62[2.10].—T. N. L. Patterson, "Gaussian Formula for the Calculation of Repeated Integrals," tables appearing in the microfiche section of this issue.

Abscissas and weights of the r-point Gaussian quadrature formula for the integral

$$(n-1)! \int_{-1}^{1} dx_1 \int_0^{x_1} dx_2 \int_0^{x_2} dx_3 \cdots \int_0^{x_{n-1}} f(x_n)dx_n \equiv \int_{-1}^{1} w(x)f(x)dx$$

are tabulated to 20 significant figures for $n = 2, 4$; $r = 2(2)16$ and $n = 3, 5$; $r = 2(1)16$. The resulting formula is exact if $f(x)$ is a polynomial of degree $2r - 1$ (or $2r$ when r is even). The weighting function is

$$w(x) = (-1)^n w(-x) = (1-x)^{n-1}, \qquad 0 < x \leqq 1 .$$

This has discontinuous even (odd) derivatives at $x = 0$.

A normal approach to such an integration might be to divide the interval into two sections and use either the Gauss-Legendre formula or better still the appropriate Gauss-Jacobi formula in each section separately. However, the existence of these tables does allow the interval $[-1, 1]$ to be treated as a whole.

J. N. L.

63[2.20, 7].—Henry E. Fettis & James C. Caslin, *More Zeros of Bessel Function Cross Products*, Report ARL 68–0209, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, December 1968, v + 56 pp., 28 cm. [Released to the Clearinghouse, U. S. Department of Commerce, Springfield, Virginia 22151.]

In this compact report the authors continue their previous 10D tabulation [1] of the roots of the equations (a) $J_0(\alpha)Y_0(k\alpha) = Y_0(\alpha)J_0(k\alpha)$, (b) $J_1(\alpha)Y_1(k\alpha) = Y_1(\alpha)J_1(k\alpha)$, and (c) $J_0(\alpha)Y_1(k\alpha) = Y_0(\alpha)J_1(k\alpha)$.

These new tables give the roots α_n and the corresponding normalized roots γ_n of all three equations, for $n = 5(1)10$ and $k = 0.001(0.001)0.3$. For equation (c) these roots are also tabulated corresponding to $k^{-1} = 0.001(0.001)0.3$.

The normalized roots are related to the others by the equation $\gamma_n = (1-k)\alpha_n/(n\pi)$ for (a) and (b), and by $\gamma_n = |k-1|\alpha_n/[(n-\frac{1}{2})\pi]$ for (c). The authors note the properties $\lim_{k\to 1} \gamma_n = 1$ (all n) and $\lim_{n\to\infty} \gamma_n = 1$ (all k).

For examples of applications of these tables, as well as details of their calculation, the user should consult the earlier report [1].

J. W. W.

1. Henry E. Fettis & James C. Caslin, *An Extended Table of Zeros of Cross Products of Bessel Functions*, Report ARL 66–0023, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, February 1966. (See *Math. Comp.*, v. 21, 1967, pp. 507–508, RMT **64**.)

64[2.40, 7, 10].—John Riordan, *Combinatorial Identities*, John Wiley & Sons, Inc., New York, 1968, xii + 256 pp., 23 cm. Price \$15.00.

This volume deals in the main with identities involving the binomial coefficients. As is well known, binomial coefficients are the simplest combinatorial entities and arise quite naturally in a wide variety of combinatorial problems.

According to the preface, "the object of this book is to present identities in mathematical setting that provide areas of order and coherence." However, the author remarks that his "initial hope that some of this order and coherence would be acquired by the identities themselves now seems illusory, $\cdots$, the age-old dream of putting order in chaos is doomed to failure." Elsewhere in the preface, the author states that the tools of combinatorialists are "recurrence, generating functions and such transformations as the Vandermonde convolution; others, to my horror, use contour integrals, differential equations and other resources of mathematical analysis." An examination of the preface reveals a confusion which is reflected in the volume itself. The point is this: A good portion of the book leans heavily on well-known results for the Gaussian hypergeometric functions ${}_2F_1(a; b; c; z)$ and its natural extensions to hypergeometric functions of higher order, for example, the ${}_3F_2$. However, this well-known literature appears to have escaped the attention of the author. Thus, such references as the volumes by A. Erdélyi et al., *Higher Transcendental Functions*, Vols. 1–3, McGraw-Hill, 1953–1955, and E. D. Rainville, *Special Functions*, Macmillan Company, 1960, are not mentioned. Consequently, many results in the first two chapters and elsewhere that could be stated once and for all are proved anew.

