9	18	18	18	18	9	18	9	18	9	9	9	9	18
					18	9	0													
10	17	9	C	C	18	10	4	2	16	14	6	12	8	8	12	6	14	16	2	4
					10	18	C													
11	15	18	0	C	18	11	17	4	14	10	12	15	16	7	6	3	1	5	13	8
					2	9	C													
12	11	3	0	C	18	12	12	6	12	6	18	18	6	6	18	18	6	12	6	12
					12	18	0													
13	3	18	0	C	18	13	7	8	10	2	6	3	14	5	12	15	11	1	17	16
					4	9	C													
14	6	9	0	C	18	14	2	10	8	16	12	6	4	4	6	12	16	8	10	2
					14	18	C													
15	12	6	0	C	18	15	15	12	6	12	18	9	12	3	18	9	3	15	3	6
					6	9	0													
16	5	9	0	C	18	16	10	14	4	8	6	12	2	2	12	6	8	4	14	10
					16	18	C													
17	10	18	0	C	18	17	5	16	2	4	12	15	10	1	6	3	13	11	7	14
					8	9	C													

N=CHAR NO. A=RELATED RES MOD K OF ORDER=H. R=RESOL MODULUS. T=REAL OR COMPLEX

TABLE II. CHARACTERS MOD K

K = 20

N	A	H	R	T	CHARACTER													
0	1	1	1	R	8	0	8	0	0	0	8	0	8	0	8	0	0	0
					8	0	8	0										
1	11	2	4	R	8	0	4	0	0	0	4	0	8	0	4	0	8	0	0	0
					8	0	4	0										
2	17	4	5	C	8	0	8	0	0	0	2	0	4	0	8	0	6	0	0	0
					2	0	4	0										
3	7	4	0	C	8	0	2	0	0	0	6	0	4	0	4	0	6	0	0	0
					2	0	8	0										
4	9	2	5	R	8	0	4	0	0	0	4	0	8	0	8	0	4	0	0	0
					4	0	8	0										
5	19	2	0	R	8	0	8	0	0	0	8	0	8	0	4	0	4	0	0	0
					4	0	4	0										
6	13	4	5	C	8	0	2	0	0	0	6	0	4	0	8	0	2	0	0	0
					6	0	4	0										
7	3	4	0	C	8	0	6	0	0	0	2	0	4	0	4	0	2	0	0	0
					6	0	8	0										

K = 21

N	A	H	R	T	CHARACTER													
0	1	1	1	R	12	12	0	12	12	0	0	12	0	12	12	0	12	0	0	12
					12	0	12	12	0											
1	8	2	3	R	12	6	0	12	6	0	0	6	0	12	6	0	12	0	0	12
					6	0	12	6	0											
2	10	6	7	C	12	4	0	8	10	0	0	12	0	2	8	0	6	0	0	4
					2	0	10	6	0											
3	17	6	0	C	12	10	0	8	4	0	0	6	0	2	2	0	6	0	0	4
					8	0	10	12	0											
4	16	3	7	C	12	8	0	4	8	0	0	12	0	4	4	0	12	0	0	8
					4	0	8	12	0											
5	2	6	0	C	12	2	0	4	2	0	0	6	0	4	10	0	12	0	0	8
					10	0	8	6	0											
6	13	2	7	R	12	12	0	12	6	0	0	12	0	6	12	0	6	0	0	12
					6	0	6	6	0											

M=CHAR NO. A=RELATED RES MOD K OF ORDER=H. R=RESOL MODULUS. T=REAL OR COMPLEX

TABLE 11. CHARACTERS MOD K

K = 21 (CONTINUED)

N	A	H	R	T	CHARACTER															
7	20	2	0	R	12	E	C	12	12	C	C	6	C	E	6	C	6	0	0	12
					12	0	6	12	C											
8	4	3	7	C	12	4	C	E	4	C	C	12	0	8	8	0	12	0	0	4
					8	0	4	12	C											
9	11	E	0	C	12	10	C	8	10	0	C	6	0	8	2	0	12	0	0	4
					2	C	4	6	C											
10	19	E	7	C	12	E	C	4	2	C	C	12	0	10	4	C	E	0	C	8
					10	C	2	6	C											
11	5	6	C	C	12	2	C	4	E	C	C	6	0	10	10	0	6	0	0	8
					4	0	2	12	C											

K = 22

N	A	H	R	T	CHARACTER															
0	1	1	1	R	10	0	10	0	1C	0	1C	0	10	0	0	C	10	0	10	0
					10	C	10	0	1C	0										
1	13	10	11	C	1C	C	E	C	4	C	7	C	6	C	0	C	1	0	2	0
					9	0	3	0	5	0										
2	15	5	11	C	10	C	6	C	E	0	4	0	2	0	0	0	2	0	4	0
					8	C	6	C	1C	C										
3	19	10	11	C	10	0	4	C	2	0	1	0	E	0	C	C	3	0	6	0
					7	C	5	C	5	C										
4	5	5	11	C	10	C	2	C	E	C	E	C	4	C	C	0	4	0	E	0
					6	0	2	0	1C	0										
5	21	2	11	R	10	C	1C	C	1C	C	5	C	10	0	0	0	5	0	10	0
					5	0	5	C	5	0										
6	9	5	11	C	10	0	E	0	4	0	2	0	6	0	0	0	6	0	2	0
					4	0	E	0	1C	0										
7	7	10	11	C	10	C	E	C	E	C	9	0	2	0	0	C	7	0	4	0
					3	0	1	0	5	0										
8	3	5	11	C	10	C	4	C	2	0	6	0	8	0	0	0	8	0	6	0
					2	0	4	C	1C	C										
9	17	10	11	C	10	0	2	0	6	0	3	0	4	0	0	0	9	0	8	0
					1	0	7	C	5	0										

N=CHAR NO. A=RELATED RES MOD K OF ORDER H. R=RESOL MODULUS. T=REAL OR COMPLEX

TABLE II. CHARACTERS MOD K

K = 23

N	A	N	R	T	CHARACTER
0	1	1	1	R	22 22 22 22 22 22 22 22 22 22 22 22 22 22 22 22 22 22 22 22 22 22 22 22 C
1	5	22	0	C	22 2 16 4 1 18 19 6 10 3 9 20 14 21 17 8 7 12 15 5 13 11 0
2	2	11	0	C	22 4 10 8 2 14 16 12 20 6 18 18 6 20 12 16 14 2 8 10 4 22 C
3	10	22	0	C	22 6 4 12 3 10 13 18 8 9 5 16 20 19 7 2 21 14 1 15 17 11 C
4	4	11	0	C	22 8 20 16 4 6 10 2 18 12 14 14 12 18 2 10 6 4 16 20 8 22 0
5	20	22	0	C	22 10 14 20 5 2 7 8 6 15 1 12 4 17 19 18 13 16 9 3 21 11 0
6	8	11	0	C	22 12 8 2 6 20 4 14 16 18 10 10 18 16 14 4 20 6 2 8 12 22 0
7	17	22	0	C	22 14 2 6 7 16 1 20 4 21 19 8 10 15 9 12 5 18 17 13 3 11 C
8	16	11	0	C	22 16 18 10 8 12 20 4 14 2 6 6 2 14 4 20 12 8 10 18 16 22 0
9	11	22	0	C	22 18 12 14 5 8 17 10 2 5 15 4 16 13 21 6 19 20 3 1 7 11 0
10	9	11	0	C	22 20 6 18 10 4 14 16 12 8 2 2 8 12 16 14 4 10 18 6 20 22 C
11	22	2	0	R	22 22 22 22 11 22 11 22 22 11 11 22 22 11 11 22 11 22 11 11 11 11 C
12	18	11	0	C	22 2 16 4 12 18 8 6 10 14 20 20 14 10 6 8 18 12 4 16 2 22 0
13	21	22	0	C	22 4 10 8 13 14 5 12 20 17 7 18 6 9 1 16 3 2 19 21 15 11 C
14	13	11	0	C	22 6 4 12 14 10 2 18 8 20 16 16 20 8 18 2 10 14 12 4 6 22 0
15	19	22	0	C	22 8 20 16 15 6 21 2 18 1 3 14 12 7 13 10 17 4 5 9 19 11 0
16	3	11	0	C	22 10 14 20 16 2 18 8 6 4 12 12 4 6 8 18 2 16 20 14 10 22 C
17	15	22	0	C	22 12 8 2 17 20 15 14 16 7 21 10 18 5 3 4 9 6 13 19 1 11 C

N=CHAR NO. A=RELATED RES MOD K OF ORDER=N. R=RESOL MODULUS. T=REAL OR COMPLEX

TABLE II. CHARACTERS MOD K

K = 23 (CONTINUED)

N	A	H	R	T	CHARACTER															
18	6	11	0	C	22	14	2	6	18	16	12	20	4	10	8	8	10	4	20	12
					16	18	6	2	14	22	C									
19	7	22	0	C	22	16	18	10	15	12	9	4	14	13	17	6	2	3	15	20
					1	8	21	7	5	11	D									
20	12	11	0	C	22	18	12	14	20	8	6	10	2	16	4	4	16	2	10	6
					8	20	14	12	18	22	C									
21	14	22	0	C	22	20	6	18	21	4	3	16	12	19	13	2	8	1	5	14
					15	10	7	17	9	11	C									

K = 24

N	A	H	R	T	CHARACTER															
0	1	1	1	R	8	C	C	C	E	C	E	C	C	C	8	0	8	0	0	0
					8	0	8	0	C	0	E	0								
1	7	2	4	R	8	C	C	C	E	0	4	0	0	0	4	0	8	0	0	0
					8	0	4	0	C	C	4	0								
2	13	2	8	R	8	0	0	0	4	0	8	0	0	0	4	0	4	0	0	0
					8	C	4	C	C	0	8	0								
3	19	2	8	R	8	C	C	0	4	C	4	0	C	0	8	0	4	0	0	0
					8	0	8	0	C	0	4	0								
4	17	2	3	R	8	C	C	C	4	C	E	C	0	0	4	0	8	0	0	0
					4	0	E	0	0	0	4	C								
5	23	2	12	R	8	C	C	0	4	0	4	0	0	0	8	0	8	0	0	0
					4	0	4	C	C	0	8	0								
6	5	2	0	R	8	0	C	C	E	C	E	C	0	0	8	0	4	0	0	0
					4	0	4	0	0	0	4	0								
7	11	2	0	R	8	C	C	C	E	0	4	0	0	0	4	0	4	0	0	0
					4	0	8	0	0	0	8	0								

N=CHAR NO. A=RELATED RES MOD K OF ORDER H. R=RESOL MODULUS. T=REAL OR COMPLEX

TABLE III. PRIMITIVE CHARACTER N'S

K	ϕ(K)	T	N
1	1	R	0
1	0	C	
3	1	R	1
3	0	C	
4	1	R	1
4	0	C	
5	1	R	2
5	2	C	1 3
7	1	R	3
7	4	C	1 2 4 5
8	2	R	2 3
8	0	C	
9	0	R	
9	4	C	1 2 4 5
11	1	R	5
11	8	C	1 2 3 4 6 7 8 9
12	1	R	3
12	0	C	
13	1	R	6
13	10	C	1 2 3 4 5 7 8 9 10 11
15	1	R	5
15	2	C	3 7
16	0	R	
16	4	C	2 3 6 7
17	1	R	8
17	14	C	1 2 3 4 5 6 7 9 10 11 12 13 14 15
19	1	R	9
19	16	C	1 2 3 4 5 6 7 8 10 11 12 13 14 15 16 17
20	1	R	5
20	2	C	3 7
21	1	R	7
21	4	C	3 5 5 11
23	1	R	11
23	20	C	1 2 3 4 5 6 7 8 9 10 12 13 14 15 16 17 18 19 20 21
24	2	R	6 7
24	0	C	
25	0	R	
25	16	C	1 2 3 4 6 7 8 9 11 12 13 14 16 17 18 19
27	0	R	
27	12	C	1 2 4 5 7 8 10 11 13 14 16 17

TABLE III. PRIMITIVE CHARACTER N'S

K	NO	T	N
28	1	R	7
28	4	C	3 5 9 11
29	1	R	14
29	26	C	1 2 3 4 5 6 7 8 9 10 11 12 13 15 16 17 18 19 20 21 22 23 24 25 26 27
31	1	R	15
31	28	C	1 2 3 4 5 6 7 8 9 10 11 12 13 14 16 17 18 19 20 21 22 23 24 25 26 27 28 29
32	0	R	
32	8	C	2 3 6 7 10 11 14 15
33	1	R	11
33	8	C	3 5 7 9 13 15 17 19
35	1	R	14
35	14	C	5 6 7 9 10 11 13 15 17 18 19 21 22 23
36	0	R	
36	4	C	3 5 9 11
37	1	R	18
37	34	C	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35
39	1	R	13
39	18	C	3 5 7 9 11 15 17 19 21 23
40	2	R	10 11
40	4	C	6 7 14 15
41	1	R	20
41	38	C	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39
43	1	R	21
43	40	C	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41
44	1	R	11
44	8	C	3 5 7 9 13 15 17 19
45	8	R	
45	12	C	7 8 10 11 13 14 16 17 19 20 22 23
47	1	R	23
47	44	C	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45
48	0	R	
48	4	C	10 11 14 15

TABLE 11B. PRIMITIVE CHARACTER N'S

K	NO	T	N
49	0	R	
49	36	C	8 2 3 4 5 6 8 9 10 11 12 13 15 16 17 18 19 20 22 23 24 25 26 27 29 30 31 32 33 34 36 37 38 39 40 41
51	1	R	17
51	14	C	3 5 7 9 11 13 15 19 21 23 25 27 29 31
52	1	R	13
52	10	C	3 5 7 9 11 15 17 19 21 23
53	1	R	26
53	50	C	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51
55	1	R	22
55	26	C	5 6 7 9 10 11 13 14 15 17 18 19 21 23 25 26 27 29 30 31 33 34 35 37 38 39
56	2	R	14 15
56	8	C	6 7 10 11 18 19 22 23
57	1	R	19
57	16	C	3 5 7 9 11 13 15 17 21 23 25 27 29 31 33 35
59	1	R	29
59	56	C	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57
60	1	R	11
60	2	C	7 15
61	1	R	30
61	58	C	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59
63	0	R	
63	20	C	7 8 10 11 13 14 16 17 19 20 22 23 25 26 28 29 31 32 34 35
64	0	R	
64	16	C	2 3 6 7 10 11 14 15 18 19 22 23 26 27 30 31
65	1	R	26
65	32	C	5 6 7 9 10 11 13 14 15 17 18 19 21 22 23 25 27 29 30 31 33 34 35 37 38 39 41 42 43 45 46 47
67	1	R	33
67	64	C	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65

TABLE III. PRIMITIVE CHARACTER N'S

K	NO	T	N
68	1	R	17
68	14	C	3 5 7 9 11 13 15 19 21 23 25 27 29 31
69	1	R	23
69	20	C	3 5 7 9 11 13 15 17 19 21 25 27 29 31 33 35 37 39 41 43
71	1	R	35
71	68	C	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69
72	0	R	
72	8	C	6 7 10 11 18 19 22 23
73	1	R	36
73	70	C	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71
75	0	R	
75	16	C	3 5 7 9 13 15 17 19 23 25 27 29 33 35 37 39
76	1	R	19
76	16	C	3 5 7 9 11 13 15 17 21 23 25 27 29 31 33 35
77	1	R	33
77	44	C	7 8 9 10 11 13 14 15 16 17 19 20 21 22 23 25 26 27 28 29 31 32 34 35 37 38 39 40 41 43 44 45 46 47 49 50 51 52 53 55 56 57 58 59
79	1	R	39
79	76	C	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77
80	0	R	
80	12	C	10 11 14 15 18 19 22 23 26 27 30 31
81	0	R	
81	36	C	1 2 4 5 7 8 10 11 13 14 16 17 19 20 22 23 25 26 28 29 31 32 34 35 37 38 40 41 43 44 46 47 49 50 52 53
83	1	R	41
83	80	C	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81
84	1	R	19
84	4	C	7 11 19 23

TABLE III. PRIMITIVE CHARACTER N'S

K	MO	T	N
85	1	R	34
85	44	C	5 6 7 9 10 11 13 14 15 17 18 19 21 22 23 25 26 27 29 30 31 33 35 37 38 39 41 42 43 45 46 47 49 50 51 53 54 55 57 58 59 61 62 63
87	1	R	29
87	26	C	3 5 7 9 11 13 15 17 19 21 23 25 27 31 33 35 37 39 41 43 45 47 49 51 53 55
88	2	R	22 23
88	16	C	6 7 10 11 14 15 18 19 26 27 30 31 34 35 38 39
89	1	R	44
89	86	C	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87
91	1	R	39
91	54	C	7 8 9 10 11 13 14 15 16 17 19 20 21 22 23 25 26 27 28 29 31 32 33 34 35 37 38 40 41 43 44 45 46 47 49 50 51 52 53 55 56 57 58 59 61 62 63 64 65 67 68 69 70 71
92	1	R	23
92	20	C	3 5 7 9 11 13 15 17 19 21 25 27 29 31 33 35 37 39 41 43
93	1	R	31
93	28	C	3 5 7 9 11 13 15 17 19 21 23 25 27 29 33 35 37 39 41 43 45 47 49 51 53 55 57 59
95	1	R	38
95	50	C	5 6 7 9 10 11 13 14 15 17 18 19 21 22 23 25 26 27 29 30 31 33 34 35 37 39 41 42 43 45 46 47 49 50 51 53 54 55 57 58 59 61 62 63 65 66 67 69 70 71
96	0	R	
96	8	C	18 19 22 23 26 27 30 31
97	1	R	48
97	94	C	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95
99	0	R	
99	36	C	7 8 10 11 13 14 16 17 19 20 22 23 25 26 28 29 31 32 34 35 37 38 40 41 43 44 46 47 49 50 52 53 55 56 58 59
100	0	R	
100	16	C	3 5 7 9 13 15 17 19 23 25 27 29 33 35 37 39

TABLE IV. $L(0,y)$, $L'(0,y)$

K	N	Re L	Im L	Re L'	Im L'
3	1	1/3	0	.31606 62755 57540	0
4	1	1/2	0	.39159 43927 06837	0
5	1	3/5	1/5	.40634 13965 70487	.07655 63771 45729
5	2	0	0	.48121 18250 59603	0
7	1	4/7	$2 \cdot 3^{1/2}/7$	.35626 78142 72506	.06410 18180 33884
7	2	0	0	.69922 47609 03196	- .19115 28599 37905
7	3	1	0	.45354 68922 61845	0
8	2	0	0	.88137 35870 19543	0
8	3	1	0	.35636 25954 30333	0
9	1	1	$1/3^{1/2}$	.38031 02934 56905	- .00620 91950 71828
9	2	0	0	.84426 07688 88490	.36947 42274 66555
11	1	1.48420 34473 86297	.25456 08168 86316	.26298 81007 60277	- .02614 54127 79897
11	2	0	0	1.15681 64899 65261	.46623 56392 21504
11	3	.06125 10980 68249	.79854 75122 67834	.51237 39768 46529	- .08234 10664 15372
11	4	0	0	.87678 19719 23957	.53173 39005 22941
11	5	1	0	.10141 57628 09549	0
12	3	0	0	1.31695 78969 24817	0
13	1	$(10 + 4 \cdot 3^{1/2})/13$	$(2 + 6 \cdot 3^{1/2})/13$	.27389 86724 34522	- .26351 27221 31772
13	2	0	0	1.44678 90655 09812	.47026 81838 16586
13	3	1	1	.30175 53778 72957	- .26033 84707 08952

TABLE IV. Continued

K	M	Re L	Im L	Re L'	Im L'
19	8	0	0	1.45277 23338 72710	-.33789 96636 42997
19	9	1	0	-.44443 25660 67851	0
20	3	0	0	1.84273 00347 01113	.67427 54776 26817
20	5	2	0	-.52625 01929 64839	0
21	3	0	0	1.96444 09741 42194	-1.21876 52614 97857
21	5	1	$3^{1/2}$	.39942 26881 75298	-1.01582 18356 44353
21	7	0	0	1.56679 92369 72411	0
23	1	2 .56791 61046 39603	.22129 64560 32212	-.85555 69149 61107	-.10689 80486 21516
23	2	0	0	1.31335 59348 46157	1.71071 71608 31962
23	3	-.06063 82875 33160	1.65762 89564 10486	1.16943 31756 55631	-.80474 29051 73310
23	4	0	0	1.87253 86710 23387	.63299 67467 32440
23	5	.26254 77940 88111	.83431 17412 80327	.29503 58305 67389	-.97811 66684 96163
23	6	0	0	.88744 62271 99693	.98306 83291 23254
23	7	1.47360 87548 34204	.53541 19005 40193	-.59005 78552 81424	-.61327 80617 03699
23	8	0	0	1.15719 00128 57308	-1.72222 09325 08895
23	9	-.72169 52355 93975	-.88001 55526 08980	1.65172 84845 96972	.17328 64703 00849
23	10	0	0	2.70735 00181 00860	-.24800 20381 40696
23	11	3	0	-.89484 16097 39472	0
24	6	2	0	-.86686 44523 98599	0
24	7	0	0	2.29243 16695 61178	0

TABLE IV. Continued

K	N	Re L	Im L	Re L'	Im L'
13	4	0	0	.80754 06058 03756	.84435 06603 13880
13	5	$(10 - 4 \cdot 3^{1/2})/13$	$(2 - 6 \cdot 3^{1/2})/13$	.34066 38637 79141	.39288 30714 04252
13	6	0	0	1.19476 32172 87109	0
15	3	0	0	1.27377 75194 39554	-.89409 62071 16943
15	5	2	0	-.05917 50802 39517	0
16	2	0	0	1.61489 09161 73095	-.40319 97191 61511
16	3	1	1	.17447 26824 35040	-.54384 09051 77312
17	1	2.02591 29159 27545	-.75928 68360 22116	-.14338 68176 07525	.31800 37514 56174
17	2	0	0	1.64415 61857 37981	-.77056 06770 45327
17	3	1.28684 99334 53867	.18312 05017 05629	-.28203 38120 71173	-.18913 58760 08623
17	4	0	0	1.06667 5544 78041	.71758 66770 87595
17	5	-.52041 24480 70169	.28894 34354 24649	.97180 22852 15067	.17809 42328 71638
17	6	0	0	.95425 36696 08270	1.25730 56447 75319
17	7	.73706 13633 94640	1.46418 31565 20432	.51461 06901 83011	-.65636 11003 32323
17	8	0	0	2.09471 25472 61101	0
19	1	2.28603 66572 30563	-.31631 86482 83414	-.42907 14301 76376	.22706 35097 41163
19	2	0	0	1.88229 24413 19401	1.04247 47347 36524
19	3	1	$3^{1/2}$	.45669 73242 17490	-.81697 33527 23669
19	4	0	0	1.89914 86669 74475	.85242 82722 99539
19	5	-.51690 49243 03635	-.24795 11401 29595	1.07330 55709 98351	.54211 56047 91505
19	6	0	0	.50194 83471 96682	1.30391 89212 73034
19	7	.75718 40565 46756	1.16229 43339 86795	.24207 46120 00999	-.71615 03247 03019

TABLE V. ϵ AND ARG(ϵ)

K	N	Re ϵ	Im ϵ	Arg ϵ
3	1	$3^{1/2}$	0	0
4	1	2	0	0
5	1	1.90211 30325 90307	1.17557 05045 84946	.55357 43588 97045
5	2	$5^{1/2}$	0	0
7	1	1.02261 87918 71794	2.40013 33583 45538	1.17394 79534 27576
7	2	2.37046 94055 76201	-1.17510 62918 84787	-.46022 35744 82810
7	3	$7^{1/2}$	0	0
8	2	$8^{1/2}$	0	0
8	3	$8^{1/2}$	0	0
9	1	1.92836 28290 59618	2.29813 33293 56934	.87266 46259 97165
9	2	2.29813 33293 56934	1.92836 28290 59618	.69813 17007 97732
11	1	3.17606 64857 52390	.95530 18779 84370	.29217 35653 84828
11	2	2.63610 55643 24835	2.01269 65627 57447	.65209 24520 84554
11	3	-2.13117 47936 52102	2.54127 80247 15501	2.26864 84002.20198
11	4	2.07016 20998 31071	2.59122 15035 42878	.89671 81747 36191
11	5	$11^{1/2}$	0	0
12	3	$12^{1/2}$	0	0
13	1	1.88269 66926 19015	3.07497 20589 95239	1.02140 74751 23185
13	2	3.09912 46837 40938	1.84266 82269 54497	.53642 24514 00742
13	3	1.04483 16069 12815	3.45084 43768 44019	1.27679 50250 21113
13	4	.91083 58324 46326	3.48860 68976 50093	1.31540 87768 96969
13	5	-.13918 92672 69219	-3.60286 36315 95992	-1.60941 00786 70752
13	6	$13^{1/2}$	0	0
15	3	2.03614 78418 20509	-3.29455 64141 85328	-1.01722 19678 97851
15	5	$15^{1/2}$	0	0
16	2	3.69551 81300 45147	-1.53073 37294 60359	-.39269 90816 98724
16	3	1.53073 37294 60359	3.69551 81300 45147	1.17809 72450 96172
17	1	3.40489 82294 56392	-2.32522 43003 72919	-.59916 08920 78066
17	2	3.04792 94083 58189	-2.77671 14221 08048	-.73886 78741 02503

