D. H. Lehmer points out [3] that computation of a table of $\phi(n)$ usually has some indirect purpose inasmuch as any desired individual value of $\phi(n)$ can be rather easily obtained. See [3] for further discussion.

D. S.

1. J. J. SYLVESTER, "On the number of fractions contained in any Farey series . . . ," *Philos. Mag.*, v. 15, 1883, pp. 251–257.
2. J. W. L. GLAISHER, *Number-Divisor Tables*, British Association Mathematical Tables, v. 8, Cambridge, 1940, Table I.
3. D. H. LEHMER, *Guide to Tables in the Theory of Numbers*, National Acad. of Sciences, Washington, D. C., 1941, pp. 6–7.

50[9].—MORRIS NEWMAN, *Table of the Class Number $h(-p)$ for p Prime, $p \equiv 3 \pmod 4$, $101987 \leqq p \leqq 166807$*, National Bureau of Standards, 1969, 49 pages of computer output deposited in the UMT file.

This is an extension of Ordman's tables [1] previously deposited and reviewed. Those tables were computed because the undersigned wished to examine all cases of $h(-p) = 25$; this extension to $p = 166807$ was computed because (you guessed it) he wished to examine all cases of $h(-p) = 27$.

Unlike Ordman's tables, all $p = 4n + 3$ are listed consecutively here; those of the forms $8n + 3$ and $8n + 7$ are not listed separately.

We may now extend the table in our previous review of the first and last examples of a given odd class number:

h	$8n + 3$		$8n + 7$	
27	3299	103387	983	11383
29	2939	166147	887	8863
31	3251	133387	719	13687

For $p = 8n + 7$ our table here could be much extended, but not for $p = 8n + 3$, since there are known $p = 8n + 3 > 166807$ with $h(-p) = 33$.

D. S.

1. EDWARD T. ORDMAN, *Tables of the Class Number for Negative Prime Discriminants*, UMT 29, *Math. Comp.*, v. 23, 1969, p. 458.

51[9].—A. E. WESTERN & J. C. P. MILLER, *Indices and Primitive Roots*, Royal Society Mathematical Tables, Vol. 9, University Press, Cambridge, 1968, liv + 385 pp., 29 cm. Price \$18.50.

To describe fully what is in this volume would be a long task; we therefore abbreviate somewhat. Let P be prime and let

$$P - 1 = \prod_i q_i^{\alpha_i} \tag{1}$$

be the factorization of $P - 1$ into prime-powers. If ξ is the smallest positive exponent such that

$$y^\xi \equiv 1 \pmod P$$

and $\nu = (P - 1)/\xi$, then ν is the *residue-index* of y (mod P). If $\nu = 1$, y is a *primitive root* of P. Let

$$(2) \qquad g, -g', h$$

be, respectively, the least positive, least negative, and least positive prime primitive roots of P. For any primitive root G and

$$y \equiv G^m \qquad (\text{mod } P)$$

we say m is the *index* of y to the base G (mod P) and write it

$$(3) \qquad m = \text{ind}_G\,(y)\,.$$

Let p_n be the nth prime: $p_1 = 2$, $p_2 = 3$, $\cdots$.

Choose $G = g$ unless $g > g'$, in which case choose $G = -g'$. The main Table 1 (216 pages) lists for each odd prime $P \leqq 50021$, the factorization (1), the data (2), and, for $y = p_1, \cdots, p_{12} = 37$, their indices (3), and residue-indices, ν. The arguments $y = 6$, 10, and 12 are also given.

Table 2 continues with only $P \equiv 1$ (mod 24) to $P < 10^5$. Table 3 continues with only $P \equiv 1, 49$ (mod 120) to $P < \frac{1}{4}\cdot 10^6$. The arguments 6, 10, and 12 are dropped here. Table 4 continues with only $P \equiv 1$ (mod 120) to $P < 10^6$. The arguments 6, 10, and 12 are replaced by $y = p_{13}, p_{14}, p_{15} = 47$. The brief Table 5 gives any further $y = p_i$ up to $h = p_n$ for any previous P such that $n > 12$ or 15, respectively.

The 45-page Introduction is very interesting. It takes up many topics, including the long, intricate history of these tables, going back at least 50 or 60 years. The ingenious, but involved techniques for computing g and the other data were developed by Cunningham, Woodall, and Western, and, of course, preceded modern high-speed machines. We forego a description of these methods ourselves, but recommend that the reader study this part of the Introduction.

