

munity, and its guests, as a meeting place and retreat. Frequent symposia are held in this congenial spot, and the present volume consists of summaries of the lectures delivered at two such gatherings in 1965.

The topic of the first collection is numerical problems in Approximation Theory, and it consists of 11 fairly full presentations, most of which deal with some aspect of Chebyshev approximation. The second symposium is entitled "Methods of Functional Analysis in Numerical Mathematics," and 16 talks are represented, some by brief abstracts.

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60[2.10].—BRUCE S. BERGER, ROBERT DANSON & ROBERT CARPENTER, **A.** *Tables of Zeros and Weights for Gauss-Laguerre Quadrature to 24S for $N = 400, 500,$ and $600,$ ms. of 4 typewritten pp. + 12 computer sheets (reduced), 28 cm. **B.** *Tables of Zeros and Weights for Gauss-Laguerre Quadrature to 23S for $N = 700, 800,$ and $900,$ ms. of 4 typewritten pp. + 18 computer sheets (reduced), 28 cm. Copies deposited in the UMT file; additional copies obtainable from Professor Berger, Department of Mechanical Engineering, The University of Maryland, College Park, Md. 20742.**

These two manuscript tables (prepared in November 1968 and January 1969, respectively) represent an impressive extension of the authors' 24S table [1] of zeros and weights for Gauss-Laguerre quadrature corresponding to $N = 100, 150, 200,$ and $300.$

As in the preparation of the earlier table, the present tables were calculated on a CDC 6600 system, using double-precision floating-point operations accurate to approximately 30S. Moreover, the same over-all checks have been applied to the computed values.

The senior author has recently applied these extensive tables to calculations relating to a problem in acoustics [2].

J. W. W.

1. BRUCE S. BERGER & ROBERT DANSON, *Tables of Zeros and Weights for Gauss-Laguerre Quadrature,* ms. deposited in the UMT file. (See *Math. Comp.*, v. 22, 1968, pp. 458-459, RMT 40.)

2. BRUCE S. BERGER, "Dynamic response of an infinite cylindrical shell in an acoustic medium," *J. Appl. Mech.*, v. 36, 1969, pp. 342-345.

61[2.10].—LEE M. HUBBELL and RALPH E. CHRISTOFFERSEN, *Tabulation of a New Set of Orthogonal Polynomials for Numerical Integration,* ms. of 8 typewritten pages & 18 typewritten pages of tables & 5 pages of figures, deposited in the UMT file.

The authors consider the orthogonal polynomials associated with the quadrature problem

$$(1) \quad \int_1^{\infty} \frac{e^{-x}}{x^k} f(x) dx = \sum_{i=0}^j w_{i,j}^{(k)} f(x_{i,j}^{(k)}) + R_j^{(k)} f,$$

where $R_j^{(k)} f = 0$ if $f(x)$ is a polynomial of degree $2j + 1$ or less. The abscissas $x_{i,m}^{(k)}$ are the zeros of a polynomial

$$(2) \quad \phi_m^{(k)}(x) = \sum_{j=0}^m a_{m,j}^{(k)} x^j$$

which is one of a set of orthogonal polynomials satisfying the orthonormal conditions

$$(3) \quad \int_1^{\infty} \frac{e^{-x}}{x^k} \phi_i^{(k)}(x) \phi_j^{(k)}(x) dx = \delta_{i,j}.$$

The method of calculation is described. As is well known such a calculation is highly unstable with respect to amplification of round-off error. An iterative refinement technique is used. The calculation is described for the more general case, associated with

$$\int_1^{\infty} \frac{e^{-\alpha x}}{x^k} f(x) dx.$$

The tabulation covers only the case $\alpha = 1$, and seems to have been carried out using double-precision (twenty significant figure) accuracy. No statement is made about the actual computer or the appropriate machine accuracy parameter.

The tabulated quantities include

$$a_{i,j}^{(k)}, x_{i,j}^{(k)}, w_{i,j}^{(k)}, \bar{\delta}_{i,j}^{(k)}, \quad i \leq j \leq 10, k = 1, 2, 3, 4, 5.$$

Here the first three are defined above. The fourth, $\bar{\delta}_{i,j}^{(k)}$ are the actual approximations to $\delta_{i,j}$ obtained from (3) by expanding the polynomials according to (2) and using the known moments to evaluate the right-hand side of (3).

Finally, for $k = 1$ only, the approximations to the exact integral (1) using the quadrature rule on the right-hand side of (1) for each of the four functions

$$f(x) = e^{-x}, x^{18}, x \ln x \quad \text{and} \quad x \sin x$$

are given.

Unfortunately there is some vagueness about the precise accuracy to which these calculations were carried out, but the presumption is that double-precision function values were used. The reviewer is very happy to note that this sort of information is being brought to the attention of a possible user. But in many applications the user will have at his disposal only single-precision function values. He might like to know, for example, how his results will be affected if he uses single-precision weights and abscissas. It seems to the reviewer that a single element of information, the value of the machine accuracy parameter, is vital to obtain a complete picture.

The method described involves the prior calculation of the moments, a well-known source of error. Recently general procedures which do not involve the calculation of moments have been constructed by Gautschi [1], [2] and by Golub and Welsch [3].

J. N. L.

1. W. GAUTSCHI, "Construction of Gauss-Christoffel Quadrature Formulas," *Math. Comp.*, v. 22, 1968, pp. 251-270.

2. W. GAUTSCHI, "Algorithm 331: Gaussian Quadrature Formulas," *Comm. ACM*, v. 11, 1968, pp. 432-436.

3. G. H. GOLUB & J. M. WELSCH, "Calculation of Gauss Quadrature Rules," *Math. Comp.*, v. 23, 1969, pp. 221-230.