TABLE V. ϵ AND ARG(ϵ) continued

K	N	Re ϵ	Im ϵ	Arg ϵ
17	3	4.07544 09935 83321	.62512 45538 45543	.15220 19312 68197
17	4	2.53740 95426 61801	3.24985 42756 26652	.90788 74949 60880
17	5	-4.01625 12407 96554	-.93259 09986 69908	-2.91343 16800 53110
17	6	.31288 60714 06026	4.11121 66455 10195	1.49483 72845 26410
17	7	-.57415 46527 45935	4.08293 35574 71906	1.71050 33088 19333
17	8	$17^{1/2}$	0	0
19	1	4.27399 13560 17271	-.85615 29586 99351	-.19770 03112 14353
19	2	2.92982 08852 83875	3.22740 60141 47339	.83369 16158 48289
19	3	-.42602 15112 48112	4.33803 01603 32438	1.66868 86383 27192
19	4	3.29627 24435 10167	2.85211 99094 98111	.71328 46216 16381
19	5	-3.34093 83767 45221	-2.79966 61877 42926	-2.44411 36561 07368
19	6	-1.33281 42433 94786	4.15013 32740 77350	1.88154 18703 60194
19	7	.23581 42830 47877	4.35251 55512 54311	1.51667 03958 27372
19	8	4.11907 60899 06456	-1.42590 74884 30066	-.33326 02262 64910
19	9	$19^{1/2}$	0	0
20	3	3.80422 60651 80614	2.35114 10091 69893	.55357 43588 97045
20	5	$20^{1/2}$	0	0
21	3	2.77458 61853 69981	-3.64714 56647 56764	-.92044 71489 65620
21	5	-.29022 72862 95607	4.57337 60092 83458	1.63417 15279 10386
21	7	$21^{1/2}$	0	0
23	1	4.73441 27753 14405	.76507 23318 35201	.16021 31282 15957
23	2	.48983 19656 11407	4.77075 09519 43024	1.46848 08808 84316
23	3	-3.70182 17124 06681	3.04901 88601 50666	2.45259 42561 56737
23	4	4.09821 12129 57684	2.49091 64686 89367	.54613 92854 56681
23	5	-.06572 77472 57658	4.79538 10967 68059	1.58450 19385 83276
23	6	1.83051 46884 20521	4.43274 36171 60451	1.17917 36983 84475
23	7	4.38851 92577 86073	1.93414 03061 93108	.41511 60562 62526
23	8	-.05647 65017 88967	-4.79549 89734 90212	-1.58257 27636 73005
23	9	-4.78553 38523 65232	-.31411 10438 46563	-3.07604 90545 95738
23	10	4.73000 87956 25147	-.79184 39197 90224	-.16587 04240 90253
23	11	$23^{1/2}$	0	0
24	6	$24^{1/2}$	0	0
24	7	$24^{1/2}$	0	0

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
3	1	8.0397371556814667&000	1.1410552501050816&000	-2.4545099621298712-001
3	1	1.1249206207772935&001	1.4497180288332200&000	7.6823791913697294-001
3	1	1.5704619176721626&001	1.7483726301350657&000	-9.7270668318265483-001
3	1	1.8261997495693128&001	1.6169871877518526&000	-6.6284598510212837-002
3	1	2.0455770801774246&001	1.7416421836903812&000	1.3572345407092832&000
3	1	2.4059414856453451&001	2.5463036884379910&000	-1.2386873493948646&000
4	1	6.0209489046959967&000	1.2964995755658179&000	1.8276509586123733-001
4	1	1.0243770304166555&001	1.7884670315788848&000	-2.9677590944832697-001
4	1	1.2588098012312423&001	2.0596941952741539&000	4.9394247631991422-001
4	1	1.6342607104587227&001	1.9704130473440680&000	-7.5489374079158495-001
4	1	1.8291993196123535&001	2.0616037769246569&000	9.6300424380553980-001
4	1	2.1450611343983460&001	2.2405940669388182&000	-1.0204929604908778&000
4	1	2.3278376520455326&001	2.2813671375174652&000	6.7425452090622498-001
5	1	6.1835781954508539&000	1.1129301656040605&000	-4.4883016541825469-001
5	1	8.4572291744232078&000	1.1766943720402576&000	1.1193407351499073&000
5	1	1.2674946417011356&001	2.0194925632212837&000	-1.3035513986612472&000
5	1	1.4825025570328428&001	2.0555437184998591&000	-4.9417873330152741-003
5	1	1.7337802106855046&001	2.0127117652657054&000	-1.2523844186990466-001
5	1	1.8998588041686145&001	1.7551465245496572&000	2.0455999455796993&000
5	1	2.2487584582028750&001	1.9187457251931970&000	-2.3480936641882605&000
5	1	2.4365279775402298&001	1.3788CC32066604556&000	-7.3585761250088798-001
5	2	6.6484533447277147&000	1.5823062438726339&000	-4.0975587766811463-001
5	2	9.8314444328866696&000	1.5949360757349336&000	-1.4983910974618315-001
5	2	1.1958845626083515&001	1.6347824341334030&000	1.5306993846003087&000
5	2	1.6033821128384236&001	1.1009474139155484&000	-1.7500726922249813&000
5	2	1.7566994292325555&001	1.5823712095660157&000	2.3102865902104122-001
5	2	1.9540732622784750&001	2.0186466915968479&000	1.4710632742659139&000
5	2	2.2227405454459411&001	3.6734607914404536&000	-9.5282201222562646-003
5	2	2.4588466217408195&001	3.1482898703720381&000	-1.0190187688757617&000
5	3	4.1329C37C5212851&&CCC	9.7758CC8357844781-001	5.3724337759261793-001
5	3	9.4429311297285091&000	1.1497638353969906&000	-1.0932761550776375&000
5	3	1.1282856441581&CC&0C1	1.3945417300634419&000	6.5988273962631774-001
5	3	1.4115464266569646&CC1	2.5766886880156658&000	8.2787570095882811-001
5	3	1.6995903942590284&CC1	3.1445056240258427&000	-5.3586474751924339-001
5	3	1.9729054786311626&CC1	1.5570626936651018&000	-1.2870472369775637&000
5	3	2.1283047157777877&CC1	1.6310893645021414&000	4.7171369611033403-001
5	3	2.2965576434791486&001	1.5574603647369691&000	2.4822269157768522&000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
7	1	5.1981161994665456E000	1.7044623582395346E000	-3.5384C85141509044-001
7	1	8.4136109914711776E000	1.4348782326274152E000	-4.4913345131562312-001
7	1	9.9798959C2C513932EEEE	1.0362555529955C46716000	1.6770552475585207E000
7	1	1.3854542874481498E001	1.9196354748122172E000	-2.1497038557227722E000
7	1	1.5746869407639415EEE1	1.6365780246176590E000	-5.9976463583996103-001
7	1	1.7161416543706C70EEE1	1.5615611501604772E000	1.4072310670968610E000
7	1	1.9651224232335955EEE1	3.5810C6495235883226000	4.1597381317057061-001
7	1	2.1652525069756426EEE1	4.0562435854847957E000	4.8846780053748188-001
7	1	2.4154664535578771EEE1	2.0995331200742514E000	-2.1213102411633026E000
7	2	4.3564C16247362E42EEEE	1.4468428588043670E000	7.2754952042008878-001
7	2	8.7855547144990754EEEE	1.4419987149317541E000	-1.3573486792893107E000
7	2	1.0736119587493393EEE1	1.5581931C978076C2E000	9.5259C81383800413-002
7	2	1.2532547822E862746EE1	1.5052819684EC46C1E000	1.9121430202125608E000
7	2	1.5937448204687960E001	2.4947515705352693E000	-1.9222154133477156E000
7	2	1.7616C53198876542E001	2.6778817979766335E000	7.3194039825201559-002
7	2	2.0030558985C82C3CEEE1	1.8547325945C552736000	-1.0209405536506408E000
7	2	2.1314647244104256E0C1	1.7223205064567077E000	1.2415212446348810E000
7	2	2.3203672461346655EEE1	3.1244260860386743E000	2.8181520434476563E000
7	3	4.4757382837286831EEEE	1.1922145913264712EE00	-2.2224215437331718-001
7	3	6.8454917124913773EEEE	1.3540833069012903E000	1.3547874147363283E000
7	3	1.1160184543115530EEE1	5.8221496082661158-001	-1.6153455785464569E000
7	3	1.2489603343033[illegible]EE1	1.5571398522817761E000	6.6549029931373495-001
7	3	1.5112882258743765EEE1	2.4143528049317490E000	-8.8623340864036703-002
7	3	1.6802876475728854EEE1	2.3831690835589761E000	1.9018715034619355E000
7	3	1.9611876056690031EE01	3.7270974677785982E000	-1.6227425026849736E000
7	3	2.1899913703318322EEE1	1.3393937EC2638361E000	-1.5606939948896658E000
7	3	2.3162971799736579EEE1	1.5553784063216535E000	3.8852500692285088-001
7	3	2.4498847555356329EEE1	1.0162142419706516E000	2.5857575301202252E000
7	4	6.2012300427558813E000	1.0382515846141589E000	-8.7797654278595348-001
7	4	7.9236308980920377E000	1.1822820353879097E000	9.2113658759916621-001
7	4	1.1010644862C7249CEEE1	3.0284982730922686E000	5.4860417921799966-001
7	4	1.3829867899861368E001	2.8071862892183569E000	-1.1198897566457166E000
7	4	1.6013727134150408EEE1	2.2672209662842396E000	-7.1644651656125532-001
7	4	1.8044857542174025EEE1	1.6429762740569988E000	-2.5252502878937193-001
7	4	1.9113885719489582EEE1	4.5101155136903742-001	2.2113377789310648E000
7	4	2.2756405955774308EEE1	9.9096734263670243-001	-2.7276546187113064E000
7	4	2.3955938435167979EEE1	2.0840556347665943E000	-7.3367040390544116-002

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
7	5	2.5093745529291197&CCC	7.7802015506034939-001	4.7456375278979857-001
7	5	7.4849317397159611&000	1.9591233027438386&000	-9.8310591274821891-001
7	5	9.8935437940977221&0CC	1.9781961951014475&000	-1.0466696366742200-001
7	5	1.2257424886489217&001	2.1239209095099556&000	2.5922190806566690-001
7	5	1.4135077759037771&001	2.3023204600505729&000	2.1044486003221876&000
7	5	1.7714092561931159&CC1	6.6003167122473383-001	-2.0565625082162205&000
7	5	1.8889097600175881&001	1.5851517011114545&000	1.7538238959247587-001
7	5	2.0604819114912533&CC1	2.0390872523863469&000	1.3964267475803352&000
7	5	2.2666356427924666&CC1	3.7276065021005911&000	1.8573257460250693&000
8	2	4.8999739970070365&CCC	1.7170121236893011&000	-1.3109412067021685-001
8	2	7.6284288417653978&C0C	2.2823256029946889&000	5.7101326811082717-001
8	2	1.0806588163861712&001	1.8586648269363792&000	-1.0589760576835358&000
8	2	1.2310542994236530&C01	1.8999292196215046&000	1.3065680619299931&000
8	2	1.5195754256451236001	2.8485401780723241&000	-1.1476632471637060&000
8	2	1.7022285974308347&001	2.6214843646673370&000	9.2687644558905237-004
8	2	1.8805958977C7148&CC1	3.0371507741830C99&000	1.1207816411126899&000
8	2	2.1131645962221344&001	3.4847275065225C67&000	-9.5228424049130173-001
8	2	2.3083849996200547&0C1	2.2087426618745683&000	-8.8861018473780075-001
8	2	2.4201963557815&C26CC1	1.8399928438665760&000	2.1192130495865589&000
8	3	3.5761548367875891&C00	1.4031926857710021&000	3.5809439815578785-001
8	3	7.4344729573702210&CC0	1.9685320693199720&000	-6.5471575374408886-001
8	3	9.5032019619729090&0CC	2.1822716264855310&000	8.3146988429904435-001
8	3	1.2340501159072212&CC1	2.8351344482622758&000	-6.4655547541873341-001
8	3	1.4490971928160547&CC1	2.4467670136185313&000	-3.2668040069947030-001
8	3	1.6164693667563410&001	2.6340244300799975&000	1.5183629555789844&000
8	3	1.9086441660C648C4&CC1	1.4015745070578185&000	-1.6894783115075504&000
8	3	2.0073638423068069&001	1.6461389949065637&000	1.3202539422251310&000
8	3	2.2374667568296C89&0C1	3.9052625351302229&000	9.8960885246074417-002
8	3	2.4419648220C606C9&CC1	3.2417869C861858296&000	-1.0141227620840128&000
9	1	4.5757357624248559&CC0	1.4404362309230145&000	-6.1533789672236972-001
9	1	6.8918030040624451&CCC	1.7C2696309528526&000	5.4332688792426453-001
9	1	9.2624089646819879&0C0	2.5902364820542747&000	1.6004504676630014&000
9	1	1.2586494690151173&CC1	1.7362245452188285&000	-2.0542170786820378&000
9	1	1.4089278964366772&CC1	2.1820461139027567&000	1.2341239601809530-001
9	1	1.6255178151532136&001	2.0691642036335091&000	-2.8102950133640556-001
9	1	1.7471836185056538&CC1	1.3027746756236386&000	2.3779394304387416&000
9	1	2.0298659166537719&CC1	4.1891940412501325&000	-2.0151858760670929&000
9	1	2.2176517902378345&C01	2.7674072494184810&000	-1.5586702626384389&000
9	1	2.3834775677556883&CC1	2.2771154352330216&000	-6.2770906240312048-001

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
9	2	5.3195751232811956&CCC	1.7665482373954112&000	-8.1641762828731242-001
9	2	7.8346714385872323&000	1.7001732164926233&000	-1.7560801972022814-001
9	2	9.6207728532469855&CCC	1.6645SC45C0839547&000	1.6637104471451606&000
9	2	1.2601813393C145298CC1	3.886C6379847834936000	-7.5993567321404946-001
9	2	1.5125930IC76545C48CC1	1.5614526515011578&000	-1.6779022321672528&000
9	2	1.6447228436287895&CC1	1.9191E2510678C455&00C	5.1120830144023878-001
9	2	1.8325315415599C19&0001	2.6299778751142678&000	1.0609935177273967&000
9	2	2.0C87C361441C7284&CC1	3.5064501367500211&000	2.4287297235557028&000
9	2	2.2722178259276694&0C1	2.9692319053552148&000	-2.8315366593734868&000
9	2	2.4364628706218757&0C1	2.1368733615867479&000	-1.1926797515551766&000
9	4	3.4440931551489439&0CC	1.2384690278491567&000	8.1492608676229538-001
9	4	7.5324330522904069&C00	2.4270740867294934&000	-1.1388353887489754&000
9	4	1.0016550422565336&CC1	1.6825779143145347&000	-7.8444097784197389-001
9	4	1.1583649082344983&0C1	1.7048743231750164&000	1.C6966179433937766000
9	4	1.3751949SC9078668&CC1	3.2357214937216097&000	1.7743615136651729&000
9	4	1.6502063704564426CCC1	3.4009358311456665&000	-1.9674998794165563&000
9	4	1.8545064758488539&C01	1.7090192448506284&000	-1.3661215243959391&000
9	4	1.9769931915979808&CC1	1.9112076364170015&000	8.9745022045923948-001
9	4	2.1698128855256938&CC1	2.9891915024587484&000	9.6301816313341604-001
9	4	2.3294892017371350&CC1	3.5375458665538197&000	2.8662244977201663&000
9	5	2.9019946C77372835&CCC	1.1285782467846780&000	1.3411885965381682-001
9	5	5.9115899586279044&000	2.0351402466075813&000	1.1686753519652130&000
9	5	9.5654429345367C37&CCC	1.9462373491614593&000	-1.6840537720289607&000
9	5	1.1407478947536364&C01	2.0338566417167783&000	-1.3362615934497436-001
9	5	1.9386373737633531&C01	2.2192991500027696&000	5.2167238758654486-001
9	5	1.5109372647488CC66CC1	2.45CC620758504292&000	2.4289807300605863&000
9	5	1.8120618073995667&001	2.2620952445797357&000	-2.5931762234924064&000
9	5	1.9550649847172474&CC1	2.6869398752730135&000	-1.8215823764597365-001
9	5	2.1679908967226200&CC1	1.5444574217204535&000	-9.0429134219827526-001
9	5	2.2956183317665745&CC1	9.1186304044075852-001	1.7822123496607847&000
9	5	2.4804459564574062&CC1	4.9246972026690413&000	1.4082306504557174&000
11	1	3.5470410917194SC18CCC	1.6965824400174771&000	-2.5098804897062705-001
11	1	6.6307304504849421&0C0	1.3966667879622451&000	-5.1044869362088833-001
11	1	7.8864346592015075&CCC	7.0814156660011963-001	1.8019813907324296&000
11	1	8.1594064282948772&C01	2.0393550619788024&000	-2.6116767763050557&000
11	1	1.3330227437882184&CC1	1.7540662448486874&000	-8.2868025518832307-001
11	1	1.4674032161018154&0C1	1.8325994658911330&000	1.1262831627617296&000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
11	1	1.6642923635179381E001	3.3417250109926379E000	1.5486065845233709E000
11	1	1.8749537566535146E001	4.3467117057830964E000	-1.7432527010296620-001
11	1	2.0507976162286870CCC1	4.5751654312655456E000	-3.6201102487418828-002
11	1	2.2959293933788770E001	4.0295577587010261?-001	-1.6086442798587337E000
11	1	2.3623699209665796ECC1	1.2157692156414?C1E000	8.0385376486425439-001
11	2	5.1336996269537762E00C	1.0759458465606086E000	-1.0731122016209135E000
11	2	6.7062197918366289E00C	1.3296049542747134E000	7.6524800153061190-001
11	2	9.0057129099586C79E0CC	2.5753957548123224E000	1.9779883675242643E000
11	2	1.2246208519892531E001	1.5950944969814059E000	-2.1523938827138447E000
11	2	1.3458305039944746CC1	2.2946202971492871E000	7.6547826819726878-001
11	2	1.6000365709033381ECC1	1.6706?124146732606E000	-1.3107453421771991E000
11	2	1.7083214281473697E001	1.5955209786327755E000	1.2233405689865213E000
11	2	1.8758430005846545ECC1	2.6429964086168353E000	3.3981222933107895E000
11	2	2.1582011399172422E001	2.4333684136419172E000	-3.6225434828643106E000
11	2	2.3103466082450674E001	1.9227969213705097E000	-1.3922897617349317E000
11	2	2.4397684653628959EC1	1.9545451198788703E000	2.0172749592338317-001
11	3	5.C7C3163793C817126C00	1.3641440585141328E000	-1.1301434451025154E000
11	3	6.8518881244?C87886000	1.6953527668134946E000	6.4785179291122514-001
11	3	9.4288253220129096E000	2.6173455395955431E000	2.3510848820658926-001
11	3	1.2599860491396326001	2.9795819104979326E000	1.9555019496243300E000
11	3	1.4316647599233678CC1	1.5144C551679131498E000	-2.4904841623601746E000
11	3	1.5681251205313450E001	2.0765602547093328E000	-2.3411048878945169-001
11	3	1.7431617935225737ECC1	1.9721455641706361E000	1.6467523662604918-001
11	3	1.8527889377871?C7E001	6.8238144090993344-001	2.8325090968757632E000
11	3	2.1598088258814195E001	2.2202422841355014E000	-3.3508012276316063E000
11	3	2.2838854063380596CCC1	2.9006?C489433C8C9E000	-3.3526817207012163-001
11	3	2.4458195812153090E001	3.3443192742152999E000	4.0381482500754134-002
11	4	4.6293536625111226ECCC	2.3544520279676538E000	-2.8698855038641281-001
11	4	7.6618576286868621E000	1.6525379456629300E000	-1.0466041815848470E000
11	4	9.3257627859556258E0CC	1.7177870795886?C4E000	5.9452452360719348-001
11	4	1.1086040907349663ECC1	1.5148181299966212E000	2.6152928056047081E000
11	4	1.4391345209913738E001	1.680C7148229C01?0E000	-2.8981458728057285E000
11	4	1.5957014739?C6785ECC1	4.5927620423642873E000	-8.1056408495872622-001
11	4	1.7C5898771290?8596CC1	1.4?554C954C4C58C1E000	1.5737?718595867758000
11	4	1.9507528806831888E0C1	2.8259654535699763E000	-8.2203320971352485-001
11	4	2.0612650543723331ECC1	2.1256581862888390E000	2.6362770652829653E000
11	4	2.3139078605782840ECC1	4.0613139014286677E000	-2.6048829639444925E000
11	4	2.4720835788552776CCC1	3.3229650385476C57E000	-1.3370831853572198E000

TABLE VI. ZEROS OF L-SERIES

K	A	ZERO	RE DERIV	IM DERIV
11	5	2.4772437112292343&CCC	1.1731?0&14&1349826000	7.8746670614746679-001
11	5	6.8007084083865180&CCO	1.8845230557163026&000	-1.6062626954780180&000
11	5	8.9712843&&49538386CCC	1.3C51189443517223&000	-5.6580114886657620-001
11	5	1.0108337357392797&CC1	7.62C6980244852331-001	1.6256683178359389&000
11	5	1.3040119328&172478CC1	4.1835679967821476&000	-5.5606727517258138-001
11	5	1.5109158246665C18C&CC1	3.4313419&C&C0685556CO	-1.0779968196665454&000
11	5	1.6990710701030143&CC1	2.8896971496310506000	-8.9478203603413470-001
11	5	1.8797246536162653&CC1	2.2027679696350021&000	-6.0839709388298840-001
11	5	2.0067593328&46C94&CC1	1.9712575506919318&000	1.4435197196864643&000
11	5	2.1638177818276574&CC1	2.7884510696173935&000	3.8896326729033440&000
11	5	2.4672836861C955488CC1	1.3447?2C4462C2645-001	-2.9941415674184877&000
11	6	2.6960040848691723&CCO	9.7369837874703254-001	9.3182377106849009-001
11	6	7.2069264712591C43&CCO	1.3755808734610953&000	-1.6562193861106620&000
11	6	8.7041610659115580&CCC	1.8921054203034679&0CO	5.7231363960265990-001
11	6	1.1275796241973477&CC1	2.1598600727623932&000	-5.2869915116070528-001
11	6	1.2623296171423352&CC1	1.7571131076046289&000	2.0139673028488257&000
11	6	1.5169105282641905&C01	4.9232648166134922&000	-3.2336036556231657-001
11	6	1.7688076037910&3C&CC1	9.8231293233481037-001	-2.2140988371571701&000
11	6	1.8893441735693521&CC1	1.5163078580848125&000	-1.5305151398670676-001
11	6	1.9990626024343921&CC1	9.9541016135363460-001	1.9850919468958110&000
11	6	2.2157147139133311C&CC1	4.9803798519514560&000	1.7601542415305485&000
11	6	2.4336616343661370&001	3.8788013545642190&000	-2.4155285218411029&000
11	7	1.2311882&C94&44456&CCC	7.9485052491489628-001	1.4403746034958717-001
11	7	4.9627151638056233&000	2.3456531553902512&000	7.2018043429811198-001
11	7	8.0893911453840612&000	2.8475982466642964&000	-1.0448769425165830&000
11	7	1.0490053638202233&0C1	1.9035382232573377&000	-1.0293044428345694&000
11	7	1.2115069806903322&0C1	1.7634552000583685&000	2.9075846344920653-001
11	7	1.3437657526437057&CC1	9.0702781475485916-001	2.7336051999279094&000
11	7	1.6949789782231107&CC1	5.2383005786470280-001	-2.5120856817508748&000
11	7	1.7874813297635343&001	2.0076824452359429&000	3.9862237754257136-001
11	7	1.9830128560533166&CC1	2.7731200114299226&000	-2.2342034191319022-001
11	7	2.1320393716311146&001	2.8369693248326187&000	1.1688071545057943&000
11	7	2.3103365240819911&001	3.8315403644251264&000	1.0575030351178301&000
11	7	2.4710602022376963&CC1	4.8785330548815352&000	2.1219001350629563&000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
11	8	3.610040431481681866C00	1.4547177680d441296000	1.5872875752616284-001
11	8	6.031809302694154066C00	2.0472338837395534&000	1.6650315431520892&000
11	8	9.968986597464014466000	3.5933550666712186-001	-1.6017257878576828&000
11	8	1.091936691352131366C01	1.2681322554881386&000	5.5252558703244757-001
11	8	1.293643640384360966001	2.4435392258523289&000	1.3009289565591369&000
11	8	1.499397619485517966C01	3.8896404714554124&000	1.3905966461850507&000
11	8	1.733108585992351066C01	3.7C2619114491045&000	-1.6534549298976610&000
11	8	1.900984021465341786001	3.3936157238582091&000	-6.2484807546458028-001
11	8	2.097103286691046566C01	2.2296439848020345&000	-1.2952783713087224&000
11	8	2.242170734775003966C1	1.5864541353033718&000	-3.5958949411698355-002
11	8	2.319330453707956366C01	-2.8333626975358338-001	2.2421198918166406&000
11	9	3.414921879322203786CCC	1.1274171152253511&000	-3.7347016761498163-001
11	9	5.273086575842089966000	1.0455626262188064&000	1.4501598530828020&000
11	9	8.953545464372326466CCC	2.8320187956034602&000	-1.6621524061181127&000
11	9	1.100919006252031266C1	2.2851458417820516&000	-7.7563474400577920-001
11	9	1.273163403755865566C1	2.4747252846062275&000	5.0778083136735079-001
11	9	1.494827047321575966C1	2.1945727288230396&000	-4.1626005707009215-001
11	9	1.600335371281118166001	1.0049898041128375&00C	2.6946729259067847&000
11	9	1.923890660918179366C01	1.3322048213559458&000	-3.3241882881417047&000
11	9	2.057905577472379766C1	1.8884678207134493&000	-8.6016897491736143-001
11	9	2.188489757248339466C1	2.0398601709196287&000	7.5316530757400522-001
11	9	2.338804121013922766C1	2.5521199445479147&000	2.2840295050382618&000
12	3	3.804627633C508651&0C0	1.931623C7434771456000	1.6954336094051564-001
12	3	6.692223320SCC1312800C	2.5208214014882378&000	-1.9633586636486070-001
12	3	8.890592958726741566CC	2.8331459783C154826000	2.8554756334335015-001
12	3	1.118839274SC749C966C1	2.9423491422908582&000	-4.3957523677605910-001
12	3	1.296617886802884566C1	3.1468571840098602&000	6.5327970085997062-001
12	3	1.518148087588821786001	2.9515670867200C536000	-9.1885QC7668340893-001
12	3	1.663263327452376286001	3.0955078493453346&000	1.1899523833112741&000
12	3	1.888436945712065166C1	2.8351195C1966525756000	-1.3993799126608499&000
12	3	2.010392819124581986C1	2.8804263041390&C776000	1.4928253281839419&000
12	3	2.228583910722688E6CC1	3.2382139207180314&000	-1.4347170161860056&000
12	3	2.356131971313784566C1	3.2650554154837919&000	1.0621748403179033&000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
13	1	4.2446093426458497E0CC	8.9251379293101517-001	-5.2820C65323180100-001
13	1	5.5771319695126146E0CC	1.0010842737127630E000	1.0206801171471176E000
13	1	8.2890924153835714ECCC	3.386C22C1116C665Z6000	1.1793673994514901E000
13	1	1.0933239191167282ECC1	3.1693959439724204E000	-1.6156824910957889E000
13	1	1.2712070365924251ECC1	2.9473030785891692E000	-5.1284367812071578-001
13	1	1.4662973932502155ECC1	2.4728440479612150E000	-7.4094598392342367-001
13	1	1.6255893625113448ECC1	1.9844323968911908E000	1.8108C911033452889-C01
13	1	1.7251385414910358ECC1	3.0628685634340177-001	2.9657667140567002E000
13	1	2.0654260416588518ECC1	1.4726018239705820-001	-2.4575C594303362306000
13	1	2.1366642269C28332ECC1	1.9201706066965152E000	3.2449472138551493-001
13	1	2.3253722350274113ECC1	2.2345567211661596E000	-7.0704547210792294-001
13	1	2.4261910613362986ECC1	1.7468905458239957E000	2.0783757510584384E000
13	2	4.4548539115E4C681ECCC	1.7807457719C819428000	-9.5629081834439570-001
13	2	6.8098310691689439E000	1.4031663722429204E000	-3.3201743119869334-001
13	2	7.9951284248480114ECCC	5.4529392971153680-001	1.9126344939846391E000
13	2	1.1530313056363383ECC1	2.2764559384599852E000	-2.6849237145130172E000
13	2	1.3035590426179138E001	2.5467890130550528E000	-4.1170179224357409-001
13	2	1.4776900321569510ECC1	2.8283015011229646E000	1.6194029422989697-001
13	2	1.6649318517863811ECC1	2.6717426518449759E000	-1.6042975198487051-001
13	2	1.7931546495663685EC01	2.3198087191090856E000	2.3339877882084664E000
13	2	1.9957427147933150ECC1	5.8414725176852590E000	1.2730669538296148E000
13	2	2.2466060863804557ECC1	5.2702524497844728-001	-2.8246583306146156E000
13	2	2.3567217346394998E001	1.4245121474295750E000	-5.5529021385232227-001
13	2	2.4473479552GC31126ECC1	1.C7748C857945657GE000	1.6782331722828214E000
13	3	3.7438215641461395E0CC	2.1691175430783883E000	-3.2631219240612595-001
13	3	6.7294196966943624E000	1.5597505387166322E000	-9.3808422669791746-001
13	3	8.2066232014560272E0C0	1.5801260513405356E0C0	1.0348324899426966E000
13	3	1.0212540844533478E001	2.8097514877379018E000	2.7128862104500753E000
13	3	1.3587144687321545ECC1	1.6920266259901C28-001	-2.0841265803994392E000
13	3	1.4468748219148105E001	1.5696566773999937E000	2.6507515968951637-001
13	3	1.6083089930495055ECC1	2.4169560296474276E000	1.3901731796694332E000
13	3	1.8106863800509828ECC1	3.5560JC81287797306C00	1.9952774046456916-001
13	3	1.9552902275290640E001	3.6896189800881423E000	2.2612915651172584E000
13	3	2.1746038258668166EC01	4.5997511916878133E000	-2.0834322176015533E000
13	3	2.3439296580966631E001	3.0517940250953697E000	-1.8230069068176839E000
13	3	2.4952022662455604EC01	2.4258798011822022E000	-9.0353087856382155-001

