

fifth chapter is devoted to functions of matrices, and the sixth to norms; then comes a chapter on perturbation theory, one on direct products and stability, and, finally, a chapter on nonnegative matrices.

The treatment is lucid, and the only prerequisites are elementary algebra and calculus. Only the real and complex fields are considered. There are a reasonable number of examples and exercises, and about three or four references per chapter for supplementary reading. The book provides an excellent background in the subject for prospective numerical analysts, as well as to the many nonmathematicians who need to use matrices in their work.

A. S. H.

67[7].—IRWIN ROMAN, *Extrema of Derivatives of  $J_0(x)$* , ms. of nine typewritten sheets (dated Jan. 1964), deposited in the UMT file.

This manuscript table gives to 10S the critical points below 100 and the corresponding extrema of the first four derivatives of the Bessel function  $J_0(x)$ . Accuracy to within two units in the final figure is claimed. Also tabulated are the (rational) values of these derivatives for zero argument.

A six-page introduction sets forth a detailed description of the method used in computing the tabular values on a desk calculator. In particular, formulas are listed relating the derivatives of  $J_0(x)$  to linear combinations of  $J_0(x)$  and  $J_1(x)$ , with coefficients expressed as polynomials in  $1/x$ . The values of these Bessel functions required in the computation of the table were obtained by interpolation in the Harvard tables [1].

A concluding page lists the six basic references cited in the introductory text.

As implied by the author, this unique table constitutes a natural supplement to published tables of extrema of  $J_0(x)$ , in particular, that compiled by the Mathematical Tables Committee of the British Association for the Advancement of Science [2].

J. W. W.

1. HARVARD UNIVERSITY, COMPUTATION LABORATORY, *Annals*, v. 3: *Tables of the Bessel Functions of the First Kind of Orders Zero and One*, Harvard University Press, Cambridge, Mass., 1947. (See *MTAC*, v. 2, 1947, pp. 261–262, RMT 380.)

2. BRITISH ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE, *Mathematical Tables*, v. 6: *Bessel Functions, Part I, Functions of Orders Zero and Unity*, Cambridge University Press, Cambridge, England, 1937. (See *MTAC*, v. 1, 1945, pp. 361–363, RMT 179.)

68[7].—T. S. MURTY & J. D. TAYLOR, *Zeros and Bend Points of the Legendre Function of the First Kind for Fractional Orders*, Oceanographic Research, Marine Sciences Branch, Department of Energy, Mines and Resources, Ottawa, Canada. Deposited in UMT file.

Let

$$P_\nu(x) = {}_2F_1\left(-\nu, 1 + \nu; \frac{1}{2}; \frac{1-x}{2}\right)$$

$$P_\nu(x_j) = 0, \quad P_\nu'(y_j) = 0.$$

The following are tabulated:

$$P_\nu(x): \nu = -0.5(0.02)0.5, x = -1.0(0.01)1.0; 7D,$$

$$P_\nu(x): \nu = -0.5(0.1)8.5, x = -1.0(0.02)1.0; 7D,$$

$$x_j, y_j, P_\nu'(x_j), P_\nu(y_j), \nu = 0.1(0.1)8.5, 7D.$$

The values  $y_j$  are called bend points. Some related tables are (1) Gray, M. C., "Legendre functions of fractional order," *Quart. Appl. Math.*, v. 11, 1953, pp. 311-318, also *MTAC*, v. 8, 1954, p. 24; (2) Ben Daniel, D. J. and Carr, W. E., *Tables of Solutions of Legendre's Equation for Indices of Nonintegral Order*, University of California Lawrence Radiation Laboratory, Livermore, UCRL-5859, September, 1960 (or from Office of Technical Services, Wash., D. C.). See also *Math. Comp.*, v. 16, 1962, pp. 117-119; (3) Abramowitz, A. & Stegun, I. A., Editors, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, AMS No. 55, U. S. Government Printing Office, 1964. See Chapter 8 and references given there. See also *Math. Comp.*, v. 19, 1965, pp. 147-149.

Y. L. L.

69[9].—FRANCIS L. MIKSA, *Table of Primitive Pythagorean Triangles, Arranged According to Increasing Areas*, ms. in five volumes comprising a total of 27 + 980 typewritten pp. (consecutively numbered), deposited in the UMT file.

The main table of this voluminous unpublished work gives in 980 pages the generators, sides, and areas of the 52,490 primitive Pythagorean triangles whose areas do not exceed  $10^{10}$ , arranged according to increasing areas.

Running counts of these triangles are given for each page, and such counts are separately tabulated for areas less than  $10^k$  at intervals of  $10^{k-1}$ , for  $k = 7(1)10$ . The author develops a formula that provides an independent check on these counts.

A preliminary section (dated May 21, 1952) contains a listing of the sides and areas of all primitive Pythagorean triangles having equal areas less than  $10^{10}$ . Included are 158 pairs and a single triple of such triangles. All equiareal triangles therein that are generated by a special formula attributed to Fermat are so indicated. Appended to this table is a list of the generators and sides of primitive triangles whose areas consist of the digits 1(1)9 or 0(1)9 appearing just once.

A foreword (dated November 11, 1961) to the main table presents the details of the underlying calculations, performed with the assistance of a 10-column desk calculator.

This elaborate census of primitive Pythagorean triangles according to areas may be considered as a companion to the manuscript tables of Anema [1] and the author [2] listing these triangles according to increasing perimeters.

It seemed appropriate here also to refer to recent pertinent computations by Jones [3] and Beiler [4] and to a book of the latter [5], wherein one finds many additional references.

J. W. W.

1. A. S. ANEMA, *Primitive Pythagorean Triangles with their Generators and their Perimeters, up to 119 992*, ms. in the UMT file. (See *MTAC*, v. 5, 1951, p. 28, UMT 111.)

2. F. L. MIKSA, *Table of Primitive Pythagorean Triangles with their Perimeters Arranged in Ascending Order from 119 992 to 499 998*, ms. in the UMT file. (See *MTAC*, v. 5, 1951, p. 232, UMT 133.)