In illustration, Example 4 on page 5, the Vandermonde convolution formula on page 8, Example 6 on page 13; and Example 2 on page 53 are all special cases of the ${}_2F_1$ of unit argument. Again, Example 7 on page 13 is a special case of a ${}_2F_1$ with argument equal to one-half which arises from a quadratic transformation formula. On page 15, Eq. (10) is a special case of a terminating Saalschützian ${}_3F_2$ or unit argument. The same is true of the last expansion on page 16. This is a rather interesting case as the author relates that in 1954 Turan observed that the formula occurs without proof in a book by the Chinese mathematician Le-Jen Shoo which is dated 1867. Riordan states that Turan's paper created a "wave of interest that is reflected in the bibliography." Saalschütz's work is dated 1890. It appears to us that the first two chapters could have been considerably shortened and made more useful on appeal to the theory of special functions.

Chapter I is titled 'Recurrence', and in the main is concerned with basic properties of series involving binomial coefficients and which are of the type ${}_2F_1$ and ${}_3F_2$ as noted above. The chapter also deals with Abel's generalization of the binomial formula and multinomial Abel identities. Chapter II uses the results of Chapter I to derive inverse relations based on the hypergeometric results already noted, for example,

$$a_n = \sum_{k=0}^{n} (-)^k \binom{n}{k} b_k \quad \text{implies} \quad b_n = \sum_{k=0}^{n} (-)^k \binom{n}{k} a_k\,,$$

and vice versa. Corresponding results associated with the name of Abel are taken up in Chapter III. 'Generating Functions' is the subject of Chapter IV. As is perhaps well known the application of such relations is useful to derive difference, differential and other properties of transcendentals. In this connection, the references by A. Erdélyi et al., and Rainville noted above contain much valuable information.

Chapter V takes up partition polynomials which are also called Bell polynomials and inverses with applications to number theory. The forward difference operators $x\,\Delta$, Δx, the analogous backward difference operators, the central difference operator, and the derivative operators xD and Dx are studied in Chapter VI.

Each chapter contains numerous examples and problems for the reader. Undoubtedly, these should be useful for self-study and to locate specific examples needed in a wide variety of problems.

Y. L. L.

65[4, 7, 8, 11, 13].—Murray R. Spiegel, *Mathematical Handbook of Formulas and Tables*, McGraw-Hill Book Co., New York, 1968, x + 271 pp., 28 cm. Price $3.95 (paperbound).

This relatively inexpensive compilation of mathematical formulas and tables is a recent addition to the popular Schaum's Outline Series of books mainly in mathematics and engineering.

The book is divided into two main parts. Part I (Formulas) consists of 41 sections, of which 39 present a total of 2309 formulas (supplemented by diagrams and graphs) selected from a wide range of topics in such fields as algebra, geometry, trigonometry, analytic geometry, calculus, differential equations, vector analysis, Fourier series, Fourier and Laplace transforms, special functions (gamma, beta, Bessel, Legendre, elliptic, and others), and probability distributions. The first and last sections of Part I consist, respectively, of a table of 27 frequently used mathematical constants (given to from 10S to 25S) and a useful table of conversion factors.

Part II (Tables) consists of 52 numerical tables, preceded by a set of sample problems illustrating their use. These tables, which generally range in precision from 3S to 7S, cover the standard elementary functions as well as a large number of the higher mathematical functions, including the gamma function, Bessel functions, exponential integral, sine and cosine integrals, Legendre polynomials, elliptic integrals, and the error function. Also included are tables for the calculation of compound interest and annuities, and a small table of random numbers. An appended index of special symbols and notations and a general index have also been included.

Despite the existence of several errors (listed elsewhere in this issue), this reviewer considers this attractively arranged and clearly printed book to be a valuable addition to the ever-increasing number of such handbooks.

J. W. W.

66[3, 8].—Peter Lancaster, *Theory of Matrices*, Academic Press, New York, 1969, xii + 316 pp., 24 cm. Price $11.00.

This book differs considerably in the material presented from most books on matrices and linear algebra and deserves wide adoption, especially in courses intended for students majoring in other areas who are interested primarily in applications. Nevertheless, only a few sections are devoted to applications as such, and then only in terms of their mathematical formulation with no discussion of the physics itself. Thus there are sections on small vibrations, differential equations, and Markoff chains.

After a rather standard introduction in the first two chapters, the third discusses the Courant-Fischer and related theorems; the Smith canonical form and the Frobenius and Jordan normal forms are developed in the next chapter; the