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
13	4	4.938590808457977186CCC	1.629C0134333146346&000	-1.2454632341153415&000
13	4	6.729314311681933186000	2.0058426560734708&000	5.4084585383375080-001
13	4	9.414175909214344C&CCC	1.760951586646297B&000	-6.7783825939058614-001
13	4	1.045772032495685186C1	8.4326714756962633-001	2.1509682955434073&000
13	4	1.361724094236895186C1	3.2016888326812185&000	-2.8959486734064270&000
13	4	1.542454877186256C&CC1	1.9190479776148575&000	-1.5134242709952891&000
13	4	1.687786511665621686C1	1.6961838754195C250&000	-1.2866255903088933-001
13	4	1.788925107706472486C01	7.4798609676648019-001	2.2841241875184396&000
13	4	2.C2991963519078756CC1	5.58E81756599C9371&000	-1.8544514403925000-001
13	4	2.222865695310673466001	3.4768335689347595&000	-2.0925251556786001&000
13	4	2.3549C792431303146CC1	3.7420978208817366&000	1.9869689487874869-001
13	5	8.8396030955C993019-CC1	7.4608435596557560-001	4.6115731734518862-001
13	5	5.8346687891686858&0C0	1.4759070149275936&000	-1.6400113518392364&000
13	5	7.5425647912180547&CCC	1.8258SC1498488335&000	1.1516093525132246-001
13	5	9.4371452978705126&000	2.309C6743996975636&000	1.2382500853396564&000
13	5	1.1706748031945682&CC1	3.6687118798009192&000	4.9928602941762369-001
13	5	1.3620424253839547&CC1	4.418C2536559743366&000	6.8206146939947749-001
13	5	1.628871078987774986C01	5.7765964261143201-001	-1.9450871571179553&000
13	5	1.714380802629295186CC1	1.5781499294327238&000	5.6642274939013161-001
13	5	1.890955342446577886C01	2.422C56457453C827&000	7.2700677621245481-001
13	5	2.011901294655637686CC1	1.4334576435909677&000	3.6538936799314503&000
13	5	2.296077375481446386CC1	2.2896332E83S7C3946C00	-4.0416225740069170&000
13	5	2.434566566161053686C01	1.9476294559415035&000	-1.3543773C245352636&000
13	6	3.119341475CC86C346CCC	1.5952159704582SC26000	1.1104428181290511&000
13	6	7.231590739418762086C0	9.9819055273667538-001	-1.8007726556080400&000
13	6	8.625426635032591686CC	1.5878184353949130&000	2.0541835040991960-001
13	6	1.033642072623153956CC1	1.9225480467506161&000	1.7051819715153447&000
13	6	1.261701279102317986C1	4.4013612361055566&000	1.1596743246903866&000
13	6	1.514833241706574486CC1	2.0095768199909352&000	-2.2805258900168206&000
13	6	1.627482605745E588&CC1	2.6771736702411C416000	9.2421462111650283-001
13	6	1.875125235623423386CC1	1.0324058689639104&000	-1.5064947294C43146000
13	6	1.954804144334704C&CC1	1.2245619541214171&000	1.0440375335050166&000
13	6	2.095918191740SC316CC1	1.3944501SC50628266C00	3.7187558797400411&000
13	6	2.359202788517293786C01	4.6427299909165638&000	-3.8172164584507992&000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
13	7	3.3298323592257066E000	1.7975109398181439E000	8.2038446445063527-001
13	7	6.6119788601771887E000	3.1709758285044482E000	-7.1270338328509926-001
13	7	9.3047042004695309E000	1.2429117523878120E000	-1.3968910870615449E000
13	7	1.0561822846541455E001	1.4721666605348988E000	6.1983296523945646-001
13	7	1.2088231214850623E001	1.3194128252275362E000	2.8239651711529581E000
13	7	1.5125320499526784E001	3.0028710722150222E000	-3.0278345380911220E000
13	7	1.6675390801915911E001	2.6912482299568765E000	-9.9774555330679324-001
13	7	1.8229826791150416E001	2.7376342697288263E000	-1.6751312215584736-003
13	7	1.9792304584161733E001	3.0883764168944195E000	8.6308050006258327-001
13	7	2.1683451216124721E001	2.7389564179806013E000	-3.8575914650697838-001
13	7	2.2606637778127121E001	1.1687944667534399E000	3.3628323768140283E000
13	8	2.2731312010133709E000	9.3215533759760848-001	7.4309535292284302-001
13	8	5.9943334826393757E000	3.1212112615455157E000	-3.3038125455860548-001
13	8	8.5343438822950019E000	2.6048693191547887E000	-1.0954126036622132E000
13	8	1.0506730485058795E001	2.3249576300354577E000	-4.5588216259439530-001
13	8	1.2325779168648482E001	2.0454240781778998E000	1.5000919535531948-001
13	8	1.3510731028223699E001	5.0598191731193781-001	2.9046403187746195E000
13	8	1.6773490865675870E001	1.3368493210365849E000	-3.4991823835551332E000
13	8	1.8177475821821411E001	1.5337904859549075E000	-1.0106964480133872E000
13	8	1.9165776869463774E001	1.5541835343434943E000	1.4577786470932750E000
13	8	2.1290723664194827E001	3.2823519763361239E000	-2.3623011383620911-001
13	8	2.2502180003961319E001	2.8363488548267602E000	2.6957838887252351E000
13	8	2.4700975914928955E001	4.6877903747580557E000	-1.8334230459206973E000
13	9	2.1955531911254145E000	1.0987768638209872E000	-3.6408581311612695-002
13	9	4.5654012450856859E000	1.4908106850466229E000	1.6413812528218807E000
13	9	8.4826985330787716E000	9.94446183667698[illegible]-001	-1.9069902723195291E000
13	9	9.5988295758840470E000	1.7851322424216980E000	7.9215419564120054-001
13	9	1.2148426919164611E001	2.1415741640449023E000	-9.2168339065126131-001
13	9	1.3397280801938934E001	1.8975677685568918E000	1.5697617330202120E000
13	9	1.5346893684388531E001	4.1683902794237123E000	2.4237638741638387E000
13	9	1.7917874765388983E001	2.7094347897138932E000	-3.2699290222856971E000
13	9	1.9597386678987031E001	1.3383013347662717E000	-1.4038072696331255E000
13	9	2.0599047034745260E001	1.5631172421352636E000	7.8511744581330618-001
13	9	2.1939834993780180E001	1.6320416184383394E000	2.9834269732837732E000
13	9	2.4211314367316313E001	5.4719888026136793E000	-9.8569564766337551-001

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
13	10	3.660974648384434460C0	1.3551818112508383&000	-2.5546528266203893-001
13	10	5.422738SC07217C536CCC	1.1602503198661469&000	1.7647349416278043&000
13	10	9.046993348C&704C5&CCC	2.5238993874627210&000	-2.3604339085695384&000
13	10	1.112452605047154&6C01	1.0643414750961463&000	-1.0198496512470478&000
13	10	1.199504392631782336C1	8.8426223138823420-001	1.3651230545443891&000
13	10	1.456208942822243360C1	3.3631766761014704&000	-3.8061473144739731-001
13	10	1.596780220625727C60C1	3.2365445677942350&000	2.2267540121539128&000
13	10	1.838297991113929466C1	3.8743314305965428&000	-2.4638237240419490&000
13	10	2.009754801517745660C1	2.4627840502124031&000	-1.6232422109671004&000
13	10	2.157530316915113&3CC1	2.0055243414878328&000	-4.5248154880088780-001
13	10	2.268077176274483086C1	1.64C4C694C1354379&000	1.7100610793625787&000
13	10	2.417186681125933C78CC1	2.5647851878665407&000	4.5186392898042322&000
13	11	2.345468533555064C&CCC	1.3455859174462242&000	-2.8265913186528001-002
13	11	5.376883025807054786000	1.9580599239669401&000	3.2511886250264041-002
13	11	7.217702604C2582886CCC	2.0358398548534415&000	2.0945760143639113&000
13	11	1.074269657978&0C396CC1	9.1397472817699145-001	-2.3272457186858725&000
13	11	1.8976C7428255922186C1	1.8269529217375172&000	-2.6258724939613827-003
13	11	1.368260616414036C&CC1	2.1942571?C7663158&000	7.8440209014381480-001
13	11	1.519982307397033086C1	2.310C2810C8617484&000	2.9610787093868972&000
13	11	1.795727240C&912636CC1	2.5825324858944213&000	-2.7808512750327210&000
13	11	1.912658986C318C796CC1	3.2438?8C8279C6855&000	6.1407813010773707-001
13	11	2.119379701492812966001	2.9027622940346687&000	-1.6887431031175380&000
13	11	2.270723393356596266CC1	2.1186818436229512&000	-5.9237442163507975-001
13	11	2.377231307382&C83&CC1	1.7151883267C864586000	1.7893411037803825&000
15	3	2.7346C37C911883736CCC	1.3741479199857884&000	3.4961335012415034-001
15	3	5.243010492754489466CC	2.3893254742755210&000	1.5972808073380262&000
15	3	8.414685249804895&6CCC	2.5777164326937115&000	-2.0833535CC1105156000
15	3	1.C1876027276453C26CC1	2.331C5892968102036000	-6.8207869717836488-001
15	3	1.190737247654667086C1	2.26C984755497108&000	1.0007575774996986-001
15	3	1.333735585541558956CC1	2.2042718832983416&000	2.0589352848054523&000
15	3	1.536613957E457782&CC1	5.12C5C401C35E56546000	1.6354456576825270&000
15	3	1.777239024921666486001	2.0462979006?C172C&000	-3.0039486505137930&000
15	3	1.897699965C0882546CC1	2.7228190281089201&000	-2.9638587542938028-001
15	3	2.062763177596965C&CC1	2.62CC844C376494366000	-3.9304422842534356-001
15	3	2.188397302420891286C1	2.4[illegible]571C3172193266000	1.4399884881CC0687&000
15	3	2.333848CC177598786CC1	3.43[illegible]6330456791356000	3.3156819139725963&000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
15	5	3.C570182C58CE6852&CCC	1.551494943351133C&000	-5.4586168927430753-001
15	5	5.3431920536759689&C00	1.8696C03211890449&000	3.50854C2935177501-001
15	5	7.26763C5562&6EEC73&CCC	2.29C2C2428ICS4654&000	2.0198742368542156&000
15	5	1.0133748155228376&CC1	3.6940664217333655&0C0	-1.9025282568235507&000
15	5	1.2159698533810771&CC1	1.9980202037778629&000	-1.5383984175712299&000
15	5	1.3489384573155316&CC1	2.2395578884132697&00C	4.8005914506429680-001
15	5	1.52404C852SC39033&CC1	2.7531C765817426C1&000	7.2806875942668338-001
15	5	1.6633556416165586&CC1	2.7C622016042628736&000	3.1802686063826382&000
15	5	1.9103497185254655&CC1	4.17844C2543C754236C00	-3.0249131899081773&000
15	5	2.0662363C5141C3C7&CC1	3.0575590195814897&000	-1.6441342569077028&000
15	5	2.2176397688213761&CC1	2.4366090689093973&000	-8.3470724741976145-001
15	5	2.3367999238674935&CC1	2.4478428C54133847&000	1.1257290564674646&000
15	5	2.4903501165042568&CC1	3.4168609375615745&000	1.6948436732645558&000
15	7	4.4C670C2356C36737&CCC	2.0037146816727C28&000	-1.1324736980575623&000
15	7	6.5907826702154?C9&C00	1.8994931941888686&000	-3.3714645190470738-001
15	7	8.2645016539C3425C&CCC	1.9556536663413688&000	1.2594234820211324&000
15	7	1.0326204250314941&0C1	3.6955383534732346&000	2.0717879888059941&000
15	7	1.3028956273591639&CC1	2.4372541253711437&000	-2.7254833751734568&000
15	7	1.4464461134463436&CC1	2.6351039966944919&000	-5.4719254758443273-001
15	7	1.6171892884989842&CC1	2.4010279489594636&000	-3.2676680476793972-001
15	7	1.7444764266C55676&CC1	2.2303629232398793&000	1.7196746554609743&000
15	7	1.9137842424152781&CC1	4.1273516545692325&00C	2.8345121581060244&000
15	7	2.1284643657?C1849&CC1	5.2736171051935383&000	-2.3357&C82381229416000
15	7	2.3148945354924?7C&CC1	1.5408030427457346&000	-2.2804500505007387&000
15	7	2.4071858315178683&CC1	2.3525811996274524&000	7.3944069063209655-001
16	2	2.842441C24C7636C7&CCC	1.783C6610097391366&000	6.8705086500706054-001
16	2	6.2422266164266771&C00	1.8088889112080561&000	-1.2552457459507903&000
16	2	7.60C2030991566475&0CC	1.9388818454530443&000	1.2937536159412062&000
16	2	1.CC704C957CC49596&CC1	3.7313571248060424&000	-4.1483940823450325-001
16	2	1.2089144523325726&CC1	2.855269C1560846146CC0	-1.0066139091183885&000
16	2	1.3553C993153E0344&CC1	2.8930443754049542&000	7.4532466153010623-001
16	2	1.5247985C01C4019E&CC1	3.9785892882842?C8&000	1.4160044997712349&000
16	2	1.7482072627817549&001	2.6836915333681534&000	-2.1784323542322675&000
16	2	1.8745128716715184&CC1	2.6132971750073389&000	1.0544608638163005-001
16	2	1.9970144302147592&CC1	2.6538310042853855&0C0	2.7034453101765074&000
16	2	2.2391552640?364C5&CC1	2.1019497169782893&000	-2.6005517451535928&000
16	2	2.3290769671135C5&CC1	2.7422C5191C85C511&C00	1.2369205949780043&000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
16	3	3.3462194046633E3C16CCC	1.9252172544395947760000	-2.9234549842674726-001
16	3	5.5641094106971897E000	2.4823385779987136E000	7.9976111975756552-001
16	3	8.1178015711951346CCC	3.2548575790502534E000	-6.1523956167085526-001
16	3	1.0249860905418859ECC1	2.1387819694311420E000	-9.1077020557198C91-001
16	3	1.1384761298857380E001	1.8062162342952299E000	2.0891004237073038E000
16	3	1.4178995431966875ECC1	1.9277961290101403E000	-2.2535946685675356E000
16	3	1.5238424483154C38EC01	2.4226291204E064546E000	9.1026986689928928-001
16	3	1.7028663692256346ECC1	3.7980708364701094E000	6.7425366809327763-001
16	3	1.8705166C21547170ECC1	4.5135103378738870E000	5.3299794143812285-001
16	3	2.0682182285207026ECC1	2.7864651596575593E000	-1.9490C128631308298E000
16	3	2.1850617642532195CECC1	2.8206597424541243E000	5.6959686191049458-001
16	3	2.3196943064786023ECC1	3.504C672E233671685000	2.4761722532998956E000
16	6	3.7721566366C675C478CCC	1.9170352275401606E000	-4.4210631925562347-001
16	6	5.7280473334358EEC5ECCC	2.3087467825921296E000	1.2195823819742288E000
16	6	8.6678664084923850ECCC	2.1397372709035429E000	-1.5801292522796212E000
16	6	1.0002590232436555ECC1	2.3490411991883268E000	9.7070266581745777-001
16	6	1.2049942431652227ECC1	3.7507831684974522E000	4.5733720962037871-001
16	6	1.3979916295SC4430ECC1	3.9918765741291903E000	-4.5780SC4178213724-001
16	6	1.6C85488CC4E475S6ECC1	1.1894925874EC21CCE000	-1.2467007523702540E000
16	6	1.6681722732861616ECC1	6.4031519540263234-001	1.7525672642616548E000
16	6	1.9234933E212647146CC1	3.9374702902845306E000	-2.2378662314678717E000
16	6	2.0727966572359244ECC1	2.9072959722647169E000	-9.2624897709438805-001
16	6	2.1927548602701529ECC1	2.9888475C567632126000	1.3972529275460878E000
16	6	2.3587555115C72546ECC1	4.7159C732833475356000	1.0034301796071282E000
16	7	1.5855837647CE5513ECCC	1.2892873238CCC1626000	3.5030633C24790892-C01
16	7	5.C1745981C1333741ECCC	2.6472728469742618E000	-4.1668280180890364-001
16	7	7.3238900CE842C5C8ECCC	2.4213126221261757E000	-4.4807QC6326436493-001
16	7	8.9108990E17324C97ECCC	2.5978957754626478E000	1.6182076792258960E000
16	7	1.1605732C15E2E246ECC1	2.3C77907C53101C17E000	-1.9172995703413842E000
16	7	1.2893129273135984ECC1	2.5011712354715340E000	5.9721167435395848-001
16	7	1.4526152CC12E3818ECC1	3.4821415904944297E000	1.7939SC3585662539E000
16	7	1.6824456244CC2775ECC1	2.920CE277E9834CE6E000	-2.0764736681269375E000
16	7	1.8062192E601C0730ECC1	3.0711C243EC2766216000	5.458C197497811247-001
16	7	1.9769378E9891387ECC1	3.5536IC72E3052849E000	3.7113455535123162-002
16	7	2.1106025338C59675ECC1	3.8734622172472CE7E000	2.1714709648386758E000
16	7	2.3404956C2885311CECC1	1.6S4CS883949257896000	-2.5677431326S648576000
16	7	2.4322953935543665ECC1	2.3427377360484965E000	6.4564734619055054-0C1

TABLE VI. ZEROS OF L-SERIES

K	A	ZERO	RE DERIV	IM DERIV
17	1	2.7670944819881642&CCC	1.C44E3547426758678000	-6.1317992608517405-001
17	1	4.2721855751740075&CCC	9.8005637720288411-C01	1.2959365731365820&000
17	1	7.1623905&C22&&&24&CCC	4.C9878C281C44021C&000	5.1369068034619728-001
17	1	9.7585480209381145&CCC	2.0314267C535136498000	-2.0574924961350870&000
17	1	1.1021316284444CC35&&C1	2.5&5C7&78508497988000	6.4371113786947907-001
17	1	1.3265774361735&44&CC1	2.0982554889123137&000	-1.2333897824850899&000
17	1	1.4424185572&&32&2&CC1	1.9134754&&&515115&CCC	1.C750C&32C599603C&000
17	1	1.5748418435905988&C01	1.5436899082182942&000	3.94626C1C0507766&&000
17	1	1.&7133105C2&&&C&3&CC1	&.45&4C7562657363&3-001	-3.8586054033082664&000
17	1	1.9827573224410462&C01	2.158720566575261&&000	-9.5155&77441794846-001
17	1	2.1C8&127129251C53&CC1	2.4713152527702624&000	5.0322435667591598-001
17	1	2.2542227927&C1551&CC1	2.96C13C5C&&&571598000	1.1609400746664463&000
17	1	2.3868131848710991&CC1	3.3833221178621532&000	3.287676C7039440806000
17	2	3.0308502175441112&CCC	1.3&&13&2C4573261&&000	-1.1804727651336986-001
17	2	4.7151287501&80&1&0CC	1.06053144542926146000	2.0142552327327563&000
17	2	&.5381C926355134336CCC	7.54C6C11590222651-001	-2.2892292185227676&000
17	2	9.5869617&&46C1364&CCC	1.926&37051203C346&000	3.9827255952650167-001
17	2	1.15747347C2211585&CC1	2.7329571595501050&000	3.6552958685542636-002
17	2	1.3115412616&2&2&1C&CC1	2.9033221178560792&000	1.4127655609485406&000
17	2	1.4963660134455729&CC1	4.25&27198&&&1&154C&0C0	1.0154549339630616&000
17	2	1.665682C539841597&CC1	5.4389337088237C90&000	1.0826277153893458&000
17	2	1.9172690328441136&&C1	&.7046&568&5&1462C6-002	-2.0314324774239957&000
17	2	1.9862604940973290&0C1	1.&545C311359442C78000	3.6496196072889730-001
17	2	2.1382492653284794&CC1	2.3346074411968788&000	7.6378689239218430-001
17	2	2.2500919C7098631&&C01	1.4&4&4&44&10C&884&000	3.4979402540079883&000
17	2	2.4625581933071316&C01	7.2254473655137C39&000	-7.0886967797906873-001
17	3	2.3450224437153C&5&CCC	1.4&1&245&&C&&52526&000	3.7589567070465346-001
17	3	4.8412668748&C4737&C0C	2.6065218469331128&000	1.7263990067613873&000
17	3	&.2571959&9C7C83&5&CCC	1.00508287523803836000	-2.3182787932879969&000
17	3	9.6026341345327768&CCC	1.6C7&C67727101775&000	-2.6206033376703348-001
17	3	1.C883533&7145159&6&C01	1.45064527401896&4&000	1.7077247568545764&000
17	3	1.2967759647&C94218CC1	4.2436208751277591&000	1.6756492154642789&000
17	3	1.5008504836026858&CC1	4.97315887C38781705000	-9.5015966281022855-001
17	3	1.6929212719&648C78CC1	2.6398712690435871&000	-2.0620608682732903&000
17	3	1.8C971261&&0C1682&&&1	3.125C1125319493546000	7.171&558738117932-001
17	3	2.0247738294575321&001	1.2448230179581854&000	-1.500&6685768495446000
17	3	2.1C03C02&3712782&&CC1	1.252&170697951430&000	1.1545250633152424&000
17	3	2.2196452901548815&CC1	&.7C7548165529162&-001	4.1851484137790601&000