Tables 6 and 7 are related to this method, and the first is of interest in its own right. One defines A_n numbers (they are much used in the foregoing techniques) as those divisible by no prime $> p_n$. Table 6 gives the number of such $A_n \leqq N$ for $n = 1(1)51$—the last representing $p_{51} = 233$ — and from $N = 10^3$ to 10^8 by varying increments.

The uses of the main tables are discussed, together with examples. In spite of the limited number of arguments y listed, usually (3) can be found fairly quickly for any y. If y is an A_{12} (or A_{15}) number, one has its index merely by addition. Consider ind 97 (mod 4933). Since

$$97\cdot 51 \equiv 14 \qquad (\text{mod } 4933)\,,$$

one has ind 97 $=$ ind 2 $+$ ind 7 $-$ ind 3 $-$ ind 17. That is, one expresses y in terms of A_n numbers. To compute y from ind y may be longer, and a desk computer is desirable.

Other minor tables of interest in their own right, and related to these methods, are given in the Introduction. Thus, $P_a(n)$ is the smallest prime P such that every $p_i \leqq p_n$ has a ν divisible by a. Tables are given for a up to 17, and varying n up to 16. In connection with $a = 5$, a long discussion of quintic residues is given by Western.

Various conjectures are examined. Artin's conjecture, as modified by the

Lehmers, Heilbronn, and Hooley, is discussed at some length, and Table 8 gives data for $P < 50000$. Let $G(x)$ denote max $g(P)$ for $P \leqq x$. One has $G(760321) = 73$, and it is conjectured that $G(x) = O\{(\ln x)^3\}$.

Reviewer's comments: The whole book is interesting, and the tables are useful. Owing to their long history, the present tables exhibit a conflict between Cunningham's original purpose, the study of g and ν, and the later purpose of printing a useful table of indices. No explanation is given for the favoritism towards certain arithmetic progressions for P in Tables 2–4. For the second purpose alone, *all* $P < 10^5$ would perhaps be preferable. Again, the varying arguments y in Tables 1–4 is not explained. The *indices* for $y = 6$, 10, and 12 are obviously redundant, but Cunningham was interested in their values of ν, e.g., that for $y = 10$ relates to the decimal expansion of P^{-1}. For the second purpose alone, a uniform $y = p_i$, $i = 1(1)15$ would perhaps be preferable. Thus, besides their interest and utility, the tables are replete with charming vestigial structures, as befits the national character.

Perhaps there are legal aspects: Besides leaving his unpublished extensions of [1] to the London Mathematical Society, Cunningham also left a legacy to pay for their completion and publication. That is a similar situation to that of Mansell's *Logarithms* in the immediately preceding volume of the Royal Society Mathematical Tables, and inverts the situation in American universities, where the slogan is "Publish or Perish."

With the use of a modern computer, one would probably select a more direct program, and not use Cunningham's intricate method. Presumably, [2] was so computed. Of course, that is no criticism of the present tables.

The conjecture for $G(x)$ seems to the reviewer unduly conservative: the data shown would justify the stronger

$$G(x) = O\{(\ln x)^2\}$$

or

$$(G(x))^{1/2} = O(\ln x)\,.$$

If $g(x)$ is the largest gap between successive primes, the reviewer [3] has conjectured

$$(g(x))^{1/2} \sim \ln x\,.$$

When one reflects upon the meaning of $G(x)$, this may be less of a coincidence than appears at first.

Two errata in the Introduction are these: On p. xlv, for $P - 1 = 2\cdot 59$, read $P - 1 = 2^2\cdot 3^3$, and, in the line below, for "P exceeding 41" read "P exceeding 109." More serious, in Section 31, p. xlvi is the implication that the tables always use $G = g$. Sometimes, in Table 1, $G = -g'$, as is indicated above. Another pertinent reference that should have been listed is [4].

D. S.

1. Lt. Col. Allan J. C. Cunningham, H. J. Woodall & T. G. Creak, *Haupt-Exponents, Residue-Indices, Primitive Roots, and Standard Congruences*, F. Hodgson, London, 1922.
2. J. C. P. Miller, *Table of Primitive Roots*, *Math. Comp.*, v. 17, 1963, pp. 88–89, UMT **2.**
3. Daniel Shanks, "On maximal gaps between successive primes," *Math. Comp.*, v. 18, 1964, pp. 646–651.
4. C. A. Nichol, John L. Selfridge & Lowry McKee, *A Table of Indices and Power Residues for all Primes and Prime Powers below 2000,* W. W. Norton, New York, 1962. (See *Math. Comp.*, v. 17, 1963, pp. 463–464, RMT **72.**)