TABLE VI. ZEROS OF L-SERIES

K	A	ZERO	RE DERIV	IM DERIV
17	4	3.3876430198OC58C8&CCC	2.3527295&839&8245&OOO	8.6536436280219015-001
17	4	6.4665033288665565&C0C	2.8792210544545554&OOO	-1.4423837515255314&OOO
17	4	8.5127310&C&2455326CCC	2.2623586625069968&OOO	-9.4918084992927891-001
17	4	1.0412833476969932&CC1	1.1628619232306777&OOO	-4.0395786887918332-001
17	4	1.1C389711276956536CC1	-2.805C926958108559-001	1.522509475210285&OOO
17	4	1.4279140983OC0657&CC1	3.3720284126114263&OOO	-3.6535568403965164&OOO
17	4	1.5854658931004722&CC1	2.31706C70376C57&O&OOO	-1.5996278100512738&OOO
17	4	1.9C9285647216109O&OC1	2.656&089772035573&OOO	5.0221326567580457-001
17	4	1.8763588643&6668&&CC1	3.3C65C136C36153516COO	2.8562949973169772-001
17	4	2.0280738951457105&OO1	3.3878861162583737&OOO	7.5352624379843857-001
17	4	2.1643775&752531C5&CC1	3.7309156195106517&OOO	2.5999535544129389&OOO
17	4	2.3417786777837554&CC1	&.8101976277821386&OOO	7.158436C842567475-001
17	5	4.4515C76C&2142334&CCC	2.4121958860997098&OOO	-1.1799281603999817&OOO
17	5	6.5213777993211879&CCC	2.5306&34085552361&OOO	-2.4751731812875365-001
17	5	8.7207727911463975&CCC	2.1166421797973281&OOO	-6.0685988575482373-001
17	5	1.0056732347947C38&CC1	1.8985999690366462&&OOO	1.6182483307004175&OOO
17	5	1.1954878961137&64&CC1	4.0967149024241187&OCO	2.9533331214&C4550&OOO
17	5	1.4847261885&C9839&C01	4.3792OC6638940961-001	-2.9501165984706135&OOO
17	5	1.5987421C&07&1476&CC1	1.4483136437729938&OOO	-6.2686275165933718-001
17	5	1.6936450452662188&C01	1.2270664359624135&OOO	1.6515786841675399&OOO
17	5	1.8947383&8C442874&CC1	3.9661394317030769&OOO	6.8229228838596360-001
17	5	2.0328974693623334&CC1	4.2433111657753450&OOO	2.7065082935232536&OOO
17	5	2.2511191482712403&OC1	3.3983707&822969836OOO	-2.97819C4733124608&OOO
17	5	2.3635873252&3736C&CC1	3.9459139501069676&OOO	3.8540758973998607-001
17	6	4.8514742042C&8716&COO	&.5427491567C95352-0C1	-1.327927C4325C3159&0C0
17	6	5.8773430906637415&CCC	1.1536251747364433&OOO	8.9157992269533458-001
17	6	8.4585921605235&7&&CCC	2.8138157350275295&OOO	-5.3360292279332997-002
17	6	1.0012387248C66384&CC1	2.846C8389576334766OOO	2.0472372922851752&OOO
17	6	1.2368557&7C1C4771&CC1	4.725117025881717&&OOO	-1.2075126830658041&OOO
17	6	1.4366075091&C3951&CC1	2.5936&56&4&585&23&OOO	-2.1447034413181023&OOO
17	6	1.5917783752&5C3&4&CC1	1.9018612469234106&OOO	-8.7097383610321687-001
17	6	1.7C795326C7&7123C&&CC1	1.73&81C&&267C[illegible]&1&OOO	9.8118068043290436-001
17	6	1.8250023351837899&OC1	7.0588393159O[illegible]15-001	3.7454747360025019&OOO
17	6	2.1108C4943703&C90&CC1	1.7458338561239477&OOO	-3.9289166480033898&OOO
17	6	2.2095826855453&0C&&C1	3.2&15744&C2481575&OOO	-6.619011986096111&-002
17	6	2.3843207480C84740&CC1	2.65765C113879987&&OOO	-1.4046142267117539&OOO
17	6	2.49C8398C473CC716&CC1	2.77068705164337C46OOO	1.2562621190462566&OOO

TABLE VI. ZEROS OF L-SERIES

K	A	ZERO	RE DERIV	IM DERIV
17	7	3.967230157164154E0CCC	1.522175641CC518598E000	-1.273464628744604260000
17	7	5.876135135475694582000	1.594454708206C1216000	-2.859042519920562-002
17	7	7.297138812SC2727C6CCC	1.153557196179175726000	2.154536464876853460000
17	7	1.026525357795C3673ECC1	3.741SC516C155C12C6000	-1.913239523589833860000
17	7	1.190056102577533962CC1	3.563656644544851626000	-3.959725C809478494-001
17	7	1.388726765877515C6CC1	2.590846560147314866000	-1.403012193124187526000
17	7	1.525827866285C4746CC1	2.6281567756644E3546CCC0	3.327738741359538O-001
17	7	1.696255332C8E99C562C01	2.442061617088583986000	1.27738C6676206382-001
17	7	1.789362486314C73E6CC1	5.724345326167 7C76-001	3.398946791150282126000
17	7	2.073601382461062360C1	2.578711297538C9C56CC0	-4.818815477062550726000
17	7	2.235274CC671266356CC1	5.072714082198877 2-001	-1.403374050882395260000
17	7	2.291608761336E5678CC1	1.045514988466460526000	9.910184356064533 8-001
17	7	2.455088429721233260C1	3.357818075011526360000	1.826831282874718160000
17	8	3.728142CE5042C446E6CC	1.585S9C5445446579E000	-8.385820882658740 -001
17	8	5.635590758 C91548&CCC	1.728927529121865 6000	4.2726000771254658-001
17	8	7.282832855534252326CC	1.5S7C6945169933596000	2.663701013709252160000
17	8	1.061732005264566726C1	1.412566615756686360000	-2.998717954388585960000
17	8	1.197767295241462C2CC1	1.923795157983031360000	-6.25783C5728048320-001
17	8	1.32C980729551C3756CC1	2.064715884927318460C0	1.581679610091721460000
17	8	1.564351453246EC746CC1	1.527442627912600C86000	-1.190545045949124960000
17	8	1.626848320767812860001	5.564490384920898 7-001	2.028215217858115660000
17	8	1.868163323E151C56CCC1	5.88CE622432170517 C00	-1.234648796089766260000
17	8	2.042753799212527260C1	4.034C7889907885366000	-2.359907C611254426 6000
17	8	2.22C0C28C53E522646CC1	1.22CC8C80804529326000	-1.822362204738857 6000
17	8	2.315205504769474160C1	1.541238124354753CC000	3.144409334027321-001
17	8	2.4C806736E2839181 6CC1	4.167592674582115 8-001	2.613548810364629160000
17	9	1.5542696C1485073786CC	1.3263E1871 EC447C66C00	-3.483109160878541 3-002
17	9	4.776887597840487 6&CC0	1.675268355385280086000	-4.525128870584472 -001
17	9	6.124541432519863286CC	1.121716985167971 6000	2.035565963247540 6000
17	9	9.285589056463884 66CCC	3.706736077296799 6000	-2.424658547847718 6000
17	9	1.145953084658036360C1	8.48641C6590023295-001	-1.513474502818788 6000
17	9	1.234958587 4C46435 6CC1	1.3305CC8C174316366000	7.918014794616530 8-001
17	9	1.391997128725079 96C01	2.151598609C168C376000	2.579233705030283 6000
17	9	1.610500264401027 76CC1	5.256903364998290 66000	-1.508899942672441 -001
17	9	1.787907737 4C631496CC1	4.387377113407438 76000	-1.272158912015751 36000
17	9	1.943553024609536 56001	4.114135562C457149 6000	-8.005352702505437 0-001
17	9	2.129930234154393 76CC1	1.736413595434841 96000	-1.739734976243375 86000
17	9	2.219566986157050C6CC1	2.00456C6C863795556CC0	1.171910685234901 76000
17	9	2.398511641 8C520C56001	2.447036080844522 46000	-6.98509C1930563653-002
17	9	2.467870157679C3136CC1	-2.936209753C734940-001	3.245286966481315 06000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
17	10	2.1221701014577989E000	1.1690513088211542E000	5.1093267470113527-001
17	10	5.0751153080965438E000	2.7923730980225229E000	6.9373617177745459-001
17	10	7.4078230543891543EEEE	3.9232559634718102E000	3.7978289396087005-001
17	10	1.0099631006022388E0C1	1.2761388659207643E000	-2.0562238405516611E000
17	10	1.1279869905220351EEE1	1.8999993251833512E000	2.7013358211561498-001
17	10	1.2937810668313018EEE1	2.3C1C8343102E29222E000	9.4911181679189671-001
17	10	1.4258617677355083E0C1	1.6933662277816013E000	3.8069721302522428E000
17	10	1.7436583911245402EEE1	-2.2679432333854346-001	-2.1651701297214388E000
17	10	1.8051317961378566EC01	1.6983907468573080E000	4.8204467298794854-001
17	10	1.9703355339789725EC01	3.3761949796167017E000	5.7790220612845140-001
17	10	2.1449297257167747EEC1	3.1794C4C3486841816E000	-6.3596019330927667-001
17	10	2.2692346790029229E001	2.8932012794477873E000	1.2318473218010210E000
17	10	2.3987378796128621EEC1	3.2730355195653067E000	3.8016110339621129E000
17	11	3.9131461302555944-CC1	8.1355826E33617512-001	-1.9765873314876630-001
17	11	3.2325203853518697EEEE	1.5951168147650880E000	1.4552895688176013E000
17	11	6.8194345706ZC5736EEEE	2.0983580C759C29626E000	-1.9810739208393284E000
17	11	8.2986753677556934E000	2.5911337959441837E000	4.0338877919352304-001
17	11	1.0638617456E6728EEC1	2.1410883456172784E000	-1.1558623781608083E000
17	11	1.1957847786912799EEC1	2.0131514792473442E000	8.3058406538654966-001
17	11	1.3333629792910JC3EC01	1.7426447490C328096E000	3.4522303654751909E000
17	11	1.6121484991735521EEC1	3.1441356146389508E000	-3.8772016824417786E000
17	11	1.7641201674892768EEC1	2.3070242455655653EEEE	-1.7585436145009685E000
17	11	1.9194451333396406EEC1	1.0865449495822748E000	-6.5886653460831987-001
17	11	1.9733914278616551EEC1	6.5312140805634941-002	1.5075414992257989E000
17	11	2.2143928924745125E001	4.7030626902203888E000	-9.2731926905665262-001
17	11	2.3383753126985355EEC1	4.5405746664279627E000	1.8807146744804125E000
17	12	1.8945628886364C06EEEE	8.1874155921503307-001	1.0236697556127150E000
17	12	4.C2106312276084196EEE	2.2617574823845865E000	-2.0775784151731661E000
17	12	7.9739522505711777EEEE	1.6482364276568977E000	-8.3510311851615801-001
17	12	9.1939026766233614EEEE	1.5891878565077736E000	1.4953280C459461386000
17	12	1.1656327353137838EEC1	3.0085752935784299E000	-6.4692818810324159-001
17	12	1.2862745743244886EEEC1	2.4955328381825936EEEE	2.6638317C995162056E000
17	12	1.5402C3147631056CEEC1	3.8442952723171225E000	-3.0129629997454016E000
17	12	1.7122130C956727526EEC1	1.8652812858961616E000	-1.8349379497003746E000
17	12	1.8306091782764297EEC1	2.0913176249018286EEEE	1.5461307C369CC098-001
17	12	1.9615854474606373EEC1	2.1717862334779341E000	1.6064043930812812E000
17	12	2.0964356893466C48EEC1	2.3831205726966815E000	4.4374216734071791E000
17	12	2.3532763967313614EEC1	2.6505761138525435E000	-4.5342945348311147E000
17	12	2.4515551762242722EEC1	1.5025332031624449E000	-1.5482295296095495E000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
17	13	1.7187971663856354&CCC	1.2627371388238259&000	9.2144373594652847-001
17	13	5.8969313427048536&CCC	1.3255974928157039&000	-1.9687029058382384&000
17	13	7.4445905350277345&CCC	1.7207C7334533172I&CCC	-1.2535418871560028-001
17	13	8.9753918588263837&CCO	1.8273855778061324&000	1.5304350454840441&000
17	13	1.C5845CC555546165&CC1	3.5532SC1114293618&CCO	2.3162268014233692&000
17	13	1.3672600734106SC36CC1	1.4559C4259&556C46&000	-2.4831323306IC8299&000
17	13	1.4566493238861758&CC1	2.3837211881096468&000	1.2237173036137094&000
17	13	1.6527498863144155&CC1	1.9118499916C446528&000	-2.0165673681968220&000
17	13	1.8094023IC4689854&001	1.9445950066530351&000	1.54761C6828201065-001
17	13	1.9162315C0535584360C1	1.2365179632684341&000	2.6036471463169991&000
17	13	2.1060809414139324&CC1	6.1754150574105781&000	3.1800793965550658&000
17	13	2.3502200478141C38&CC1	6.9399009481819927-001	-3.7104465780106109&000
17	13	2.4530287255563366&CC1	2.1154522587867835&000	-8.7090978144365406-001
17	14	3.7323453139703861&CCC	2.6386048692929067&000	-3.2296EC5786191853-0C1
17	14	6.7619483210414847&CCC	7.7C870373094362?7-001	-1.1367316109111278&000
17	14	7.6449721119141667&CCC	8.855669946332&878-CC1	1.0205441048978519&000
17	14	9.6339813179936796&CCC	2.5931831782424293&000	3.0495297996989360&000
17	14	1.2501117117C7371598CC1	2.8217C2C3474817420&000	-3.2841331355335547&000
17	14	1.4140530825829674&0C1	1.7707281346595771&000	-1.3659481654583317&000
17	14	1.5199466723475588&CC1	2.0145585490901361&000	1.1961139298182840&000
17	14	1.7095396259645680&0C1	3.6406732942764768&000	3.558705462631971?-001
17	14	1.8759036754209SC6&CC1	3.2086433019770712&000	3.4006557040989387-002
17	14	1.9830315101233428&CC1	2.2179367208799629&000	3.7252224288385241&000
17	14	2.2528305282?C7935&0C1	1.1339631383252215C&C00	-3.8552646523105152&000
17	14	2.3554354933821881&C01	2.6235225202218549&000	-7.0711782719548766-001
17	15	3.1671107502799315&CCC	1.8836341654706317&000	-6.8136360751636998-001
17	15	5.4320512780649534&C00	2.2449953977253022&000	7.1231387407532265-002
17	15	7.7197213354465208&CCC	2.0623210C898755836&000	-2.3307091981565649-001
17	15	8.8929128466147751&C00	1.0222196355675819&000	2.6624281141211516&000
17	15	1.2363568354503510&CC1	2.4295510880805087-001	-2.5143129243728816&000
17	15	1.3259265C892C2888&CC1	1.9212780349791710&000	1.5286997515388968-001
17	15	1.5089480431809652&0C1	2.1729?C60465836876&000	-2.5181269314499549-001
17	15	1.6137062550524379&CC1	1.3627915619626586&000	2.4641791993483653&000
17	15	1.8296587116035684&CC1	5.6858192422550169&000	3.4422530382206168-001
17	15	2.0168053652932932&CC1	3.8702434643444613&000	-2.0913447263997193&000
17	15	2.1445728875163CC1&CC1	4.1331940914430790&000	2.9572347528658513-001
17	15	2.3372561692137720&CC1	2.4414243136829561&000	-2.2347588494081022&000
17	15	2.4675367454723598&CC1	1.7271599392955967&000	-5.6992811854230095-001

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
19	1	2.3927644441546036&000	1.9762063390591040&000	-1.9938812918170782-001
19	1	5.3701190037218313&000	1.4423028977330661&000	-9.4981385859315429-001
19	1	6.5914277258732388&000	1.3185158679514228&000	1.3390150447551376&000
19	1	8.6735453472623649&000	3.5653306094668582&000	2.8296041985717032&000
19	1	1.1635351451206206&001	1.0284172452019736&000	-3.1261084340001715&000
19	1	1.2957993166777428&001	1.4513066340940312&000	-7.9689157857579427-001
19	1	1.3886795099461003&001	1.2360754438641395&000	1.6224492808482215&000
19	1	1.5984677143266472&001	3.9449364131720507&000	2.5917204985038450-001
19	1	1.7401930689589502&001	4.0624865690879845&000	1.8561922947241852&000
19	1	1.9180575876783693&001	5.7910874329632204&000	1.0435224900662197-002
19	1	2.1071723502346222&001	3.0927504864436988&000	-2.8303573339367218&000
19	1	2.2314254366910893&001	3.2333917005550946&000	-6.5035999611003648-001
19	1	2.3971525211878446&001	1.7017684240024946&000	-1.1074344392064458&000
19	1	2.4763826382247812&001	1.2727773751980129&000	1.4824631859676150&000
19	2	4.2706237172182306&000	9.6716044712491072-001	-1.3583562587385153&000
19	2	5.5895410371669134&000	1.4031954884693190&000	6.1132587549862347-001
19	2	7.4293253506636694&000	2.1100236687982411&000	2.2404232958535682&000
19	2	9.9385382554403015&000	4.9166277748670650&000	-6.8304467307587259-001
19	2	1.2139788696582056&001	1.8447446195756648&000	-2.1222812143258156&000
19	2	1.3185286175582667&001	2.4776497846202510&000	9.9277623256796199-001
19	2	1.5313347240163891&001	2.6293665069553942&000	-1.3699756838995644&000
19	2	1.6690837499954326&001	1.7913373457351758&000	-1.8356776608089497-002
19	2	1.7410842300857295&001	3.5789188075161525-001	2.5173442475400113&000
19	2	2.0316720611636871&001	2.9693958328410571&000	-4.8365180706403895&000
19	2	2.1634875387190644&001	2.7382906984360988&000	-1.9283174981772820&000
19	2	2.3082291527704359&001	1.7528322534737275&000	-1.0066420471097187&000
19	2	2.3903418131483480&001	1.4471373865641128&000	1.7153393694944488&000
19	3	3.5223377716927161&000	1.0749916191622703&000	-1.3061662897874438&000
19	3	5.2109868212391312&000	1.4902990598271777&000	9.5799751138921116-001
19	3	7.9123768084737213&000	2.2360704184547426&000	-7.2041751350799442-001
19	3	9.0504466423712858&000	1.5585997567384744&000	2.3464311569040450&000
19	3	1.1652112305602106&001	4.9227165205269141&000	-1.7948130422813354&000
19	3	1.3651673001434042&001	2.0420618304032379&000	-2.3259108989926880&000
19	3	1.5034164677704515&001	1.8575300992714610&000	-6.0368274757419147-001
19	3	1.6139476030544795&001	1.6653606620603796&000	1.3977863667652663&000
19	3	1.7515836839225565&001	1.9129075088554394&000	4.1175500228909136&000
19	3	2.0098493415482387&001	2.8846394825470567&000	-3.9443513640076681&000
19	3	2.1233633074286513&001	3.5451455460371498&000	-5.1609193619439632-001
19	3	2.2719775693147161&001	3.8501781832637710&000	-4.6853055239973908-001
19	3	2.4336752950447823&001	2.6601244567666738&000	-1.2112981773110594&000

TABLE VI. ZEROS OF L-SERIES

K	A	ZERO	RE DERIV	IM DERIV
19	4	-3.627151410265863960000	2.554469272846491460000	-5.915380917166145O-001
19	4	6.144738739176663860000	2.091425491058582560000	-1.003177087422731160000
19	4	7.963838721052741260000	1.592552319061033060000	-1.368892768451957O-001
19	4	8.919819492265253600000	1.436408349356353-001	2.277479206731821560000
19	4	1.221923325840852160001	2.026602991204813060000	-3.605226635937969660000
19	4	1.352356622082939760001	2.663465899932834160000	-8.373650599839347 2-001
19	4	1.515475284576543360001	2.261837704261857860000	-5.459185138612502 1-001
19	4	1.623174168174972360001	1.940351844412600360000	2.106713977944214560000
19	4	1.839960854982453560001	3.300026646424260O6000	-1.098953250692781160000
19	4	1.936470707827198C60001	2.506761395152736660000	2.961901354861256260000
19	4	2.145392796599038960001	6.299828768849238960000	-1.920326412825386260000
19	4	2.340258715286654660001	8.301890902422172O-001	-2.607230735074513260000
19	4	2.426218787655207560001	2.046532618814744576000	8.315388364C922389-002
19	5	1.855635900802261430-002	5.534621024159914O-001	3.393924928255149 -001
19	5	4.624084659866367960000	2.109866474150458960000	-1.754302059519795360000
19	5	6.801991311416967560000	1.033179699620072360000	-7.800594917280283 -001
19	5	7.638132353866393760000	5.376656484382213 -001	1.409496869756539760000
19	5	1.019493963410200560001	4.576423836092838360000	6.514482491182569 -001
19	5	1.220896579445814460001	4.249652706957574260000	-1.142640438600453460000
19	5	1.389960291282813360001	3.855923C796572926O000	-8.929377532918997 -001
19	5	1.575307297528780760001	2.268599572144401760000	-1.583693279318693360000
19	5	1.694315331881772560001	2.335107117139692260000	5.011196329896944 -001
19	5	1.836053081531241C60001	2.626346886875698160000	1.554860312395043460000
19	5	1.961454129002855860001	2.247636707652589060000	4.848677156992752460000
19	5	2.257185973340E9C360001	-7.600302126960771 -001	-2.509135168536129360000
19	5	2.325949236454999360001	1.692187140007331160000	-3.088160297990382 -001
19	5	2.437305466914140460001	2.291852537955304160000	1.501093921770883260000
19	6	4.408834450957595660000	2.176934832798542360000	-1.508179417538604160000
19	6	6.297654896424681360000	2.313060355663770260000	-2.026777294671767 -001
19	6	8.187250542454015460000	2.652302797125268560000	3.778618577854317 -001
19	6	1.010796142252987860001	2.779513056301886660000	2.702608984247111 -001
19	6	1.139632539976407660001	2.180936232251033660000	3.369219835732668160000
19	6	1.457003800972870C860001	-2.404422549442385 -001	-2.156516684436644460000
19	6	1.525502838825822560001	1.676678783631815260000	2.626227416240557 -001
19	6	1.697134418412441660001	2.337418238142884660000	-8.761451821535483 -002
19	6	1.804739280159696860001	1.696358436681195 0000	2.468471271646089060000
19	6	1.986600550830137060001	5.642229270286869160000	2.582367983697614460000
19	6	2.200336127785626960001	2.908919045017393860000	-3.388844622766158660000
19	6	2.308483532794826560001	3.567464249148478260000	-9.913145222596821 -003
19	6	2.457786949789481660001	4.302977074645482460000	-2.338112908177316 -001

TABLE VI. ZEROS OF L-SERIES

K	A	ZERO	RE DERIV	IM DERIV
19	7	2.8096442001322564E000	2.3251C86S1C5816296DC0	7.913984277740635E-001
19	7	6.0094594014709937E000	2.3967623710440686E000	-1.7626969677498837E000
19	7	7.9020031834157377E000	1.9022676939886551 1E000	-7.406659067550947 4-001
19	7	9.3429682E2C3185E6E000	1.8584552595941E42E000	8.3476846801792582-001
19	7	1.0750584C87254323E001	1.4775932736401916E000	3.4104872243892973E000
19	7	1.3751392698154822E001	1.8457085219252028E000	-3.7363493532713535E000
19	7	1.5093582638731825E001	1.9785SC6521593318E000	-1.0904246690243829E000
19	7	1.6129506767839594E001	2.1572776879808748E000	1.6116752935711746E000
19	7	1.8462916614166614E001	1.2676445S542745466000	-1.3928730954177000E000
19	7	1.9042380304046326E001	8.0041352758068151-001	1.7383858156621956E000
19	7	2.0942295198247801E001	5.3313910839294731E000	2.1585299591563546E000
19	7	2.2679798736 1C8111E001	6.705C493251213231E000	-7.5803480159327707-001
19	7	2.4695283153067234E001	9.6646998284858323-001	-3.1736159092255085000
19	8	2.1381806344C62C2EE000	1.1227396E843E943 3E000	1.3041157988464274E000
19	8	6.3847157065322C2E000	4.5633369948602949-001	-1.8062109920838995E000
19	8	7.3563S597EE4667E4E001	1.4919032002676794E000	5.3641464550326798-001
19	8	9.3777937365665315E000	2.6364C395583C2126E000	6.0173170203562906-001
19	8	1.0971758619635417E001	3.0455482159046 7C8E000	2.2500245355571319E000
19	8	1.3240677 7CC16S541E001	4.3740310644474131E000	-1.3746491377349949E000
19	8	1.4732841201251666E001	4.4597856259047456E000	1.9534271763360238-001
19	8	1.6888013117718583E001	1.5973056985142335E000	-2.3861191680170031E000
19	8	1.8014930C422549428001	1.9403140439264829E000	-1.4344294798814041-001
19	8	1.9145690653656579E001	1.7477705395674C5E000	1.7647331136675714E000
19	8	2.0534431121788145E001	2.4212755865958C45E000	4.6316326617787819E000
19	8	2.2973483731743350E001	3.8755E75224C46C8E000	-4.6359258481323495E000
19	8	2.4327744476555450E001	2.9661093637823260E000	-2.1332964029911278E000
19	9	1.5160837531ECC535E000	9.7756E8536 71E32121-001	9.4817897474832177-001
19	9	5.4766141708670591E000	2.4092340514933773E000	-1.7653529952518357E000
19	9	7.1606708245CC7C75E000	2.7192561866456220E000	7.7740225372216302-002
19	9	9.3833263276732467E000	2.2952C754745 18C70E000	-9.8022010822361075-001
19	9	1.0785810625337691E001	2.1625229052678180E000	7.7051626776228532-001
19	9	1.2176226C8334556E001	2.0236544653C7C326E000	3.2833305463990464E000
19	9	1.4695812206650674E001	5.3152980726347SE56E00	-2.9364844742091574E000
19	9	1.6863613131C13534E001	2.6854230187304136 002	-1.4072160360409676E000
19	9	1.7415448573335CC5E001	1.0403011E6EC6C138E000	5.6919405853584983-001
19	9	1.8760555921276365E001	1.8041786483486C65E000	2.5285210111058171E000
19	9	2.0801408834151412E001	4.5584822794465539E000	-3.7303042292177C8-001
19	9	2.2011813252776533E001	4.4233S144BC8354C5E00C	2.6288741112936125E000
19	9	2.4129364658251340E001	3.0551150357514784E000	-3.2503841SC6394237E000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
19	10	2.8745471243761237&CCC	1.8&&&1CC&178C3253&CCC	9.1613110261411793-001
19	10	5.6564522975662C68&CCC	3.8639792506552595&000	2.5502722374144176-001
19	10	8.5689894461612326&CCC	5.8CE&&25&2466582&5-001	-1.74480C45132080656000
19	10	9.5275124706334125&CCC	1.38&7618&108C3512&000	4.4012589749926196-001
19	10	1.10C7C5C894&1C777&CC1	1.7062035&23973C14&000	2.1842496075787777&000
19	10	1.3021447131154&43&CC1	5.C961C51549656716&000	2.3512866964874055&000
19	10	1.5499265?C8953159&CC1	1.6484887550180547&000	-3.1420103264460794&000
19	10	1.6564528&56&&2582&CC1	2.69C5400255860396&000	-3.5133489991717769-002
19	10	1.80721579C17C8848&CC1	3.44&7C8C&372C11C3&0C0	5.0386C45871261618-001
19	10	1.9840182815156lC9&CC1	3.0193723595610907&000	-9.0774540441754120-001
19	10	2.1093&9C41&673&54&CC1	2.46&73882&C2782&9&000	6.6790009991601498-001
19	10	2.2042430974C1595&&CC1	5.9768893151529525-001	4.0416148590798168&000
19	10	2.484C5515479C6C55&CC1	7.9354588C30555860-001	-4.4670398679955440&000
19	11	1.4600880579&&75&6&CC0	1.1301462876164765&C00	6.3345499988245035-002
19	11	3.7658954CC7479C88&000	1.6654432790750704&000	1.9282211394496975&000
19	11	7.3833072143185&54&CCC	1.3837&8C&4C6548186000	-2.5505506279702587&000
19	11	8.8273821816106853&CCC	1.8168812602039&116000	-4.4567637626314886-001
19	11	1.C1311247&&42&2&16CC1	1.8824654631536167&000	1.6324907753164291&000
19	11	1.2493432718C&&2598CC1	2.7822175787C888216000	-9.0331702354382035-001
19	11	1.3511263136421336&CC1	1.9499840336476884&000	2.7819334729049814&000
19	11	1.5985542601&&9445&CC1	4.2839865846484318&000	-2.9078453048925066&000
19	11	1.7511353504250385&CC1	3.3381239322764126C00	-1.6038&C31270605136000
19	11	1.9149072210397073&CC1	1.9744614379447989&000	-1.3327283444706363&000
19	11	2.C3C103836C&C315&&CC1	1.6564C2791503926&&CC0	3.2392016164603879-001
19	11	2.1090854991441845&001	-3.4677273941789543-001	2.6433936300663278&000
19	11	2.3581148832222556&CC1	6.97CC9883633247628000	-2.9627691295132472&000
19	12	1.5732C80845521767&CCC	8.689C8953264258&2-C01	5.5115387675465281-001
19	12	4.6396456305455342&CC0	3.0877148297780166&000	1.1359343319823945&000
19	12	7.498422600723132&&CCC	2.5C175365248525106&000	-1.9014619004183544&000
19	12	8.9722860857229436&00C	2.9100703940981692&000	3.6241011775509932-001
19	12	1.1428432955263788&CC1	8.6692648412820593-001	-1.1922098744757923&000
19	12	1.2090800761532047&CC1	7.578145240059417&-001	1.2253619274508668&000
19	12	1.3790808299851813&C01	2.4440208077946353&000	4.1868976C111072088000
19	12	1.6673473290744574&CC1	8.9656362181433476-001	-3.7123162526483781&000
19	12	1.7714719249995466&001	2.4853044250831135&000	-6.3228874486344491-001
19	12	1.9210047350478449&001	2.4866171200128502&000	-2.8613018948332164-001
19	12	2.0356056510579721&CC1	2.3594197CC5334499&000	1.9186676240203303&000
19	12	2.2027878512283233&0C1	4.6536236503032270&000	1.6835825697829535&000
19	12	2.3738671C669746726CC1	4.8075263284849560&000	-6.5444325649747729-001
19	12	2.4938901948185709&CC1	5.C5959033152267136000	2.3005747471085916&000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
19	13	3.4944712547392242E000	1.8925136804535294E000	4.1401299826507170-002
19	13	5.3308864285752CCECCO	2.C8C74C2783526587E000	2.1131671765421127E000
19	13	8.4757649520212046E000	2.4985202932265647E0CO	-2.7212E7409C282760E000
19	13	1.0169545590702874ECC1	1.9756489353916445E000	-1.1117728703669775E000
19	13	1.1643487187361453ECC1	1.7665627589701366E000	9.4535948077379482-002
19	13	1.2666858490210416E001	5.7955451709290057+-001	2.5949004102522464E000
19	13	1.5315438627328522ECC1	4.6895996434439126E000	-2.3499532109613115E000
19	13	1.6772092040073936ECC1	4.2297695C490178186000	-6.3859074241532109-001
19	13	1.8499053195706C88ECC1	3.2966060965088C57E000	-1.5576396125850666E000
19	13	1.9858103758563C81ECC1	3.2268151170957411E000	-1.8964935066113551-001
19	13	2.1463659889232753ECC1	2.5190886195887498E000	-6.0622574927024329-001
19	13	2.2449262516754855ECC1	1.8981509087818295E000	2.1087367298542688E000
19	13	2.3854216770162871ECC1	3.3841725E946115C9E000	5.2140974331638040E000
19	14	3.C9878C796516243CECCC	1.1828849967565757E000	-3.88614C2831045607-001
19	14	4.4032E503236599746CCO	5.7791300761906187-001	1.7135051134967393E000
19	14	7.8704624516134714ECCC	3.2817122512265169ECOO	-2.4989IC1377855683E000
19	14	9.7847055719442990ECCO	1.8474576669823626E000	-1.4056598822074430E000
19	14	1.0977320814446541ECC1	2.141C92CE957615336000	9.9894341273791695-001
19	14	1.2961483732421BC2ECC1	3.2647589334769578E000	6.2244037645890386-002
19	14	1.4459297408973326ECC1	3.2755178114373824E000	1.2769761339617405E000
19	14	1.5971832704106773ECC1	4.3688EC39629E2338ECOO	3.1065338632177238E000
19	14	1.8423184072294892ECO1	2.1343659107091929E000	-4.01438CEE2262C625E000
19	14	1.9956338911879226ECC1	6.7350171269134264-001	-1.1830656600304552E000
19	14	2.0500211947805143ECC1	7.9215315E77016556-C01	1.1552535199139165E000
19	14	2.2379925201404823ECC1	3.2282985073006C84E000	6.2345907352847534-001
19	14	2.3389031813ECCC556CC1	1.69C1121595557310160000	4.2792367358773848E000
19	15	1.8448202825345782EECCC	1.1695011321597476E000	-3.9979799939081543-001
19	15	3.8572146514434287ECCC	1.5343486588613806E000	1.2178308598430858E000
19	15	6.5905717449119BC2EOOC	2.6675936E111734216COC	2.1628655420211191-001
19	15	8.6615833445924035ECCC	4.1141686211446751E000	-1.8571275830884300-001
19	15	1.1014870833329291ECC1	1.6C955C8733331302EC00	-1.9967074105073174E000
19	15	1.2150394771448908EEC01	2.1442914146712409E000	5.0354475716311143-001
19	15	1.4028747937224515ECC1	1.9738548C08837C92E000	-1.6096229750657293-001
19	15	1.4781881464248477ECC1	-1.8095C1179E1C3554-CC1	2.6441SC30281939156000
19	15	1.7934165061231363ECC1	8.4760299687400465-001	-4.07178C3806349118E0CC
19	15	1.9041512272105272ECC1	2.1363C65E52267775ECCO	-1.0362301739608571E000
19	15	2.0174786712617324ECC1	2.5385994C45922C545ECCC	1.0047292362979230E000
19	15	2.1804621C4465686TECC1	3.7680083513665C66E000	5.1676537744640909-001
19	15	2.3371012862547E8CECC1	3.3591634812912655E000	-1.0110485534009857-001
19	15	2.4389523571406487ECC1	2.5293E49163417CC6E000	3.4CC815C95CE112216000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
19	16	2.733275C435C4C437&CCC	1.747214517207523786000	3.335063172336615D-002
19	16	5.363651804185472786CCC	1.96C351667865555486000	-2.084720101354221D-001
19	16	6.685133907682E5548CCC	1.228295657677285286000	2.499351623683133686000
19	16	1.CC667236543C3952&CC1	1.242435176631725&C00	-3.105502598282662&6000
19	16	1.141801885446957486C1	1.8C55824850625651&0CC	-7.629321475984906D-001
19	16	1.265578C57&6&45496CC1	1.959673002343839786000	1.097337480255668&6000
19	16	1.4345293C625711117&CC1	3.2C3277637446&1C28C00	1.862417551878576286000
19	16	1.606175781C774846&CC1	5.0679692612053&C28C00	2.044526703658C3796000
19	16	1.834795253217231186C1	1.741867074468066&6000	-2.661412595436835&6000
19	16	1.913339674365395&6CC1	2.5541&C1CC&841572&000	1.64439C76259034606000
19	16	2.156008653367&C12&CC1	5.861151254584&E45-C01	-2.039535&9576912266C00
19	16	2.227726142&C&4952&CC1	1.547617550347104286000	5.593164377985403D-001
19	16	2.350089571596&5451&CC1	1.875&&7&&1172C25C&CCC	2.3498&281653241266000
19	16	2.495713C514329295&CC1	3.821&0C5852&C6094&000	4.929015841980589286000
19	17	3.0C729C64822357636CCC	1.395348&3&733233C36CCC	-8.474386&2C12851647-001
19	17	4.844020882&5C8314&CC0	1.634656912713452086000	5.687273209871212D-001
19	17	6.6C2&&56&5C54C574&CCC	1.82545C7446722255&C00	2.690715825259877186000
19	17	1.0021916437768433&CC1	3.9C3585951161974&-C01	-2.C595072812828807&000
19	17	1.C749520184356177&CC1	1.638487939163027286000	9.043342991309845D-001
19	17	1.296375432162&5&C4&CC1	2.99258C81976C6423&000	-9.751548650082050D-001
19	17	1.446497014729291786C1	2.586C827515425721&000	8.047583224583436D-C02
19	17	1.5707C037828C4773&CC1	2.332C&948104653346000	2.403605673555914386000
19	17	1.7494760C17C&175&&CC1	5.742&2545&1C567266000	2.719207464527825&6000
19	17	1.988767911326715&&CC1	1.293390005149093466000	-3.7568834756020&56&6000
19	17	2.103384&&05&781CC&CC1	2.1&419183898410136000	-1.080110363050733&&6000
19	17	2.23667C8763152C67&CC1	1.7421888C77737186&CCC	-1.946782675931344D-001
19	17	2.312152452133C731&CC1	3.277087771180&793-C01	2.373526817689134186000
20	3	3.5310956&39C976711&CCC	2.02&8C795&377&1236CC0	-5.6566615322463683-001
20	3	5.3709221726934881&CCC	2.398372401496639786000	1.128565885060987386000
20	3	7.8&2C2&717552244&CCC	3.55&333&167355C746C00	-7.424468484357673D-001
20	3	9.8856902&151615146CCC	2.26021&292665316C&000	-1.069030659&42464C&000
20	3	1.1C01C1457711C963&CC1	2.C833508545177467&000	1.926595239588249386000
20	3	1.327477102305&&59&CC1	4.296422013558423386000	-1.163459984259679786000
20	3	1.496782345013431286C1	3.129289293823035&6000	-1.119868628202671586000
20	3	1.630388879511554286C1	3.0722109442425056&000	5.843657195675061D-001
20	3	1.769750&C&991&&5&&CC1	3.813950754416481&6000	2.332C&911948756846000
20	3	1.988215321457659&&CC1	2.702&875336&C5516&000	-2.661&&45292&C6265&000
20	3	2.0537972C&52&3471&CC1	3.C87526969561337&6000	5.000&2393195351514-001
20	3	2.243454019131547&&CC1	3.76&&499717254C126000	5.069659010&221534-001
20	3	2.372434386&4&C5687&CC1	4.356334198750587186000	2.404955962818638&6000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
20	5	2.358934994C8E656C8CCC	1.787259573528135416000	-5.1754596666937735-002
20	5	4.675507749842079686C00	2.669927279464448966000	9.4622292449058895-001
20	5	7.429109774584178586CCC	2.319156908742570C6000	-1.509769334669832666000
20	5	8.804527424544950C2ECCC	2.488EC11C278228CC6C00	8.5106405115472001-001
20	5	1.066330073495362260C1	3.843798375007580266000	1.113177C9C89605176000
20	5	1.280240081CE6662586CC1	2.441476281230C7836000	-1.648607814594806266000
20	5	1.433616900C750C5156CC1	2.719598112357756266000	-5.2578285C26500735-001
20	5	1.549353496124302786CC1	2.596140570085324866000	2.256926149692644066000
20	5	1.760073479127C6346CC1	4.535E71643846C4316000	-1.496647334376321466000
20	5	1.912025280279851266C1	3.575237456561723266000	-9.801846440249183S-001
20	5	2.C5421C87C4454735E6C1	3.065975742552585866000	-2.222237851171939l-001
20	5	2.162220640C6481371ECC1	2.8C6536C82E1330396C6000	2.877145551859737666000
20	5	2.378616710015895566CC1	3.449548473505151346000	-2.855537C8814574416000
20	5	2.490661C2955354136CC1	3.567892884715495766000	4.677975813086827S-002
20	7	2.439704710751657ECC0	1.747725849C9135866C000	7.3342E28E795S5886-0C1
20	7	5.554135366741861866CC	2.647312756694764866000	-1.042765572030069866000
20	7	7.3526217C8EC7S442ECCC	2.625C272518184846EE6CC0	1.37368E8136550020-001
20	7	9.036030348328647C8CCC	3.214464726537575866000	1.5561C122893447976000
20	7	1.1459596CC5614C866CC1	2.7174435E14C299216000	-1.969727781187013666000
20	7	1.276351941187002366C1	2.794789368235775960C0	3.7651C43169346286-001
20	7	1.428943C822339522ECC1	3.535135465757132266000	1.5174386819772055E000
20	7	1.615266427433562366C1	4.8E432828E71416532E000	-3.2819498492915172-001
20	7	1.810099096196119566C1	1.728373537508782866000	-1.853446985725104266000
20	7	1.888172CC55C3748S6CC1	1.859941772833544966000	1.666400717347820566000
20	7	2.064653162256021766C1	4.925C676286EC871646000	1.090322328231953766000
20	7	2.238526032224391266C1	4.610556521482276886000	-1.500921225961873066000
20	7	2.389570024257888566C1	3.037527724550112866000	-1.356497213681761766000
20	7	2.496338092959904466C1	3.027374559366EC376C00	1.271264760260588066000
21	3	2.713661C5453C6215E6CC	1.535732585076321766000	-2.1559332998390799-001
21	3	4.616766976122208166CC	1.856676688188562C66000	1.359993484296717266000
21	3	6.963051664257C832ECCC	4.164284565720C9836000	1.323424234389580866000
21	3	5.533060566676CCC76CCC	2.5C8415E3C11851976000	-2.634924567808071866000
21	3	1.107663219778136CC6C1	2.155227456E8255E776CC0	-8.996585297487453?-001
21	3	1.234049354654957666C1	2.325842998627509766000	1.064897665836697166000
21	3	1.416776444266C3C976CC1	3.113772172E5422C166000	3.387076823828867?-001
21	3	1.531430467617495C66CC1	2.266641108651503266000	3.6543CE5C982274686000
21	3	1.780999355321264C6CC1	3.536C776957919315C000	-4.021861553988252666000
21	3	1.921531186CC32C175ECC1	2.521215565807565CC6000	-1.748187446871565986000

TABLE VI. ZEROS OF I-SERIES

K	N	ZERO	RE DERIV	IM DERIV
21	3	2.0332033245743268E001	2.9157385110065073E000	5.8964861518969016-001
21	3	2.2124205580624671E001	2.0284819337147910E000	-1.0606656203134677E000
21	3	2.2661507945443077E001	1.1905922832915009E000	2.1571144804498286E000
21	3	2.4468572757102559E001	4.9437861464475170E000	4.1403745702807982E000
21	5	3.3812379813045183E000	2.3021568076274118E000	-1.0279147209287640E000
21	5	5.7008652167753359E000	1.7818455454796844E000	-7.9023505227691097-001
21	5	7.0805047625760218E000	1.7790862698677451E000	1.2549509272564756E000
21	5	9.0272959620587943E000	3.5176327187498608E000	2.1749196126734631E000
21	5	1.1250093648102490E001	5.1949117712317455E000	-1.1476172722949018E000
21	5	1.3359792070585712E001	1.5320666275981928E000	-2.3586769571932807E000
21	5	1.4418334235736917E001	2.2595589123625535E000	2.6261011300297789-001
21	5	1.5920538698552721E001	2.9012710601210762E000	9.2663434853412694-001
21	5	1.7453137841415817E001	3.4618100979766931E000	1.3412419806026453E000
21	5	1.8780761496526625E001	3.9218872116279188E000	3.9384867378045468E000
21	5	2.1176021983805444E001	2.1173565328709609E000	-4.1771705160926078E000
21	5	2.2291733747416153E001	2.8580184637219210E000	-1.0453695686745337E000
21	5	2.3542918426366384E001	3.2285722414941324E000	2.4880609318949141-001
21	7	2.3151870643031412E000	1.4187753833051514E000	1.3034129413192003E000
21	7	5.7803682735700495E000	2.9479518007220875E000	-1.8750178725178431E000
21	7	7.6546324851636847E000	2.5464621381835138E000	-8.9139025700046883-001
21	7	9.4646732073701131E000	2.0302874999478644E000	-4.7127851215897142-001
21	7	1.0610544854674604E001	1.5891914502861290E000	2.0081650220115409E000
21	7	1.2628728375934724E001	4.9673658960709257E000	1.8239854448215055E000
21	7	1.4670821575144728E001	4.9560306264749290E000	-2.1310420750598059E000
21	7	1.6465154717935669E001	2.1941871448529965E000	-2.3017707617969723E000
21	7	1.7663898928548905E001	2.3757673467362037E000	-2.1720096280135162-001
21	7	1.8881891826340693E001	2.5595694071893302E000	1.5568046879078999E000
21	7	2.0539837853202217E001	3.8219933524492971E000	8.4151585032445782-001
21	7	2.1712361174362695E001	3.4951226068086661E000	4.0814863631968018E000
21	7	2.3951752676810717E001	3.9412369576395782E000	-4.4766328722960595E000
21	9	1.6120193910544471E000	1.3853360393320051E000	-2.8156195371472093-001
21	9	4.1320220463240865E000	1.9695065750354801E000	2.7639406778191976-001
21	9	5.8751261712862967E000	2.1467995149529811E000	2.3567676897394371E000
21	9	8.8449363843428606E000	2.7874660746486725E000	-2.7028987517829459E000
21	9	1.0329513358619588E001	2.9265918784503472E000	-5.5825568865144404-001
21	9	1.2177749735394196E001	2.1167369915165216E000	-9.2198879956490679-001
21	9	1.3284026327106051E001	1.9201256387324561E000	1.4794078610891089E000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
21	9	1.4872636453152523E001	2.4316567742766700E000	3.1306782826515834E000
21	9	1.6956976157574420E001	5.8554587814290422E000	-1.5539002104598404E000
21	9	1.8628519715440E71E001	3.9754857858879842E000	-2.3393513723404754E000
21	9	2.0268990641182220E001	1.6366237526725467960C0	-1.7332462728607176E000
21	9	2.1124022844212676E001	1.9760022487861887E000	1.1221625815969457E000
21	9	2.2666672205770742E001	3.4593549290655798E000	1.2512622041030741E000
21	9	2.4007182817457235E001	4.0201434887140565E000	2.5694743674758912E000
21	11	3.9367913195573236E000	1.7301841184857755E000	-1.4157446670258188E000
21	11	5.6322724151859550E000	2.0771623882100173E000	2.3348588823711264-001
21	11	7.6334752017547236E000	2.4673181400614715E000	3.9254902395555797-001
21	11	9.0563388860332442E000	2.2133124901358467E000	2.9640434470361295E000
21	11	1.1837647972C51642E001	2.7870788457837157E000	-3.2496137285888090E000
21	11	1.3192412518183080E001	3.0113790697654937E000	-7.6515604194685211-001
21	11	1.4811913119155300E001	2.7935088187135155E000	-6.2708860018314267-001
21	11	1.6258332678566155E001	2.4171558721171336E000	1.5659122144380427-001
21	11	1.7292485754105020E001	1.4570404757857095E000	3.0443011124329716E000
21	11	1.9378901146721630E001	6.6766C2513653233B6000	-1.7779451932764763-001
21	11	2.1135487947194246E001	4.2585728970315910E000	-2.8319928214816884E000
21	11	2.2538454285313227E001	3.3736613410502334E000	-1.6487412737570723E000
21	11	2.4056654195714254E001	1.6657896676820125000	-1.1858679609443131E000
21	11	2.4762093333290446E001	1.2589269259560182E000	1.8387218813280242E000
23	1	2.3541116878835242E000	2.2646778245050565E000	-2.4669260217842719-001
23	1	5.1201071545619040E000	1.8668261795273465E000	-1.1374166943158355E000
23	1	6.6621229250666671E000	1.8160626885574367E000	3.6393588355809202-001
23	1	7.9666968547632455E000	1.0316531730460177E000	2.8937054028492738E000
23	1	1.0803334505573203E001	4.4461585761845385E000	-3.3115190693203919E000
23	1	1.2714436799713680E001	1.0404851586981166E000	-2.0056115291021233E000
23	1	1.3643570951060140E001	1.7784950572280917E000	4.5184297843346804-001
23	1	1.4926729745555355E001	2.1633708763550990E000	2.4731391880753403E000
23	1	1.7218062566277537E001	1.9751317755665960E000	-1.5758028666957900E000
23	1	1.7817180234258853E001	9.2552400952114992-001	2.5436100131033458E000
23	1	2.0100925562777436001	5.6111946514753815E000	-2.9011796657009455E000
23	1	2.1588262943451303E001	3.7495585455395106E000	-2.3453171117955510E000
23	1	2.3052113632461652E001	2.2250632704456875E000	-1.6563499021076529E000
23	1	2.4104330042350770E001	2.2780566001606064E000	3.5923449068740387-001
23	2	4.1075763734547231E001	1.6784372896645337E000	-1.7997744114195724E000
23	2	5.5362585CC6833685165000	1.4271834332352059E000	-4.9537168084726898-001

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
23	2	6.9944568772C755C6CCCC	F.405748364366570B-C01	1.8031740881298365E000
23	2	9.5184731517195531ECCC	4.6533995375E9C6856ECCO	-9.2068157390550529-002
23	2	1.1240424457534107E0C1	4.7453367754047974E000	-4.0887729837653800-002
23	2	1.3252999991714CEE8CC1	2.7246282E6621315E6000	-2.2597187879394556E000
23	2	1.4468191536307210E0C1	3.1318128CE94635806000	1.9976615775242695-001
23	2	1.6382351797378C346CC1	1.573C3772062133C2E000	-1.2068533595991190E000
23	2	1.7120434266531612E0C1	1.1124369185E256846E000	1.6171623C429535196000
23	2	1.8493474549756556C6CC1	1.7653611555878CC66E000	5.18C3295794910575E000
23	2	2.1224731C2254777C6CC1	4.6658763495174623-001	-4.5358756514265169E000
23	2	2.2235002485632681E0C1	2.192846C252213C47E0C0	-1.3C6231534546592766000
23	2	2.3335879CC7361755ECC1	2.5944458267101431E000	4.4391883723022493-001
23	2	2.4556724443523443E0C1	3.2939386C5115934C6000	1.7590663151206629E000
23	3	3.6C81C71C722134436CCC	2.5194765952168387E00C	-1.2271573245278162E000
23	3	5.6785203171E44186ECCC	2.4136662462783851E000	-5.7436232598577000-001
23	3	7.4101066866574821ECCC	2.6782354936523396E00C	4.3614500C99308866-001
23	3	9.4598791657441446C6CC	2.1086791789053057E000	-4.4768293237664801-001
23	3	1.0298272457426571E0C1	3.552556E51725126-C01	2.7612992567102603E000
23	3	1.3432107675C705696CC1	9.3536682101614181-001	-3.8751334877885728E000
23	3	1.4590522827E235C0E0C1	2.0617825787346284E000	-1.0542987798798732E000
23	3	1.5833174984461C28ECC1	2.2862143992720231E00C	3.963278C1C8625794-001
23	3	1.7132093475415417ECC1	2.5595264588853270E000	1.9489CC3740525616E000
23	3	1.8675603945C3428ECC1	4.4474843189340578E0C0	2.8178021439859504E000
23	3	2.0466175467275176ECC1	6.4455828696C255876000	-7.6160659595747453-001
23	3	2.2287419386138C63ECC1	1.8343144683298068E000	-2.5429971086308122E000
23	3	2.2989069737378635ECC1	2.5243154E955C3C33E000	1.9032942333884554E000
23	4	2.6950636E5456561E0C0	2.3328775161294129E000	1.2492583131366814E000
23	4	6.1327170663825226ECCC	1.1029853736C849706000	-2.1623362588452919E000
23	4	7.3867360917391463ECCC	1.8514142541467189E000	5.9526831948612725-002
23	4	9.0236C7357C21C2716CCC	2.1953787357754839E000	9.0493028743569799-001
23	4	1.0403815911663341ECC1	1.9259386CE73812557E000	3.5165926995781634E000
23	4	1.3126318388262656ECC1	2.9162553869855794E000	-3.7359691552715803E000
23	4	1.4487594727372744ECC1	2.6236158912097C734E000	-1.2223084909978390E000
23	4	1.5657304C476C893ECC1	3.043526C2354836568E000	1.1466633187671766E000
23	4	1.7601238378C962796CC1	2.8855258848006480E000	-1.5428C66605343387E000
23	4	1.8860793235C312746CC1	2.1068283744500312E000	1.1096336226405428-002
23	4	1.9631495117607664ECC1	1.8922932735709316-001	2.9755972448402046E000
23	4	2.1899117064346873ECC1	7.6654572426019084E000	-2.0463048135461219E000
23	4	2.3618231747749362ECC1	2.9224E4C1812366376E000	-3.8415680723175520E000
23	4	2.4959998697416947ECC1	1.78826C4720658799E000	-1.8026688810290115E000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
23	5	2.1034098104138515&CCC	1.312657C887750176&C00	1.5432476C625312046000
23	5	6.1337997810C42018&COC	3.791620596446584 71-001	-1.8347604613656005&000
23	5	6.9875830911545447&CCC	1.4953540719909397&000	6.653799479593827 2-001
23	5	9.0143386166991563&CCO	3.080C5441C27842516000	4.3707659628799851-001
23	5	1.0842997390816332&CC1	3.0088649815255268&000	-2.9552306015256770-003
23	5	1.2058908424329C52&CC1	2.53&C8C1421953653&000	3.0058161308707792&000
23	5	1.4355339C81803839&0C1	5.24611C529925036 2&000	-2.4693305550702835&000
23	5	1.6031078844183C8&CC1	3.C6932092832064636000	-2.4053830940857453&000
23	5	1.7554314776131283&CC1	1.733C821440171523&000	-1.3437134349620015&000
23	5	1.8585874261213C33&CC1	1.6215036423161719&000	5.3757753811793335-001
23	5	1.9450684813776517&CC1	-1.6578888439251436-001	2.9412563319483058&000
23	5	2.1853313097358CC2&CC1	6.3435E35&4&0536278000	-2.8840487641383442&000
23	5	2.3241633422985458&CC1	4.9505491748034822&000	-1.7983641283476572&000
23	5	2.4825141253384SC5&CC1	2.3445248138263S30&C00	-2.1905805748017220&000
23	6	3.0119744257C2876S&CCC	2.3449835435432C55&000	1.0964692656466676&000
23	6	5.7801C85&06SCE774&CCC	3.6SC2647863664244&000	-9.0088581734175738-001
23	6	7.8132061C1545526S&CCC	3.144946E5015739E66C00	-1.1849453C838072298000
23	6	9.7687184201328 4C3&CCC	1.7106970116018261&000	-1.1964086862750015&000
23	6	1.0884838626555572&CC1	1.62C8C357446655226000	8.9538128263780045-001
23	6	1.2078704991296782&CC1	5.68C1255236271814-001	3.6209669553950550&000
23	6	1.5061337E83C1543S&CC1	1.23E74C61C89253506000	-3.9060711405301393&000
23	6	1.6027724641741428&CC1	3.C622C458433C6C446000	-2.3056124086587414-001
23	6	1.7875360718738428&CC1	1.4264376232483041&000	-1.3194755799053704&CC0
23	6	1.8523494636E33315&CC1	1.C6E4599482121786&000	1.7972234009151874&000
23	6	2.0581103866434685&CC1	3.9C758114C7735167&000	-8.6200CC9645322983-001
23	6	2.1698606732467S18&CC1	3.4596469306918871&000	1.8662952362472606&000
23	6	2.3C732096213E6CE5&CC1	5.28E5E77C378C4456&0C0	3.6819627316205234&000
23	7	2.4228C7245193995S5&CCC	1.4933288144255432&000	-8.2703176C53039393-002
23	7	4.C6368017955E575S5&CCC	1.2176418553822C963&000	2.2171481831585918&000
23	7	7.5765898355920986&CCC	1.3625711C7158751&6CCC	-2.91C228C4021307306000
23	7	8.8795585C13545CC3&CCC	2.1844928206280788&000	-4.6251676183067752-001
23	7	1.0489122351E61352&CC1	2.24S8E5643766225S&000	1.4118855373694273-001
23	7	1.1705498327673877&CC1	1.8524473421682360&000	2.60C602C6131723796000

TABLE V1. ZERCS CF L-SERIES

K	N	ZERO	RE CERIV	IM CERIV
23	7	1.396328593621444660C1	4.590241341215470980000	-1.175828535747364080000
23	7	1.528336328E58CE5160C1	4.625C15C5465961510000	1.251641893217064080000
23	7	1.720644616703926960C1	4.3245C3231671EC480C0C	-2.486938SC455245010000
23	7	1.889371584858S3C50C1	1.432195749397629280000	-1.862869798756643880000
23	7	1.9719614CE22C3233CCC1	1.922533971183315580000	9.7160995114968328-001
23	7	2.1326135671226976CCC1	2.599244516220560280000	3.67374C2729224459-0C1
23	7	2.2115372747764500C0C1	5.766168195975400081-002	3.709317770385097880000
23	7	2.4559845745SC2674CCC1	5.8C155C3C2544728180000	-5.283161523037986080000
23	8	2.C44762235517227460CCC	1.262EEC85E372915120000	1.5927484098396397-001
23	8	4.035281175034574C0CCC	1.641264722665445180000	2.040333333404478580000
23	8	7.178383719762419160CCC	2.768189111784775800C0	-2.285989346285943460000
23	8	8.58C8913413E48C57800C	3.211278580497353180000	3.0654941626908192-001
23	8	1.075645338483C3630CC1	2.178620C58506C4452600C0	-1.622145684885569160000
23	8	1.194789351CCE3386C0C1	2.316099556104434660000	6.704589271915948 4-001
23	8	1.352059537635145580CC1	2.8827127235194754C0000	1.228931850370381080000
23	8	1.477864637462672480CC1	2.477938378466185980C00	4.442048254615839360000
23	8	1.758711533427474660CC1	1.687534755337634 7-001	-3.759776471380282260000
23	8	1.855761841830637980CC1	2.052824598C18C9618C000	-9.12809726781774 4-001
23	8	1.977526761173816080CC1	2.264773452810101660000	3.08495C3847946C50-001
23	8	2.C872080353871727800CC1	2.075590947657826360000	2.690971230531771960000
23	8	2.284970431975458960CC1	3.971375520381483360000	-1.131154402402795460000
23	8	2.375888055 CE37226CC1	3.071034150272810260000	3.446636SC80317604C000
23	9	5.954288544195SCC2-CC1	1.195625043431956060000	-3.156911676709240 -001
23	9	3.556840265818296280C00	2.052445087705598960000	-2.0708821711537887-002
23	9	5.424949925362048E8CCC	2.425931522655013460000	1.406160315711323760000
23	9	7.523772029316228C8CC0	4.562358342155214860000	1.669200710970953860000
23	9	1.039764921993346C98CC1	1.147916713903720 -001	-2.103674590569549060000
23	9	1.122050426241422C8CC1	1.53E883620343966198000	1.44497235799630 3-001
23	9	1.261097450498563860C1	2.035578296C456663C6000	1.483148645220465360000
23	9	1.416792048252C4176C01	3.300C49478032189916000	3.025794453616650160000
23	9	1.613280970C46E50C36CC1	6.25E2S791483335436C00	-3.826564376445705 -001
23	9	1.800311216792705660C1	2.864995852461770160000	-2.863677203201239980000
23	9	1.912407163545371960C1	3.347585009483019660000	-2.6731206029943991-002
23	9	2.099192156171817580C1	3.965552537735043460000	-4.729665720192624 -002
23	9	2.232197690845275360C1	1.922334157128727160000	-1.445356C16936638560000
23	9	2.314700247498C2578CC1	1.623145815561704980000	1.293949323749932760000
23	9	2.413727446519501360C1	-1.580211033210227 -001	4.479105448570923960000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
23	10	3.461093698942710100	9.1903992845228121-001	-9.259797361C911060-001
23	10	4.537260273255612600	8.8669856600045776-001	1.1440700965979726E000
23	10	6.776039654769416700	3.3961862C65335C44E000	2.6891031494824970E000
23	10	9.614494561531617400	1.9808025054850718E000	-3.1374970702317452E000
23	10	1.096302855C572C72E01	2.27459C52C581232CE000	-7.3377627727093155-001
23	10	1.222204862691147600C1	2.5934186697571843E000	1.31966C71356976006000
23	10	1.426452315E7405726C01	2.8821536901247393E000	-1.0519984845595947E000
23	10	1.538415553822810600C1	2.5111912511917653E000	1.5604689716549678E000
23	10	1.673628140793976800001	3.2334169644108293E000	4.4614751740357849E000
23	10	1.942429341204198C6CC1	7.4875394158407524-002	-3.2841697131966528E000
23	10	2.016997259286C7C956CC1	2.4990525574817C17E000	-8.1882164374070521-003
23	10	2.185239216763676600C1	2.0408825653740793E000	-1.1536131365603695000
23	10	2.276816376270C04CC6CC1	1.7E2488C4991597286000	1.2491818990359323E000
23	10	2.385940471327194560C1	1.0469817270164C28E000	4.2048073253752219E000
23	11	2.871339848930367960CC	1.0823776881CC03306000	-1.0511143259337675E000
23	11	4.215189804229719160000	1.3607312365069163E000	1.0458380167709040E000
23	11	6.731189150719542360CC	3.1488844382565262E000	1.2340883882282351-001
23	11	8.334849030124396160CC	3.4474587542913144E000	1.9889511161749209E000
23	11	1.063387123021858660C1	4.5391076614505753E000	-1.9694652974638448E000
23	11	1.258169678E69625600C1	1.5537316416533779E000	-2.0312479962812850E000
23	11	1.360413172473855560C1	2.1523749C923967786CC0	5.399198556C544465-001
23	11	1.538556073355858160C1	1.67162C35158541586000	-2.5141731446824112-001
23	11	1.593810136734756360C1	-6.4452147309014191-001	2.0616503233673303E000
23	11	1.898593120269587460C1	1.1494046292431175E0C0	-4.4561495997832668E000
23	11	1.998776664788673680001	2.7683582754871202E000	-8.9978463356899156-001
23	11	2.120360225636256560C1	3.3964635546151621E000	6.8763428830062461-001
23	11	2.287407602346312060C1	3.1874888621632237E0CC	-1.0759259461514956E000
23	11	2.394644729526781E60C1	3.C516C7993941815660000	1.3983946912896110E000
23	12	3.180067760838514260CC	2.3406046195555E126000	-6.5519665934911241-001
23	12	5.515448961757735760CC	2.0289502023904412E000	-7.4402163625097698-001
23	12	7.014657341751075760CC	1.9653?C2142C6B6C46C00	7.2419220910730014-001
23	12	8.451117852174963260CC	1.3755889377859557E000	3.4604767156856401E000
23	12	1.183174132560553160C1	-3.8071931455772562-001	-1.9207440953624767E000
23	12	1.243425355313810160C1	1.5295342106825E426C00	3.6201374263581822-001
23	12	1.407430606E42985580C1	2.9536417905795567E000	5.7241480379326347-001
23	12	1.565507743525252C6CC1	3.3205180211273128E000	5.8600600485104511-001
23	12	1.697064077543365260C1	3.5310134718543529E000	2.5556933159823179E000
23	12	1.884448191645396376CC1	5.3013369702351667E000	-8.0303710906124224-001

TABLE V1. ZERCS CF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
23	12	2.016C290868E37845ECC1	5.6232099991279EE7ECCC	1.0431144786924801E000
23	12	2.2281227628E68897ECC1	8.6057531644188796-001	-2.9865062743272914E000
23	12	2.3144521534450C736CC1	2.2467123777771163CE000	-1.2753668506282292-001
23	12	2.4563821855943560ECC1	1.8402256980116600E000	-1.6011761939374330-001
23	13	4.3122314147166655ECCC	5.1177526EEC6333524-CC1	-1.7122555396606076000
23	13	5.5642430560997998E000	1.6135192E360657416000	4.1342821647945340-001
23	13	7.3827678138516EEC5CCC	2.4076181493136084E000	1.3728829530717469E000
23	13	9.2088867172173223ECCC	3.929E84741276766976000	2.3857576903655436000
23	13	1.1739384771525188ECC1	1.9851874117305959E0C0	-2.623145C41C1561786C00
23	13	1.2668884426248E51ECC1	2.8122345765658810E000	1.3692642104927522E000
23	13	1.5046403022225572ECC1	1.1913780870538523ECOC	-2.182823C9136432106000
23	13	1.5953324775945233E001	1.8455536C793366216000	4.5021400928159606-001
23	13	1.7158867262705295EECC1	2.0277315748300115E0CC	2.1918345454558360E000
23	13	1.8641062309865635ECC1	3.5342291333446473E00C	4.9361C4839344141716000
23	13	2.1063977242264053ECC1	2.5558390718324141E000	-5.0260133918728147E000
23	13	2.2312733657EC1890ECC1	2.4021321085C43543E000	-1.9821440684629984E000
23	13	2.3547230926070886EC01	2.0601397451740145E000	-5.98271C596C745216-001
23	13	2.4395936537SC3C15ECC1	1.5515637342883C86E000	2.2179652402813476E000
23	14	4.8216813469EC0682ECCC	1.6400677797278288E000	-1.6300078883063417E00C
23	14	5.5272585249417258E0CC	2.2017817001615581E000	7.9160746732634215-001
23	14	8.2345574497312755EECC	8.837C765E53671366-001	-1.04867C9942528063E000
23	14	8.8285602298210C5E0CC0	4.5338165292278681-001	1.3606911765460410E000
23	14	1.1006920648114335ECC1	5.03C790738245478886000	2.6264527155513394E000
23	14	1.3380213450669046E0C1	3.1116894886244396E000	-3.7836897159677940E000
23	14	1.5042510515EC6677E0C1	9.861C454791961456-001	-1.6226467033043409E000
23	14	1.5852532433126571ECC1	1.4927CE59647526876000	7.8232471856930913-001
23	14	1.7168919448810792E001	2.0563762813914CE76000	2.7284414342352096E000
23	14	1.9005056243568313ECC1	5.7164344687854065E000	1.2648738904234076E000
23	14	2.0674944300326958EECC1	5.3593798732813651E000	-1.1978338878428150E000
23	14	2.2066231640167673E0C1	5.1916C885007429516000	-5.6962272658240988-001
23	14	2.3761886814407437EEC1	2.8073097742484004E000	-2.4759906080646869E000
23	14	2.4830335049032483E0C1	3.1529867716258250E0C0	6.7300513248877601-002
23	15	1.3797846159596C658ECCC	1.4334484989210462E000	8.4758935012498179-001
23	15	4.7681940983720558ECC0	3.4830944622754906E000	-1.1762556877091574E000
23	15	7.2469105252120EC78ECC	9.6675087113574577-001	-1.5192288549869460E000
23	15	8.2691558869E542446ECC	1.3737103814067416E000	6.2642051957947425-001
23	15	9.6343052441614568ECC0	1.1544903250749EC8E000	2.8724520301882871E000

TABLE V1. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
23	15	1.2112566981802272&CC1	5.8419834610185531&000	-1.3772179436775139&000
23	15	1.4044459561179976&0C1	2.3383562242325830&000	-2.5346421061800675&000
23	15	1.5140311256924652&CC1	3.0101391285028947&000	4.1332955861453861-001
23	15	1.6874925313993121&001	3.1843647795593296&000	-8.1959397724882935-001
23	15	1.8141811882842957&6CC1	3.1291701680725338&000	8.1919356929180195-001
23	15	1.9725151936370182&CC1	2.9460332627785266&000	1.2078410897316360-001
23	15	2.0539451934818267&C01	5.6429194712445918-001	4.0370125255882956&000
23	15	2.3274507660989998&CC1	7.7536043772609722-001	-5.1478662248310767&000
23	15	2.4535718985603221&0C1	7.2700637235600961-001	-1.4265421727269747&000
23	15	2.5085663704080202&001	1.1534214543409734&000	1.1931346090601742&000
23	16	1.2765387824514558&CCC	5.8930656789094969-001	9.2179182114217172-001
23	16	5.3165587218944782&0C0	2.6004155794657805&000	-2.2857975509182622&000
23	16	7.1921790424105734&CCC	1.8635977717584495&000	-1.0451806121569572&000
23	16	8.4382534633698752&CC0	2.0283697225457713&000	1.2216114633129408&000
23	16	1.0520026874184326&0C1	3.3441672195075503&000	-1.8262201639795125-002
23	16	1.1925284118577153&CC1	3.3345669125911960&000	1.9369036141091004&000
23	16	1.3693546641776523&CC1	5.8258944657279258&000	1.5919061739904049&000
23	16	1.6125151922336176&CC1	3.2735324763466090-001	-2.7619282743209027&000
23	16	1.6995979977655939&CC1	1.8955507213871235&000	-1.8699764166593453-001
23	16	1.8245093772940C63&CC1	2.2956866120182546&000	1.0839764624514375&000
23	16	1.9628953553776792&CC1	2.9536424347031126&000	2.2456687894538259&000
23	16	2.0981267500681210&QC1	4.1106310234613546&000	4.9586779163786669&000
23	16	2.3746162420970420&001	-3.6555882655221327-001	-6.7276142181280890-001
23	16	2.3906971402001199&CC1	5.4610C5198116C538-001	4.7299253772922017-001
23	17	6.8927167805141C68-CC1	9.7265133764923947-001	4.5097881484587953-001
23	17	4.0180046103450257&CCC	3.2141920232504185&000	4.2105129966447456-001
23	17	6.3551351693220441&0CC	3.8474775444385563&CCC	-6.0555188391049599-001
23	17	8.6347581759667273&000	1.8510808965279965&000	-1.8462949442956600&000
23	17	9.9591432275917C43&CCC	1.5787446754263C53&000	4.4761091277408017-002
23	17	1.1255781240002592&C01	1.8807355638759157&000	1.9078028979471354&000
23	17	1.2881065736644216&CC1	3.6513829865812731&000	4.2112678339164873&000
23	17	1.5588736033456662&CC1	8.8662477899533528-001	-3.9815888741298940&000
23	17	1.6834595677625526&CC1	1.1583522619955961&000	-1.0866&C725832063C&000
23	17	1.7475485457949C25&CC1	5.1097607077971665-001	1.6315290210126624&000
23	17	1.9712265520366556&CC1	2.8290541C540750266000	-1.6053626954561316&000
23	17	2.0654144817223324&CC1	2.5155347310549900&000	1.7185146617196977&000
23	17	2.2070472761755575&CC1	4.3116320435265556&C00	3.3054397507607620&000
23	17	2.3865746C661749326CC1	6.9586619086677956&000	-1.2815352411666496&000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
23	18	2.4703917896683253&CCC	1.57C82C2C3512E7136C000	3.1351146801747915-001
23	18	4.4321176219734415&CCC	2.0C396979839441596000	2.3673035621169442&000
23	18	7.7207993255ZCCSCC&CCC	1.6932447&145C4396&000	-3.0815300322328606&000
23	18	9.3932567340605768&C00	8.753C505637606971-001	-9.107174C784809633-001
23	18	1.0039596774182388&CC1	5.4095349531824533-001	1.3220059296306668&000
23	18	1.2425109771331571&CC1	3.633758853951472168000	-3.87165219C8890173-001
23	18	1.3619448753730128&CC1	3.2057400&60679739&000	2.858345828220674&000
23	18	1.5898632104934C598&CC1	3.6450929105414522&000	-2.9191193763196992&009
23	18	1.7076380101436559&CC1	4.1093321573347467&000	2.140646073645598&-001
23	18	1.8988998C73C869C5&CC1	2.1977938478052314&000	-2.3204440226773629&000
23	18	2.0217152993523482&CC1	1.6742274910C397796000	-5.0858886979232069-001
23	18	2.0977937859874897&001	7.65C6994040624807-001	1.9818213171840663&000
23	18	2.2467845953C13&55&CC1	3.0223045750983993&000	5.7140388789462740&000
23	18	2.4936495208536258&CC1	1.4943918C973406338000	-5.0390850916503298&000
23	19	1.2337771581815&23&CCC	1.0268842?C86783498000	-3.9682645936675646-001
23	19	3.1477471373316694&CCC	1.2875472894492843&000	1.3623172278009227&000
23	19	5.8861172534409946&000	4.2911921400484563&000	5.9894567082421753-001
23	19	8.4557569353962612&CCC	1.6928779392064520&000	-2.1779030967768627&000
23	19	9.5160951504673241&CCC	2.4136951483185145&CC0	9.6432815843385583-001
23	19	1.1764563110274C72&CC1	2.2561557940275240&000	-1.5205776968813891&000
23	19	1.3016062555098128&CC1	1.8739903205262335&000	3.0635947109273701-001
23	19	1.3906373355556695&001	5.7922402709629666-003	2.9924288536546618000
23	19	1.6638334886OC07666C01	4.6203780054126251&000	-4.6048867397409004&000
23	19	1.8284525223C47748&CC1	1.3456447655984890&000	-2.4228492235186212&000
23	19	1.9232406081366334&0C1	2.2180358012348931&000	1.6323595328419863-001
23	19	2.0566815316417324&CC1	2.7404362042968703&000	1.0129812399123338&000
23	19	2.1834517033OC72578CC1	3.1657396040672933&000	2.8762065530482616&000
23	19	2.3558786387379652&CC1	6.0207103048709577&000	4.1543368841172654-001

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
23	20	1.969771541049190460C0	1.459630275839747160000	3.8311698738696398-001
23	20	4.880221735492892660CC	2.014232180563395860000	-4.6671761648263891-001
23	20	6.094388119466841560C0	1.254954084248477460C00	2.4337909C876626136000
23	20	9.197926778524802360C0	2.480591337614674160000	-3.270973007769878660000
23	20	1.067742555619705760CC1	2.437251406257519060000	-1.141444289788970260000
23	20	1.235530432162033260001	1.354499856253809860000	-6.280833955836216O-001
23	20	1.303008807756226360CC1	2.8312457404809672-001	1.772506601247444160000
23	20	1.515861404994762060001	5.962501670903479760000	2.233610045249564860000
23	20	1.733455318923654660CC1	2.344173513527234660000	-3.236089497914890160000
23	20	1.829531846C883C2760CC1	3.371637845693052560000	6.5175434823354781-001
23	20	2.000084804266995060CC1	4.066153959702473560000	-1.218597270367972260000
23	20	2.165108789935308560C01	1.586358529256988960000	-1.406023859466939660000
23	20	2.234674159C3C41906CCC1	1.347267746923858060000	1.663180503543222660000
23	20	2.390795606859496160001	3.9C978613181028656O000	2.489132470617011260000
23	21	2.473712372400738060CC0	1.697E4435030505136O000	-7.1515794406887639-001
23	21	4.746353172138165560CC0	1.479933853367880960000	-2.495409863868O312-001
23	21	5.8C015224206707446CCC0	3.28C5659679971067-001	2.108796615464478060000
23	21	9.147293456021989760CC0	2.2977444CCC2262336O000	-3.312092231634793560000
23	21	1.052705860820054060001	2.596626071879606160000	-7.8679994499496702-001
23	21	1.196008485C822E960CC1	2.991080426193022660000	5.5828087855157653-001
23	21	1.379395777511727C0CCC1	2.938599581407483560C0	-6.1371521138225952-001
23	21	1.499691157482120260001	2.690729392585554060000	1.654999094291922860000
23	21	1.65C933709345707460CC1	4.25E7558E4265295460000	3.087302036283375860000
23	21	1.842768886194495460CC1	6.853892039479579960000	-1.524066321785724260000
23	21	2.050746815465770660CC1	1.459C016953533452-001	-2.528340465335294460000
23	21	2.138467987556147660CC1	1.334373629791830760000	-3.762726908173506S-001
23	21	2.214658019725729960C1	7.180524515l143352-001	1.841578551570174O000
23	21	2.391896783424C515E0CC1	4.630269522996441760000	2.018190195728675760000

TABLE VI. ZEROS OF L-SERIES

K	N	ZERO	RE DERIV	IM DERIV
24	6	1.977190514437953006CC0	1.924998696466710560000	3.070805107572191 9-001
24	6	4.722709505812344360CC0	2.802789227910478460000	-4.343200577178146 0-001
24	6	6.558917159470904460CC0	3.141C9C4352519CC360000	5.534831374514391 5-001
24	6	8.631859564899361460000	3.430527867C1705026C000	-5.908252289605008 3-001
24	6	1.023111037559675816CC1	3.495534098340489360000	3.732218836016000 7-001
24	6	1.193585936955933360CC1	4.C597523860171300600C0	2.292116701871348 6-001
24	6	1.375222966580168160C1	3.217283959427019760000	-1.125907602630283 0600C
24	6	1.488654143965881360C1	3.236546551325005760000	1.920931995031471 6600C
24	6	1.703040557143405860C1	2.37C61416716053C80C0C	-2.111542516460587 5600C
24	6	1.787462416651816160001	2.501907974464859560000	1.867895585826341 5600C
24	6	1.972401532444155260C1	4.814854433246537760000	-8.203508809847078 1-001
24	6	2.123481315500759860C1	3.401999847304547060000	-1.127963996251249 6600C
24	6	2.224400758954810980C1	3.350826931191313860000	2.251150260547862 9600C
24	6	2.418502420176868260C1	3.238405265964113560000	-2.307888804856889 3600C
24	7	2.688658132467559760CC0	2.227453040085972560000	3.792995131424430 3-001
24	7	5.292431176177198260CC0	2.805238429629807760000	-5.931304144066859 2-001
24	7	6.971924243791727560000	3.120683328857471860000	9.261094672049649 0-001
24	7	9.224637439935561260CC0	2.836226750402029360000	-1.304185023729048 0600C
24	7	1.044572151841252560C1	2.911826773075705260000	1.580688398015999 7600C
24	7	1.263115371816651160C1	3.081475513365929060000	-1.667276153077110 5600C
24	7	1.377715270577345660C1	3.171610812000330860000	1.338652905652916 2600C
24	7	1.560365697037025460C1	4.328507563565261360000	-5.934686917124960 0-001
24	7	1.709898808691940360001	3.909085798206824360000	-3.350424306283529 4-001
24	7	1.846325885012140060C1	4.130990316700605660000	7.466110566933568 3-001
24	7	2.003461157675011880001	4.633507245582810160000	-2.586312511084619 6-001
24	7	2.155547824218793760C1	3.767378976655853860000	-9.248857748770432 2-001
24	7	2.266785732205079660C1	3.863603575632383360000	1.788546156355838 5600C
24	7	2.440914964223159460C1	4.441040311990446260000	-1.578313734548455 4600C

TABLE VII: E(K)

K	E(K)	N
1	14.13473	0
3	8.03974	1
4	6.02095	1
5	4.13290	3
7	2.50937	5
8	3.57615	3
9	2.90199	5
11	1.23119	7
12	3.80463	3
13	.88396	5
15	2.73460	3
16	1.58558	7
17	.39131	11
19	.01896	5
20	2.35893	5
21	1.61202	9
23	.59543	9
24	1.97719	6

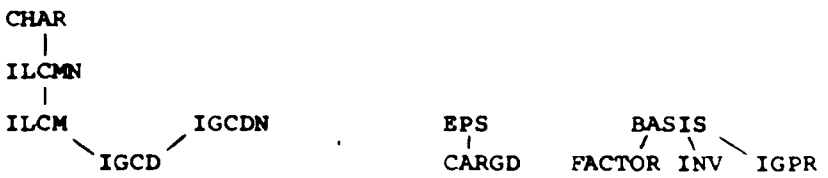
DESCRIPTION OF PROGRAMS

There are nineteen programs given, in double precision versions. These are:

1.	FACTOR	, factors K
2.	JPHI	, $\varphi(K)$
3.	INV	, inverses in M(K)
4.	IGCD	, G.C.D. 2 numbers
5.	ILCM	, L.C.M. 2 numbers
6.	IGCDN	, G.C.D. n numbers
7.	ILCMN	, L.C.M. n numbers
8.	IGPR	, least primitive root of p^{α}
9.	BASIS	, basis of M(K)
10.	RES	, residue coder (used for CHAR)
11.	CHAR	, character generator
12	DIVS	, proper divisor generator
13.	IRESΘ	, resolving modulus of a character
14.	TRCH	, character translator (into complexes)
15.	CARGD	, Arg
16.	EPS	, ϵ
17.	NGEN	, primitive N generator
18.	L	, $L(S,\chi)$
19.	LP	, $L'(S,\chi)$.

Single precision versions of these programs are easily constructed.

The following chart shows which subroutines are called by other subroutines.



To use CHAR:

CALL: 1. BASIS, 2. RES, 3. CHAR.

To use L,LP:

CALL: 1. BASIS, 2. RES, 3. CHAR, 4. TRCH, 5. L,LP

We next list some regularly used dimensioned variables, and then describe the input and output for each program.

Dimensioned Variables

IB	:	Basis elements
IH	:	Orders of basis elements
ISN	:	Coding of residues
ICH	:	Coded character values
CHZR	:	Real and imaginary parts of
CHZI	:	double precision values of χ.
IPS	:	Factors p_j of $k = \pi p_j^{\alpha_j}$
IALF	:	Exponents α_j of $k = \pi p_j^{\alpha_j}$
ID	:	Proper divisors of k

There are also other locally used dimensioned variables.

Input and Output

1. SUBROUTINE FACTOR (K, IPS, IALF, IR)

   Input: $K \in [1, 10^6]$

   Output: $K = \pi_{J=1}^{IR} IPS(J)^{**} IALF(J)$,

2. FUNCTION JPHI(K, IPS, IALF, IR)

   Input: $K = \pi_{J=1}^{IR} IPS(J)^{**} IALF(J)$,

   Value: $\varphi(K)$, the Euler totient function of K.

3. FUNCTION (IA, IM, IPHI)

   Input: IA, IM, IPHI = φ(IM)

   Value: IA^{-1} (mod IM) or 1 and error message if (IA, IM) > 1.

4. FUNCTION IGCD(IA, IB)

   Input: IA, IB

   Value: G.C.D.(|IA|, |IB|), 0 if IA = IB = 0.

5. FUNCTION ILCM(IA, IB)

   Input: IA, IB

   Value: L.C.M. (|IA|, |IB|), 0 if IA * IB = 0.

6. FUNCTION IGCDN(IA,N)

   Input: Array name IA, N

   Value: G.C.D. (IA(1),...,IA(N))

7. Function ILCMN(IA,N)

   Input: Array name IA, N

   Value: L.C.M. (IA(1),...,IA(N)) or 0 if one entry is 0.

8. SUBROUTINE IGPR(IP,IG)

Input. IP, a prime

Output: 3 if IP = 2 the least primitive root mod IP^2 otherwise.

9. SUBROUTINE BASIS (K, IB, IH, IR, IPHI)

Input K

Output: IPHI = $\varphi(K)$, IR basis elements IB(J) of orders IH(J) for the residue group M(K).

10. SUBROUTINE RES (K, IPHI, IB, IH, IR, ISN)

Input: K, IPHI = $\varphi(K)$: basis of IR elements in array IB with orders in IH.

Output: Coding of residues in the array ISN.

11. SUBROUTINE CHAR(K, N, IB, IH, IR, ICH, IPHI, ISN, IA, IORD)

Input: K, N = character number, IB, IH, IR, IPHI, ISN as above

Output. Coded character χ_N in array ICH, its corresponding residue IA of order IORD, (in the group M(K)). Code of character value at J is 0 if $\chi_N(J) = 0$, and I if $\chi_N(J) = \exp(2\pi iI/\varphi(K))$, $1 \leq I \leq \varphi(K)$

12. SUBROUTINE DIVS(K, ID, ITAU)

Input: $K \geq 1$

Output ITAU = no. of proper divisors of K. proper divisors in array ID.

13. FUNCTION IRESO(K, ID, ITAU, ICH)

Input: K, character in array ICH, ITAU proper divisors in array ID.

Value: resolution modulus of the character in ICH

14. SUBROUTINE TRCH(K, ICH, CHZR, CHZI)

Input: K, character in array ICH

Output: Decoding of character in double precision, ICH(J) $\longrightarrow$ CHZR(J) + i CHZI(J)

15. SUBROUTINE CARGD(X, Y, ANS)

Input: X,Y

Output: ANS = Arg(X + iY) in double precision

16. SUBROUTINE EPS(K, ICH, CHZR, CHZI, E8)

Input: K, ICH, CHZR, CHZR

Output: E8 = Arg ϵ of functional equation for character ICH.

17. SUBROUTINE NGEN(K, IPS, IALF, IN, IH, IR, IPHI, NCOD)

Input: $K = \pi_{J=1}^{IN}$ IPS(J)**IALF(J), $K \geq 3$.

IR = number of basis elements, of orders IH(J) J = 1,...,IR, IPHI = $\varphi(K)$

Output: NCOD(N) = 1 if χ_N primitive, 0 if χ_N imprimitive, N=1,...,IPHI

18. SUBROUTINE L(SIG, T, K, ICH, CHZR, CHZI, ANSR, ANSI)

Input: SIG, T, K, character in array ICH, translation of ICH in arrays CHZR and CHZI

Output: ANSR + iANSI = double precision value of $L(\sigma + it, \chi)$ where χ is determined by ICH.

19. LP similar to 18, but double precision first derivative.

Unusual Outputs

Output	From	Reason
BASERR, k	BASIS	k not in range $[1,10^6]$.
CHAR, N,k	CHAR	N out of proper range for character mod k or k > 2048
DIVSERR, K,J	DIVS	J = more than 200 divisors of K
FACTERR, K	FACTOR	K not in interval $[1,10^6]$ or more than 170 primes tried
IGPR, IP	IGPR	IP not a prime
IGCDN, n	IGCDN	n = number of numbers of which C.C.D. to be found not in range [1,100]
ILCMN, n	ILCMN	n = number of numbers of which L.C.M. to be found not in range [1,100]
INV ERR, a,m,Φ	INV	m < 1 or a too large or error in routine
IR TOO LARGE	CHAR	More than 7 basis elements
L ERROR, σ,t,k	L	input out of range
LPERROR, σ,t,k	LP	input out of range
NGEN K = k	NGEN	input out of range
TRCH ERR, k	TRCH	k not in range [3,2048]

PROGRAMS

```
      SUBROUTINE FACTOR(K,IPS,IALF,IP)
      DIMENSION IPS(20),IALF(20)
      DIMENSION ID(200)
      DATA (ID =
     1          2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67
     2         ,71,73,79,83,89,97,101,103,107,109,113,127,131,137,139
     3         ,149,151,157,163,167,173,179,181,191,193,197,199,211,223
     4         ,227,229,233,239,241,251,257,263,269,271,277,281,283,293
     5         ,307,311,313,317,331,337,347,349,353,359,367,373,379,383
     6         ,389,397,401,409,419,421,431,433,439,443,449,457,461,463
     7         ,467,479,487,491,499,503,509,521,523,541,547,557,563,569
     8         ,571,577,587,593,599,601,607,613,617,619,631,641,643,647
     9         ,653,659,661,673,677,683,691,701,709,719,727,733,739,743
     1         ,751,757,761,769,773,787,797,809,811,821,823,827,829,839
     2         ,853,857,859,863,877,881,883,887,907,911,919,929,937,941
     3         ,947,953,967,971,977,983,991,997,1009)
      IF (K-1)  40,72,41
   40 PRINT 42,K
      GO TO 69
   41 IF (K - 1000000) 43,43,40
   42 FORMAT ( 9H FACTERR ,  I20)
   43 K1 = K $IP = 0 $J=1 $L=1 $ICW=0 $IP1 = 2 $IP12 = 4
   44 IX = MOD(K1,IP1)
      IF (IX)  46,45,46
   45 IF (ICW)  48,47,48
   46 IF (IP12 - K1)  50,51,51
   50 IF (ICW)  49,70,49
   47 IPS(L) = IP1   $IALF(L)=0 $ICW=1 $IR=IR+1
   48 IALF(L) = IALF(L) + 1
   71 K1=K1/IP(J)
      GO TO 44
   49 ICW=0 $L=L+1
   70 J = J+1 $IF (J-149)  74,74,60
   74 IP1 = IP(J)
      IP12 = IP1*IP1
      GO TO 44
   51 IF (K1 - 1)  95,52,95
   95 IP = IP + 1
      IF (ICW)  96,97,96
   96 L = L + 1
   97 IALF(L) = 1
      IPS(L) = K1
   52 RETURN
   72 IP=0 $GOTO52
      END
```

PROGRAMS

```
      FUNCTION JPHI(N,IP,IALF,IR)
      DIMENSION IP(20),IALF(20)
C CALCULATES EULER TOTIENT FUNCTION FOR N = PROD (IP**IALF  J=1,IR
      IPHI = 1
      DO 1 J = 1,IR
      IPHI=IPHI*(IP(J) - 1)
      IF(IALF(J) - 1)  2,1,2
    2 IPHI = IPHI*(IP(J)**(IALF(J) - 1))
    1 CONTINUE
      JPHI = IPHI
      RETURN
      END

      FUNCTION INV(IA,IM,IPHI)
      DIMENSION IE(40)
      IF(IM-1)  60,60,61
   61 IF (IA)  90,91,92
   92 IA1 = IA $ GO TO 62
   91 IA1 = IA + IM $GO TO 62
   90 IX = -IA$IQ = IX/IM+ 1$IA1 = IA +IQ*IM
   62 IQ = IPHI - 1 $I1 = 1
   64 IF (IQ)  60,63,62
   62 IQ1 = IQ/2 $IE(I1)=IQ-2*IQ1 $I1=I1+1 $IQ=IQ1
      GO TO 64
   60 PRINT70,IA,IM,IPHI
   64 IAI = 1 $GOTO52
   70 FORMAT (8H INV ERR ,3I20)
   63 IF (I1-40)  21,21,60
   21 DO95J=I1,40
   95 IE(J)=0
      I1 = I1-1
      IF(I1)  41,40,41
   40 IAI=MOD(IA1,IM) $GOTO51
   41 IAI = 1 $IX=IA1 $DO95J1=1,I1 $IF(IE(J1))  96,95,96
   96 IAI = MOD(IAI*IX,IM)
   95 IX = MOD(IX*IX,IM)
   51 IT = MOD(IA1*IAI,IM) - 1
      IF(IT) 60,52,60
   52 INV = IAI
      RETURN
      END
```

```
C                DORRAYS
C
      FUNCTION IGCD(IA,IB)
      IC = IABS(IA)
      ID = IABS(IB)
      IR (IC-ID)  1,2,3
    2 IGCD = ID $RETURN
    1 IX = ID
      ID = IC SIC = IX
    3 IF(ID)  4,4,3
    4 IGCD = IC $RETURN
    3 IR = MOD(IC,ID) $IF(IR) 4,2,4
    4 IC = ID $ID = IR$ GOTO3
      END

      FUNCTION ILCV(IA,IB)
      IC = IABS(IA*IB)
      IF (IC)  1,2,1
    1 ILCV = IC/IGCD(IA,IB) $RETURN
    2 ILCV = 0 $ RETURN
      END

      FUNCTION IGCDN(IA,N)
      DIMENSION IA(100)
      IF (N-100) 1,1,2
    2 PRINT 3,N
    3 FORMAT (8H IGCDN   ,I20)
      RETURN
    1 IF(N) 2,2,4
    4 IF(N-2) 5,6,7
    5 IGCDN=IABS(IA( 1)) $RETURN
    6 IGCDN = IGCD(IA( 1),IA( 2))
      RETURN
    7 IX = IGCD(IA(1),IA(2))
      DO8 J=3,N
    8 IX = IGCD(IX,IA(J))
      IGCDN = IX $ RETURN
      END
```

PROGRAMS

```
      FUNCTION ILCMN(IA,N)
      DIMENSION IA(100)
      IF (N-100) 1,1,2
    2 PRINT 3,N
    3 FORMAT (18H ILCMN    ,I20)
      RETURN
    1 IF (N) 2,2,4
    4 IF (N-2)  5,6,7
    5 ILCMN = IABS(IA(1))
      RETURN
    6 ILCMN = ILCM(IA(1),IA(2)) $RETURN
    7 IX = ILCM(IA(1),IA(2))
      DO 8 J=3,N
    8 IX = ILCM(IX,IA(J))$ILCMN = IX $RETURN
      END

      SUBROUTINE  IGPR(IP,IG)
DELIVERS PRIMITIVE ROOT FOR P**2
      IF(IP - 2)  21,22,21
   22 IG = 3
      RETURN
   21 IP1 = IP $IP12=IP1*IP1 $IPH=IP1-1 $IG1=2
    1 IJ = 1 $IPOW = IG1
    4 IF(IPOW - 1)  2,3,2
    2 IJ = IJ + 1 $IPOW = MOD(IPOW*IG1,IP1)
      GOTO4
    3 IF(IJ - IPH)  5,6,6
    5 IG1 = IG1 + 1
      IF(IG1- IP1)  1,23,23
   23 PRINT 24,IP1
      STOP
   24 FORMAT (8H IGRR       ,I20)
    6 I = 1 $IPW=IG1
   16 IPW = MOD(IG1*IPW,IP12)
      I = I  +1
      IF (I - IPH)  16,17,17
   17 IF(IPW - 1)  19,18,19
   18 PRINT 20,IP1,IG1
   20 FORMAT(8H IG CASE    ,2I20)
      GOTO5
   19 IG = IG1
      RETURN
      END
```

PROGRAMS

```
      SUBROUTINE BASIS(K,IR,IH,IP1,IPH2)
      DIMENSION IPS(20),IALF(20),IM(20),IPA(20),IA(20)
     1                  ,IPH(20),IR(20),IH(20)
      IF (K-1) 60,63,61
   60 PRINT 70,K
      RETURN
   70 FORMAT (8H BAS ERR  ,I20)
   61 IF (K - 1000000) 62,62,60
   62 IF (K-2)  63,63,64
   63 IP1 = 1 $IR(1) = 1 $IH(1) = 1
      IPH2=1 $GOTO54
   64 CALL FACTOR (K,IPS,IALF,IR)
      DO 5 J = 1,IR  $ IF (IALF(J) - 1) 3,4,3
    4 IPA(J) = IPS(J) $IPH(J) = IPS(J) - 1 $GOTO5
    3 IPA(J) = IPS(J) **(IALF(J) - 1)
      IPH(J) = IPA(J)*(IPS(J) - 1)
      IPA(J) = IPA(J)*IPS(J)
    5 IM(J)= K/IPA(J)
      DO 6 J = 1,IR
    6 IA(J) = INV(IM(J),IPA(J),IPH(J))
      IX = 1
      IF (IPS(1) -2) 8,9,8
    9 IF (IALF(1) -2)  11,8,13
   13 IV = MOD(IA(1)*IM(1),K)
      IV = MOD(IV*(IPA(1)-2),K)
      IR(1) = MOD(IX + IV,K)
      IR(2) = MOD(IX + MOD(IV*4,K),K)
      IH(1) = 2 $IH(2) = IPH(1)/2 $IR1 = IR + 1
      IF(IP1-3)  52,20,20
   20 DO 14 I1=3,IP1 $I2=I1-1
   31 CALL IGPR(IPS(I2),IPRIM)
   32 IV = MOD(IA(I2)*IM(I2),K)
      IV = MOD(IV*(IPRIM  -1),K)
      IR(I1)= MOD(IV+IX,K)
   14 IH(I1) = IPH(I2)
      GO TO 52
   11 IP1 = IP-1
      DO 15 I1 = 1,IR1$I2=I1+1
   41 CALL IGPR(IPS(I2),IPRIM)
   42 IV = MOD( IA(I2)*IM(I2),K)
      IV = MOD(IV*(IPRIM  -1),K)
      IR(I1) = MOD(IX+IV,K)
   15 IH(I1) = IPH(I2)
      GOTO52
```

```
      UR = UR11
      UI = UI11
      VR = VR11
      VI = VI11
      SR = SR11
      SI = SI11
      TR = TR11
      TI = TI11
      ZLRF = ZLRF + TR
      ZLIM = ZLIM + TI
      IF (M - J)  99,77,77
   77 IF(ABS(TR) + ABS(TI)-1F-25)  99,55,55
   00 IF(ICH(IF) - IPHI)  27,28,27
   28 ANSR = ANSR -ZLRE *ANSI = ANSI+ ZLIM $GOTO10
   27 IF (ICH(IF) - IPH2)  29,30,29
   30 ANSR = ANSR -ZLRF *ANSI = ANSI -ZLIM $GOTO10
   29 ANSR = ANSR + CHZR(IE) * ZLRE - CHZI(IE)*ZLIM
      ANSI = ANSI + CHZR(IE) * ZLIM + CHZI(IE)*ZLRE
      GO TO 10
      END
```

R CONSTANTS

```
.01666666666666666666666666666667
.02380952380952380952380952380
.02500000000000000000000000000
.02525252525252525252525252533
.02531135531135531135531135531
.02532561505065123010130246020
.02532913165266106442577030812
.02533000550409748750735931155
.02533022339183393190573987072
.02533027778648292122591391052
.02533029138045679631354746884
.02533029477815223633102542636
.02533029562748746696293024780
.02533029583981142936379787343
.02533029589289132619652705592
.02533029590616117888423206286
.02533029590947862855445557920
.02533029591030798947185783901
.02533029591051532953452688346
.02533029591056716453167411373
.02533029591058012927890314871
.02533029591058336296548176659
```

```
          PROGRAMS

   IF(N) 31,32,33
33 IF(N-IPH1)  34,31,31
31 PRINT 35,N,K$RETURN
35 FORMAT (9H CHAR    ,2I20)
32 DO36I1=1,IPH1
   IA=ISN(I1)
36 ICH(IA) = IPH1
   IA = 1 $IORD = 1 $RETURN
34 IQ = N $DO 111I=1,IR $IQ1=IQ/IH(I1) $IFTA(I1) = IQ-IQ1*IH(I1)
   IQ = IQ1 $IPHV(I1) = IPH1/IH(I1)
11 IRPH(I1) = MOD(IPHV(I1),IFTA(I1),IPH1)
   IHH1 = IH(1) $DO 81 I1=2,IR
81 IHH(I1) = IH(I1)*IHH(I1-1)
   IA =1 $DO40I1=1,IR $ILIM = IFTA(I1) $IF(ILIM)  41,40,41
41 DO40 J1=1,ILIM $IA = MOD(IA*IR(I1),K)
40 CONTINUE
   DO 42 I1=1,IR
42 IRD(I1)= IH(I1)/(IGCD(IFTA(I1),IH(I1)))
   IORD = ILCM(IORD,IR)
   IF (IR-7)  10,12,12
12 PRINT 13,IR
13 FORMAT (13H IR TOO LARGE     ,I20)
   RETURN
10 GO TO (1,2,3,4,5,6,7 ),IR
 1 IH1=IH(1)
   DO 21 I1=1,IH1$J1=I1-1 $IA=ISN(I1)
   IC       = MOD(J1*IRPH1, ,IPH1)
   IF (IC) 21,94,21
94 IC = IPH1
21 ICH(IA) = IC
   RETURN
 2 IH1=IH(1) $IH2=IH(2)
                  DO22I2=1,IH2 $J2=I2-1$N1=J2*IH1
   IX2 = MOD(J2*IRPH 2 ,IPH1)  $DO22 I1=1,IH1 $J1=I1-1 $N1=N1+I
   IA=ISN(N1)$IX1 = MOD(J1*IRPH 1 ,IPH1)
   IC       = MOD(IX1+IX2,IPH1)
   IF (IC)  22,95,22
95 IC = IPH1
22 ICH(IA) = IC
   RETURN
```

```
      IH1=IH(1) $IH2=IH(2) $IH3=IH(3)
                                                    DO23 I3=1,IH3
      J3 = I3-1 $N3=J3*IH   H2$IX3=MOD(J 3*IRPH 3 ,IPH1)
      DO 23 I2 = 1,IH2$J2=I2-1$N2=J2*IH1 +N3
      IY2=MOD(J2*IRPH 2 ,IPH1)
      IX2 = MOD(IY2+IX3,IPH1) $DO23I1=1,IH1
      J1=I1-1                 $IX1=MOD(J1*IRPH 1 ,IPH1)
      N2 = N2+1 $I6=ISN(N2)
      IC      =MOD(IX1+IX2,IPH1)
      IF (IC)  23,96,23
   96 IC = IPH1
   23 ICH(I6) = IC
      RETURN
      IH1=IH(1) $IH2=IH(2) $IH3=IH(3) $IH4=IH(4)
      DO24 I4=1,IH4 $J4=I4-1 $N4=J4*IHH3
      IX4 = MOD(J4*IRPH4,IPH1)
      DO24 I3=1,IH3 $J3=I3-1 $N3=J3*IHH2+N4
      IX3 = MOD(J3*IRPH3,IPH1) $IX3=MOD(IX4+IX3,IPH1)
      DO24 I2=1,IH2 $J2=I2-1 $N2=J2*IHH1+N3
      IX2 = MOD(J2*IRPH2,IPH1) $IX2=MOD(IX2+IX3,IPH1)
      DO24I1=1,IH1 $J1=I1-1 $N2=N2+1 $I6=ISN(N2)
      IX1 = MOD(J1*IRPH1,IPH1)
      IC = MOD(IX1+IX2,IPH1)
      IF(IC) 24,204,24
  204 IC = IPH1
   24 ICH(I6) = IC
      RETURN
      IH1=IH(1) $IH2=IH(2) $IH3=IH(3) $IH4=IH(4) $IH5=IH(5)
      DO25 I5=1,IH5 $J5=I5-1 $N5=J5*IHH4
      IX5 = MOD(J5*IRPH5,IPH1)
      DO25 I4=1,IH4 $J4=I4-1 $N4=J4*IHH3+N5
      IX4 = MOD(J4*IRPH4,IPH1) $IX4=MOD(IX4+IX5,IPH1)
      DO25 I3=1,IH3 $J3=I3-1 $N3=J3*IHH2+N4
      IX3 = MOD(J3*IRPH3,IPH1) $IX3=MOD(IX4+IX3,IPH1)
      DO25 I2=1,IH2 $J2=I2-1 $N2=J2*IHH1+N3
      IX2 = MOD(J2*IRPH2,IPH1) $IX2=MOD(IX2+IX3,IPH1)
      DO25I1=1,IH1 $J1=I1-1 $N2=N2+1 $I6=ISN(N2)
      IX1 = MOD(J1*IRPH1,IPH1)
      IC = MOD(IX1+IX2,IPH1)
      IF(IC) 25,205,25
  205 IC = IPH1
   25 ICH(I6) = IC
      RETURN
```

```
C PROGRAMS
  6 IH1=IH(1) $IH2=IH(2) $IH3=IH(3) $IH4=IH(4) $IH5=IH(5) $IH6=IH(6)
    DO26 I6=1,IH6 $J6=I6-1 $N6=J6*IHH5
    IX6 = MOD(J6*IRPH6,IPH1)
    DO26 I5=1,IH5 $J5=I5-1 $N5=J5*IHH4 +N6
    IX5 = MOD(J5*IRPH5,IPH1) $IX5=MOD(IX6+IX5,IPH1)
    DO26 I4=1,IH4 $J4=I4-1 $N4=J4*IHH3+N5
    IX4 = MOD(J4*IRPH4,IPH1) $IX4=MOD(IX4+IX5,IPH1)
    DO26 I3=1,IH3 $J3=I3-1 $N3=J3*IHH2+N4
    IX3 = MOD(J3*IRPH3,IPH1) $IX3=MOD(IX4+IX3,IPH1)
    DO26 I2=1,IH2 $J2=I2-1 $N2=J2*IHH1+N3
    IX2 = MOD(J2*IRPH2,IPH1) $IX2=MOD(IX2+IX3,IPH1)
    DO26I1=1,IH1 $J1=I1-1 $N2=N2+1 $X6=ISN(N2)
    IX1 = MOD(J1*IRPH1,IPH1)
    IC = MOD(IX1+IX2,IPH1)
    IF(IC) 26,206,26
206 IC = IPH1
 26 ICH(X6) = IC
    RETURN
  7 IH1=IH(1) $IH2=IH(2) $IH3=IH(3) $IH4=IH(4) $IH5=IH(5) $IH6=IH(6)
    IH7=IH(7)
    DO27 I7=1,IH7 $J7=I7-1 $N7=J7*IHH6
    IX7 = MOD(J7*IRPH7,IPH1)
    DO27 I6=1,IH6 $J6=I6-1 $N6=J6*IHH5 +N7
    IX6 = MOD(J6*IRPH6,IPH1) $IX6=MOD(IX6+IX7,IPH1)
    DO27 I5=1,IH5 $J5=I5-1 $N5=J5*IHH4 +N6
    IX5 = MOD(J5*IRPH5,IPH1) $IX5=MOD(IX6+IX5,IPH1)
    DO27 I4=1,IH4 $J4=I4-1 $N4=J4*IHH3+N5
    IX4 = MOD(J4*IRPH4,IPH1) $IX4=MOD(IX4+IX5,IPH1)
    DO27 I3=1,IH3 $J3=I3-1 $N3=J3*IHH2+N4
    IX3 = MOD(J3*IRPH3,IPH1) $IX3=MOD(IX4+IX3,IPH1)
    DO27 I2=1,IH2 $J2=I2-1 $N2=J2*IHH1+N3
    IX2 = MOD(J2*IRPH2,IPH1) $IX2=MOD(IX2+IX3,IPH1)
    DO27I1=1,IH1 $J1=I1-1 $N2=N2+1 $X6=ISN(N2)
    IX1 = MOD(J1*IRPH1,IPH1)
    IC = MOD(IX1+IX2,IPH1)
    IF(IC) 27,207,27
207 IC = IPH1
 27 ICH(X6) = IC
    RETURN
    END
```

PROGRAMS

```
      SUBROUTINE DIVS(K,ID,ITAU)
      DIMENSION ID(200)
      ID(1) = 1 $ITAU = 1
      IF (K-9)  47,47,50
   50 IX = MOD(K,2)
      IF (IX)  1,2,1
    2 ID(2) = 2 $ ITAU = ITAU + 1 $ ICW2 = 1  $ GO TO 3
    1 ICW2 = 0
    3 IX = MOD(K,3) $IF (IX)  4,5,4
    5 ITAU = ITAU + 1
      ID(ITAU) = 3
      ICW3 = 1 $IF(ICW2)  9,6,9
    4 ICW3 = 0 $IF(ICW2)7,8,7
    6 IT = 5 $KV5 = K/5
   14 IF(IT-KV5)  10,10,11
   10 IX = MOD(K,IT) $IF(IX)  12,13,12
   13 ITAU = ITAU + 1
      ID(ITAU) =   IT
   12 IT = IT + 2 $GOTO14
   11 IF(K-9)  47,47,18
   18 ITAU = ITAU +1 $ID(ITAU) = K/3
      GO TO 47
    7 IT = 4 $ICW = 0 $KV4 = K/4
   24 IF (IT - KV4)  20,20,21
   20 IX = MOD(K,IT) $IF (IX)  22,23,22
   23 ITAU = ITAU +1 $ID(ITAU) = IT
   22 IF(ICW) 24,25,24
   24 ICW=0 $IT = IT +2 $GOTO26
   25 ICW=1 $IT = IT+1$ GOTO26
   21 IF(K-4)  47,47,28
   28 ITAU = ITAU+1 $ID(ITAU) = K/2
      GO TO 47
    8 KV5 = K/5 $ IT =5$ICW=0
   36 IF(IT-KV5)30,30,47
   30 IX = MOD(K,IT) $IF(IX)  32,33,32
   33 ITAU = ITAU+1 $ID(ITAU) = IT
   32 IF (ICW)  34,35,34
   34 ICW=0 $IT          =IT+4 $GOTO36
   35 ICW=1 $IT=IT+2$GOTO36
    9 KV4 = K/4 $IT=4
   46 IF(IT-KV4)  40,40,41
   40 IX =MOD(K,IT)
      IF (IX)  42,43,42
   43 ITAU = ITAU +1
      ID(ITAU) = IT
```

PROGRAMS

```
42 IT = IT +1
   GO TO 44
43 IF(K-5)  47,47,48
48 ITAU = ITAU +2 $ID(ITAU) = K/2 $ID(ITAU-1) = K/3
47 IF(ITAU-200)  17,17,27
17 RETURN
27 PRINT 21,K,ITAU $STOP
21 FORMAT (8H DIVERR  ,2I16)
   END

   FUNCTION IPERO(K,ID,ITAU,ICH)
   DIMENSION ID(200)
   DIMENSION ICH(2048)
   J=1
 8 KK = ID(J) $I=1
10 ICH=0 $L=1
 3 IF (ICH(L)).1,2,1
 2 I = I+KK $IF (I-K)  3,3,4
 1 IF(ICH-I)  5,5,6
 6 IPE = ICH(L)
   ICH=I $GOTO2
 5 IF(IPE-ICH(L))  7,2,7
 7 J = J + 1 $ IF(J-ITAU)  8,8,9
 9 IPERO = 0 $RETURN
 4 I = I + 1 $IF(I-KK)  10,10,11
11 IPERO = KK $RETURN
   END

   SUBROUTINE TRCH(K,ICH,CH2R,CH2I)
   DOUBLE PRECISION PI2, CH2R(2048),CH2I(2048),FLCH,FLK,X,Y,FLPH,FL
   DIMENSION ICH(2048)
   DATA (PI2 = 6.28318530717958647692528700)
   IF (K-8)  1,2,2
 1 PRINT 3,K $RETURN
 3 FORMAT (8H TRCHERR    ,I10)
 2 IF (K-2048)  8,8,1
 8 IPHI = ICH(1) $IPH2 = IPHI/2 $ FLPH = IPHI $FL = PI2/FLPH
   DO 4 J = 1,K $ IF (ICH(J))  5,6,5
 6 CH2R(J) = 000 $CH2I(J) = 000 $GO TO 4
 5 IF (ICH(J) - IPHI)  7,9,7
 9 CH2R(J) = 100 $CH2I(J) = 000 $GO TO 4
 7 IF (ICH(J) - IPH2)  10,11,10
11 CH2R(J) = -100 $ CH2I(J) = 000 $GO TO 4
10 FLCH = ICH(J) $X = FLCH*FL
   CH2R(J) = DCOS(X) $CH2I(J) = DSIN(X)
```

```
          ,PROGRAMS

 4 CONTINUE
   RETURN
   END

   SUBROUTINE CARGD(X,Y,ANS)
   DOUBLE PRECISION X,Y,ANS,Z,PI2,PI
   DATA(PI2 =
  1       1.5707963267948966192313222D0)
   DATA(PI=3.14159265358979323846264D0)
   IF (X)  1,2,3
 1 IF (Y)  4,5,6
 2 IF (Y)  7,8,9
 3 IF (Y)  10,8 ,12
 4 Z = X/Y
   IF (Z - .2D0)  13,13,14
13 ANS = -PI2 - DATAN(Z)
   RETURN
14 ANS = -PI + DATAN(Y/X)
   RETURN
 5 ANS = PI
   RETURN
 6 Z = -X/Y
   IF (Z - .2D0)  15,15,16
15 ANS = PI2 + DATAN(Z)
   RETURN
16 ANS = PI - DATAN(-Y/X)
   RETURN
 7 ANS = -PI2
   RETURN
 8 ANS = 0.D0
   RETURN
 9 ANS = PI2
   RETURN
10 Z = -X/Y
   IF (Z - .2D0)  17,17,18
17 ANS = -PI2 + DATAN(Z)
   RETURN
18 ANS = DATAN(Y/X)
   RETURN
12 Z = X/Y
   IF (Z - .2D0)  19,19,19
19 ANS = PI2 - DATAN(Z)
   RETURN
   END
```

PROGRAMS

```
      SUBROUTINE FPS(X,ICH,CHZR,CHZI,FA)
      DOUBLE PRECISION PI2,CHZR(2048),CHZI(2048)  ,FR,FI,FLK,FLJ,X,C,F
      DIMENSION ICH(2048)
      DATA PI2 = 6.2831853071795864769252867D0
      FR = 000 $ FI = 000 $ FLK = K $ FLJ=K-1
      IF (ICH(1) - ICH(K-1))  1,2,1
    1 ICH = 0 $ GO TO 5
    2 ICH=1
    5 DO3 J=1,K1 $IF(ICH(J))  4,3,4
    4 SI J=JX*(PI2*FLJ)/FLK  $ C=CI(ICH) 7,6,7
    7 C = DCOS(X) $GOTO8
    6 C=SIN(X)
    8 FR = FR + CHZR(J)*C
      FI = FI + CHZI(J)*C
    3 CONTINUE
      CALL CARGO(FR,FI,FA)
      RETURN
      END

      SUBROUTINE NGEN(K,ID,IALF,IW,IM,IR,IPHI,NCOD)
      INTEGER BETA1,BETA2,BETA3,BETA4,BETA5,BETA6
      DIMENSION NCOD(2048),ID(20),IALF(20),IM(20)
      IF(K-2)1,1,2
    1 PRINT 3,K $RETURN
    2 FORMAT (8H K NGEN K=  ,I20)
    4 DO 5 J=1,2048
    5 NCOD(J) = 0
      IF(ID(1) - 2)  6,7,6
    7 IF(IALF(1) -2)  10,8,8
    8 IGO = 0 $BETA1 = 1
    9 IF (ID - 1)  10,11,10
   11 BETA1 = BETA1 + 1
      NCOD(N) = 1
   12 BETA1 = BETA1 + 1
      IF (BETA1-IM(1)) 13,14,14
   14 RETURN
   13 IF(NCOD(BETA1,ID(1)))  9,12,9
   10 BETA2=1
   15 IF(ID-2)  16,17,16
   17 N =BETA1 + IM(1)*BETA2
      NCOD(N) = 1
   18 BETA2 = BETA2 + 1
      IF (BETA2-IM(2)) 19,12,12
   19 IF(NCOD(BETA2,ID(2)))  18,18,15
```

```
                    PROGRAMS

    1  ICW =1 :RETA1=0
   20  RETA2 = 1
   21  IF(IR-2)  16,24,16
   24  N =RETA1 + IH(1)*RETA2
       NCOD(N) = 1
   22  RETA 2 = RETA2 + 2
       IF (RETA2-IH(2)) 21,23,23
   23  RETA1 = RETA1 + 1
       IF (RETA1-IH(1)) 20,14,14
   14  RETA3 =1
   25   IR(IR-3)  26,27,26
   27  N =RETA1 + IH(1)*RETA2 + IH(1)*IH(2)*RETA3
       NCOD(N) = 1
   28  RETA3 = RETA3 + 1
       IF (RETA3-IH(3)) 29,30,30
   29  IP33 = 9 - ICW
       IF(MOD(RETA3,IP(IP33)))  25,28,25
   30  IF(ICW)  22,18,22
   26  RETA4=1
   35  IF(IR-4)  36,37,36
   37  N =RETA1 + IH(1)*RETA2 + IH(1)*IH(2)*RETA3
      1  + IH(1)*IH(2)*IH(3)*RETA4
       NCOD(N) = 1
   39  RETA4=RETA4 + 1
       IF (RETA4-IH(4)) 39,28,28
   38  IP44 = 4 - ICW
       IF(MOD(RETA4,IP(IP44)))  35,38,35
   36  RETA5 = 1
   45  IF(IR - 5)  46,47,46
   47  N =RETA1 + IH(1)*RETA2 + IH(1)*IH(2)*RETA3
      1  + IH(1)*IH(2)*IH(3)*RETA4
      2 + IH(1)*IH(3)*IH(2)*IH(4)*RETA5
       NCOD(N) = 1
   48  RETA5 = RETA5 + 1
       IF (RETA5-IH(5)) 49,38,38
   49  IP55 = 5 - ICW
       IF(MOD(RETA5,IP(IP55)))  45,48,45
   46  RETA6 = 1
   59  IF(IR-6)  1,57,1
   57  N =RETA1 + IH(1)*RETA2 + IH(1)*IH(2)*RETA3
      1  + IH(1)*IH(2)*IH(3)*RETA4
      2 + IH(2)*IH(3)*IH(2)*IH(4)*RETA5
      3 + IH(1)*IH(3)*IH(2)*IH(4)*RETA6*IH(5)
       NCOD(N) =1
```

DOPGRAMS

```
50 NETA6 = NETA6 + 1
   IF(NETA6 - IHI(6)) 90,60,60
60 I06A = 6 - ICY
   IF(MOD(NETA6,IDI(I06A))) 55,50,55
   END

   SUBROUTINE LISIG,T,K,ICH,CHR,CHZ,ANSR,ANSI)
   DOUBLE PRECISION
  1FLI1,FN,ALGN,ZLRF,ZLIM,FLN,AL,Z,ARG,[illegible],FLUS,A,SIGNI,[illegible],FLK,
  2THIRD,THI,[illegible],FLMU,SIGN,C,D,THUR,[illegible],CARG,SARG,DI
  3,PLOG(500)
   DOUBLE PRECISION
  1                SIG,T,CHZR(2048),CHZI(2048),ANSR,ANSI
   DIMENSION ICH(2048)
   DATA(LCW=0)
   DATA(K=6)
   IF(LCW) 13,14,13
14 DO 15 II=2,500
   FLII=II
15 PLOG(II) = DLOG(FLII)
   ICW = 1
   PLOG(1) = 0.0
   READ 10,(P(J),J=1,50)
10 FORMAT (5F2.30)
13 IF (K-1) 1,2,2
 1 PRINT 3,SIG,T,K   SRETURN
 3 FORMAT (9H L ERROR  ,2D16.8,I10)
 2 IF (K -2048) 4,4,1
 4 IO1 = K/6 $IO= K - 6*IO1 $IF(IO - 2) 5,1,5
 5 IF(ABS(T) - 100.) 6,6,1
 6 IF (SIG+ .5) 1,7,7
 7 IF (SIG - 2.5) 8,8,1
 8 ANSR = 0.0 $ANSI = 0.0 $N = .4*ABS(T) + 10. $IF = 1
   FLK = K $IPHI = ICH(1) $IPH2 = IPHI/2
11 IF (ICH(IF)) 9,10,9
13 IF = IF + 1 $IF(IF-K) 11,11,12
12 RETURN
 9 NN = K*N+IF $FN=NN $ IF(NN-500) 17,17,16
17 ALGN = PLOG(NN) $GOTO18
16 ALGN = DLOG(FN)
18 N1 = [illegible] IF $ZLRF = 0.0 $ZLIM = 0.0
20 FLN = N1
   IF (N1-500) 21,21,22
21 AL = PLOG(N1) $GOTO23
```

PROGRAMS

```
22 AL = DLOG(FLN)
23 F = 1D0/DEXP(AL*SIG)
   ARG = T*AL
   CARG = DCOS(ARG)$SARG = DSIN(ARG)
   ZLRE = ZLRE + E*CARG $ZLIM = ZLIM - E*SARG
   N1 = N1 + K $IF (N1-NN)  20,24,24
24 FLUSQ = (-(FN*FN))/(FLK*FLK)
   F = 1D0/DEXP(ALGN*SIG)
   ARG = T*ALGN
   CARG = DCOS(ARG)$SARG = DSIN(ARG)
   A = F*CARG $B1=F*SARG$ SIGM1=SIG-1D0
   Q6 = SIGM1*SIGM1 + T*T $W1 =(12D0*FN)/FLK
   TMURE = (SIG*A + T*B1)/W1
   TMUIM = (T*A - SIG*B1)/W1
   W1 = FN/(Q6*FLK)
   ZLRE = ZLRE + (SIGM1*A -T*B1)*W1 + A/2D0 + TMURE
   ZLIM = ZLIM - (SIGM1*B1 + T*A)*W1 - B1/2D0 + TMUIM
   MU = 0
34 MU = MU + 1 $IF(MU-N)  25,25 ,26
25 FLMU = MU $W=B(MU)/FLUSQ $SIGPM = SIG + 2D0*FLMU
   C = SIGPM*(SIGPM-1D0) - T*T $D=(2D0*SIGPM - 1D0)*T
   TMUR1 = (C*TMURE - D*TMUIM)*W
   TMUI1 = (D*TMURE + C*TMUIM)*W
   ZLRE = ZLRE + TMUR1
   ZLIM = ZLIM + TMUI1
   TMURE = TMUR1 $TMUIM = TMUI1
   IF (ABS(TMURE)+ ABS(TMUIM)  - 1E-25            ) 26,26,34
26 IF(ICH(IF) - IPHI)  27,28,27
28 ANSR = ANSR +ZLRE $ANSI = ANSI+ ZLIM $GOTO10
27 IF (ICH(IF) - IPH2)  29,30,29
30 ANSR = ANSR -ZLRE $ANSI = ANSI -ZLIM $GOTO10
29 ANSR = ANSR + CHZR(IE) * ZLRE - CHZI(IF)*ZLIM
   ANSI = ANSI + CHZR(IE) * ZLIM + CHZI(IE)*ZLRE
   GO TO 10
   END

   SUBROUTINFLP(SIG,T,K,ICH,CHZR,CHZI,ANSR,ANSI)
   DOUBLE PRECISION
  1FLI1,FN,ALGN,ZLRE,ZLIM,FLN,AL,E,ARG,B(50),FLUSQ,A,SIGM1,Q6,FLK,
  2TMURE,TMUIM,W1,W,FLMU,SIGPM,C,D,TMUR1,TMUI1,CARG,SARG,B1
  3 ,ALOG(500) ,SGMSQ,Q,Q1,FNSQ
   DOUBLE PRECISION TR,TI,TI11,TR11,SI,SI11,SR,SR11,VI,VI11
  1,VR,VR11,UI,UI11,UR,UR11,C1,D1,C11,D11,E11,WR,WI,
  2TEMP1,TEMP2,TEMP3,TEMP4,A1,Z1,Z2
```

PROGRAMS

```
      DOUBLE PRECISION
     1                SIG,T,CHZR(2048),CHZI(2048),ANSR,ANSI
      DIMENSION ICH(2048)
      DATA(LCW=0)
      DATA(M=40)
      IF(LCW) 13,14,13
   14 DO 15 I1=2,500


      FLI1=I1
   15 ALOG(I1) = ALOG(FLI1)
      LCW = 1
      ALOG(1) = 000
      READ 10,(R(J),J=1,50)
   10 FORMAT (D52.30)
   13 IF (K-1)  1,2,2
    1 PRINT 3,SIG,T,K  $RETURN
    3 FORMAT (2H LPERROR    ,2D16.8,I10)
    2 IF (K -2048)  4,4,1
    4 I01 = K/4 $I0= K - 4*I01 $IF(I0 - 2)  5,1,5
    5 IF(ABS(T) - 100.)  6,6,1
    6 IF (SIG + .5)  1,7,7
    7 IF (SIG - 2.5)  8,8,1
    8 ANSR = 000 $ANSI = 0D0 $N = .4*ABS(T) + 10. $IE = 1
      FLK = K $IPHI = ICH(1) $IPH2 = IPHI/2
   11 IF (ICH(IE))  9,10,9
   10 IE = IE + 1 $IF(IE-K)  11,11,12
   12 RETURN
    9 NN = K*N$IF $EN=NN $ IF(NN-500)  17,17,16
   17 ALGN = ALOG(NN) $GOTO18
   16 ALGN = ALOG(FN)
   18 N1 =    IE $ZLRE = 000 $ZLIM = 000
   20 FLN = N1
      IF (N1-500)  21,21,22
   21 AL = ALOG(N1) $GOTO23
   22 AL = ALOG(FLN)
   23 E =  AL/DEXP(AL*SIG)
      ARG = T*AL
      CARG = DCOS(ARG)$SARG = DSIN(ARG)
      ZLRE = ZLRE + E*CARG $ZLIM = ZLIM + E*SARG
      N1 = N1+ K $IF (N1-NN)  20,24,24
   24 E = 100/DEXP(ALGN*SIG)
      ARG = T*ALGN
      CARG = DCOS(ARG)$SARG = DSIN(ARG)
      A1= E*CARG              $ SIGM1=SIG-1D0
```

```
      A1 = -F*SARG *SGMSQ + SIGM1*SIGM1
      C1= SGMSQ - T*T
      D1 = 2D0*T*SIGM1
      Q = SGMSQ + T*T
      Q1 = C1*C1 + D1*D1
      FLUSQ = FLK*FLK
      Z1=.5D0*ALGN+((SIGM1   *FN*ALGN    )/Q + (FN*C1)/Q1)/F
      Z2 =((T*FN*ALGN    )/Q + (FN*D1)/Q1)/FLK
      ZLRE = ZLRE - (A1*Z1) -(B1*Z2).
      ZLIM = ZLIM - B1*Z1 + A1*Z2
      FNSQ = FN*FN
      UR     = SIG
      UI     = T
      VR     = 1.0D0
      VI     = 0.0D0
      TEMP3 =(12D0*FN)/FLK
      TEMP4 = 1D0- SIG*ALGN
      TR     = (TEMP4*A1 +     ARG *B1)/TEMP3
      TI     = (TEMP4*B1 -     ARG *A1)/TEMP3
      C = ((SIG + 2D0)*(SIG + 1D0) - T*T)/FNSQ
      E = (2D0*SIG + 3D0)/FNSQ
      D     = T*E
      SR     = -(ALGN.   *SIG)+ 1.0D0
      SI = -ARG
      ZLRE = ZLRE + TR
      ZLIM = ZLIM + TI
      J = 1
   55 F11 = F +4D0/FNSQ
      D11 =     T*E11
      C11 = C +2D0*E +4D0/FNSQ
      UR11 = C*UR - D*UI
      UI11 = D*UR + C*UI
      VR11 = C*VR - D*VI + F*UR -(2D0*T*UI)/FNSQ
      VI11 = D*VR + C*VI + E*UI +(2D0*T*UR)/FNSQ
      SR11 = (-ALGN    )*UR11 + VR11
      SI11 = (-ALGN    )*UI11 + VI11
      TEMP1 = SR*SR + SI*SI
      WR = (SR*SR11 + SI*SI11)/TEMP1
      WI = (SR*SI11 - SI*SR11)/TEMP1
      TR11 = -(WR*TR - WI*TI)*B(J)*FLUSQ
      TI11 = -(WI*TR + WR*TI)*B(J)*FLUSQ
      J = J + 1
      E = E11
      D = D11
      C = C11
```

PROGRAMS

```
      UR = UR11
      UI = UI11
      VR = VR11
      VI = VI11
      SR = SR11
      SI = SI11
      TR = TR11
      TI = TI11
      ZLRF = ZLRF + TR
      ZLIM = ZLIM + TI
      IF (M - J)  99,77,77
   77 IF(ABS(TR) + ABS(TI)-1F-25)  99,55,55
   00 IF(ICH(IE) - IPH1)  27,28,27
   28 ANSR = ANSR +ZLRE $ANSI = ANSI+ ZLIM $GOTO10
   27 IF (ICH(IE) - IPH2)  29,30,29
   30 ANSR = ANSR -ZLRF $ANSI = ANSI -ZLIM $GOTO10
   29 ANSR = ANSR +  CHZR(IE) * ZLRE - CHZI(IE)*ZLIM
      ANSI = ANSI +. CHZR(IE) * ZLIM +  CHZI(IE)*ZLRE
      GO TO 10
      END
```

R CONSTANTS

```
.0166666666666666666666666666667
.0238095238095238095238095238010
.0250000000000000000000000000000
.0252525252525252525252525252533
.0253113553113553113553113553113
.0253256150506512301013024602033
.0253291316576610644257703081233
.0253300055040974875073599115599
.0253302233918339319057398707257
.0253302777864829212259139105227
.0253302913804567963135474688847
.0253302947781522363310254263627
.0253302956274874669629302478088
.0253302958398114293637978738397
.0253302958928913261965270559277
.0253302959061611788842320628688
.0253302959094786285544555792047
.0253302959103079894718578390153
.0253302959105153299345268834653
.0253302959105671645316741197337
.0253302959105801232789031487197
.0253302959105833629654817665977
```

PROGRAMS

.02533029591058417288710101 6562
.02533029591058437536750300 6332
.02533029591058442598760319 0139
.02533029591058443864262820 1243
.02533029591058444180638445 0147
.02533029591058444259732351 1942
.02533029591058444279505827 7343
.02533029591058444284449196 8688
.02533029591058444295685039 1524
.02533029591058444299939997 233
.02533029591058444286071239 8660
.02533029591058444286090549 9017
.02533029591058444286095377 4106
.02533029591058444286096582 8278
.02533029591058444286096886 0071
.02533029591058444286096961 4170
.02533029591058444286096980 2944
.02533029591058444286096985 0088
.02533029591058444286096986 1874
.02533029591058444286096986 4820
.02533029591058444286096986 5557
.02533029591058444286096986 5741
.02533029591058444286096986 5787
.02533029591058444286096986 5799
.02533029591058444286096986 5801
.02533029591058444286096986 5802
.02533029591058444286096986 5802
.02533029591058444286096986 5802

The editorial committee would welcome readers' comments about this microfiche feature. Please send comments to Professor Eugene Isaacson, MATHEMATICS OF COMPUTATION, Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, New York 10012.

Mathematics of Computation

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